THE UNIVERSITY OF CHICAGO

The Charge Radius of the Proton

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Based on Richard J. Hill, GP:

PRD **82** 113005 (2010) [arXiv:1008.4619]

[arXiv:1103.4617]

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)| \sum_q e_q\, \bar{q}\gamma^\mu q |N(p_i)\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2^N(q^2) q_\nu\right] u(p_i)$$

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- Sachs electric and magnetic form factors ($t = q^2 = -Q^2$)

$$G_E^N(t) = F_1^N(t) + \frac{t}{4m_N^2} F_2^N(t)\,, \quad G_M^N(t) = F_1^N(t) + F_2^N(t)\,.$$

$$G_E^p(0) = 1,\; G_E^n(0) = 0,\; G_M^p(0) = \mu_p \approx 2.793,\; G_M^n(0) = \mu_n \approx -1.913$$

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- The slope of G_E^p

$$\langle r^2\rangle_E^p = 6 \left.\frac{dG_E^p}{dq^2}\right|_{q^2=0} \quad \text{or} \quad G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \ldots,$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

Charge radius from atomic physics

- $G_E^p(t)$ and $G_M^p(t)$: input for precision QED observables for bound proton lepton systems

$$\langle p(p_f)| \sum_q e_q \,\bar{q}\gamma^\mu q|p(p_i)\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1^p(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2^p(q^2) q_\nu\right] u(p_i)$$

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- For a charged point particle: $F_1(0) = 1$ and $F_2(0) = 0$
  Amplitude for $p + \ell \to p + \ell$

$$i\mathcal{M} \approx \frac{ie_\ell\, e_p}{q^2}\chi_p^\dagger\chi_p\chi_\ell^\dagger\chi_\ell \quad \Rightarrow \quad U(r) = -Z\alpha/r$$

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- Including q^2 corrections from proton

$$i\mathcal{M} \approx \frac{ie_\ell\, e_p}{q^2}q^2\left[\frac{F_1^p(0)}{8m_p^2} + \left.\frac{dF_1^p}{dq^2}\right|_{q^2=0} + \frac{F_2^p(0)}{4m_p^2}\right]\chi_p^\dagger\chi_p\chi_\ell^\dagger\chi_\ell$$

- Proton structure corrections

$$U(r) = 4\pi Z\alpha\,\delta^3(r)\left(\left.\frac{dF_1^p}{dq^2}\right|_{q^2=0} + \frac{F_2^p(0)}{4m_p^2}\right) = \frac{4\pi Z\alpha}{6}\delta^3(r)(r_E^p)^2$$

Charge radius from atomic physics

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- The change in the energy $\Big(m_r = m_\ell m_p/(m_\ell + m_p) \approx m_\ell \Big)$

$$\begin{aligned} \Delta E_{r_E^p} &= \int d^3r\, \psi(r)^\dagger\, U(r)\, \psi(r) = \frac{2\pi Z\alpha}{3}(r_E^p)^2 |\psi(0)|^2 \\ &= \frac{2(Z\alpha)^4}{3n^3} m_r^3 (r_E^p)^2 \delta_{\ell\, 0} \end{aligned}$$

- Charge radius effects $\propto m_r^3$

Charge radius from atomic physics

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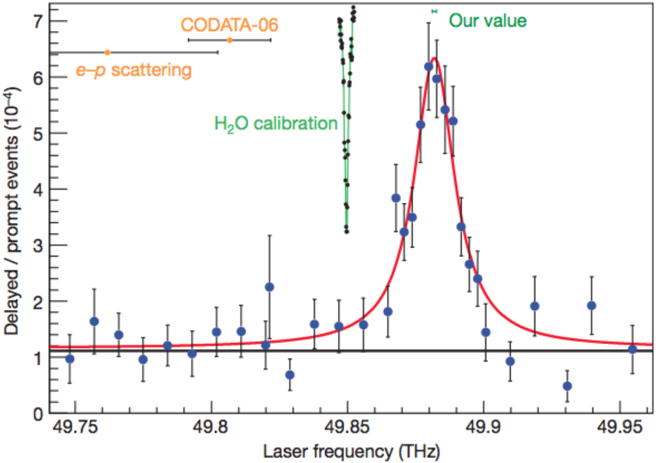
$$= \frac{2(Z\alpha)^4}{3n^3} m_r^3 (r_E^p)^2 \delta_{\ell 0}$$

- Charge radius effects $\propto m_r^3$

- **Muonic hydrogen can give the best measurement of $\mathbf{r_E^p}$!**

Charge radius from Muonic Hydrogen



- CREMA Collaboration measured for the **first time** $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ transition in Muonic Hydrogen
  [Pohl et al. Nature **466**, 213 (2010)]

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.8768(69)$ fm
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  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**
- We can also extract it from electron-proton scattering data

The recent discrepancy

- [Hill, GP PRD **82** 113005 (2010)] showed
  previous extractions are model dependent
  underestimated the error by a factor of 2 or more

- Based on a model-independent approach
  using scattering data from proton, neutron and $\pi\pi$
  [Hill, GP PRD **82** 113005 (2010)]
  $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

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- They measured [Pohl et al. Nature **466**, 213 (2010)]

$$\Delta E = 206.2949 \pm 0.0032 \text{ meV}$$

- Comparing to the theoretical expression [Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3 \text{ meV}$$

- They got

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- How reliable is the theoretical prediction?

The Theoretical Prediction

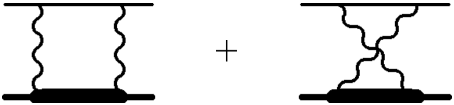
- Is there a problem with the theoretical prediction?

[Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = \quad 209.9779(49) \quad - \quad 5.2262(r_E^p)^2 \quad + \quad 0.0347(r_E^p)^3 \ \mathrm{meV}$$

$\uparrow$	$\uparrow$	$\uparrow$
mostly μ QED	already discussed	where does this term come from?

Two-photon amplitude: "standard" calculation



- "standard" calculation
  - separate to proton and non-proton
  - non-proton $\leftrightarrow$ DIS, polarizability
- For proton
  - Insert form factors into vertices

$$\mathcal{M} = \int_0^\infty dq^2\, f(G_E, G_M)$$

  - Using a "dipole form factor"

$$G_E(q^2) \approx G_M(q^2)/G_M(0) \approx [1 - q^2/\Lambda^2]^{-2}$$

  - $\mathcal{M}$ is a function of $\Lambda \Rightarrow (r_E^p)^3$ term

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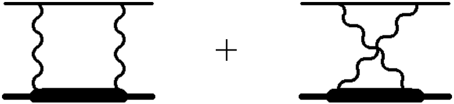

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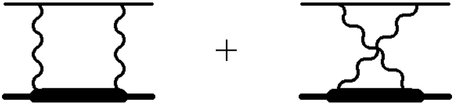
- $\mathcal{M}$ is a function of $\Lambda \Rightarrow (r_E^p)^3$ term

- Using, for example, $\Lambda^2 = 0.71\,\mathrm{GeV}^2$
  it contributes 0.018 meV to $E(2p) - E(2s)$
  [K. Pachucki, PRA **53**, 2092 (1996)]

Two-photon amplitude: “standard” calculation



- Is insertion of form factors in vertices valid?

- Even if it is, result looks funny
  two-photon amplitude ⇔ the charge radius
  only for one parameter model for G_E and G_M

"Standard" Calculation: Summary

- Using

  $r_E^p = 0.871(10)$ fm [Hill, GP PRD **82** 113005 (2010)]

  or

  $r_E^p = 0.8768(69)$ fm [Mohr et al. RMP **80**, 633 (2008)]

- The measured interval in muonic hydrogen lies

  $0.258(90)\,\mathrm{meV}$ or $0.311(63)\,\mathrm{meV}$ above theory.

- Using $\Lambda^2 = 0.71\,\mathrm{GeV}^2$,

  the proton contribution from the two-photon amplitude

- 0.018 meV to $E(2p) - E(2s)$

  [K. Pachucki, PRA **53**, 2092 (1996)]

- Is there a problem with the theoretical prediction?

NRQED

- Model Independent approach: use NRQED
- Up to $\mathcal{O}(1/m^3)$

  [Caswell, Lepage PLB **167**, 437 (1986); Kinoshita Nio PRD **53**, 4909 (1996); Manohar PRD **56**, 230 (1997)]

$$
\begin{aligned}
\mathcal{L}_e \;=\; & \psi_e^\dagger \Big\{ iD_t + \frac{\mathbf{D}^2}{2m_e} + \frac{\mathbf{D}^4}{8m_e^3} + c_F e \frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2m_e} + c_D e \frac{[\partial\cdot\mathbf{E}]}{8m_e^2} \\
& + i c_S e \frac{\boldsymbol{\sigma}\cdot(\mathbf{D}\times\mathbf{E}-\mathbf{E}\times\mathbf{D})}{8m_e^2} + c_{W1} e \frac{\{\mathbf{D}^2, \boldsymbol{\sigma}\cdot\mathbf{B}\}}{8m_e^3} \\
& - c_{W2} e \frac{D^i \boldsymbol{\sigma}\cdot\mathbf{B} D^i}{4m_e^3} + c_{p'p} e \frac{\boldsymbol{\sigma}\cdot\mathbf{D}\mathbf{B}\cdot\mathbf{D} + \mathbf{D}\cdot\mathbf{B}\boldsymbol{\sigma}\cdot\mathbf{D}}{8m_e^3} \\
& + i c_M e \frac{\{\mathbf{D}^i, [\partial\times\mathbf{B}]^i\}}{8m_e^3} + c_{A1} e^2 \frac{\mathbf{B}^2-\mathbf{E}^2}{8m_e^3} - c_{A2} e^2 \frac{\mathbf{E}^2}{16m_e^3} \Big\} \psi_e
\end{aligned}
$$

NRQED

- Need also

$$\mathcal{L}_{\text{contact}} = d_1 \frac{\psi_p^\dagger \boldsymbol{\sigma} \psi_p \cdot \psi_e^\dagger \boldsymbol{\sigma} \psi_e}{m_e m_p} + d_2 \frac{\psi_p^\dagger \psi_p \psi_e^\dagger \psi_e}{m_e m_p}$$

- Matching

- Operators with one photon coupling:
  c_i given by $F_i^{(n)}(0)$

- Operators with only two photon couplings:
  c_{A_i} given by forward and backward Compton scattering

- d_i from two-photon amplitude

Two-photon amplitude: matching

$$\frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \mathbf{k},s|\,T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\mathbf{k},s\rangle$$

$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

- Matching

$$\frac{4\pi m_r}{\lambda^3} - \frac{\pi m_r}{2m_e m_p\lambda} - \frac{2\pi m_r}{m_p^2\lambda}\left[F_2(0)+4m_p^2F_1'(0)\right]$$

$$-\frac{2}{m_e m_p}\left[\frac{2}{3}+\frac{1}{m_p^2-m_e^2}\left(m_e^2\log\frac{m_p}{\lambda}-m_p^2\log\frac{m_e}{\lambda}\right)\right]+\frac{\delta d_2(Z\alpha)^{-2}}{m_e m_p}$$

$$=-\frac{m_e}{m_p}\int_{-1}^{1}dx\sqrt{1-x^2}\int_0^\infty dQ\,\frac{Q^3}{(Q^2+\lambda^2)^2(Q^2+4m_e^2x^2)}$$

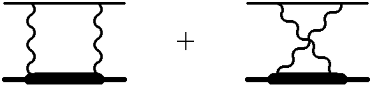
$$\times\left[(1+2x^2)W_1(2im_pQx,Q^2)-(1-x^2)m_p^2W_2(2im_pQx,Q^2)\right]$$

δd_2

- Relation between δd_2 and energy shift

$$\delta E(n,\ell) = -\delta_{\ell 0}\frac{m_r^3(Z\alpha)^3}{\pi n^3}\frac{\delta d_2}{m_e m_p}$$

- In order to determine δd_2 need to know W_i

- Im  $\sim$ Im W_i

  can be extracted from on-shell quantities:

  Proton form factor and Inelastic structure functions

- In order to find W_i need dispersion relations

  but W_1 requires subtraction...

Dispersion relation

- Dispersion relations ($\nu = 2k \cdot q,\ Q^2 = -q^2$)

$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi} \int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{\mathrm{Im}\, W_1(\nu', Q^2)}{\nu'^2(\nu'^2 - \nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2 \frac{\mathrm{Im}\, W_2(\nu', Q^2)}{\nu'^2 - \nu^2}$$

- Decompose

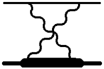
$$W_1(\nu, Q^2) = W_1(0, Q^2) + W_1^{p,1}(\nu, Q^2) + W_1^{c,1}(\nu, Q^2)\,,$$

$$W_2(\nu, Q^2) = W_2^{p,0}(\nu, Q^2) + W_2^{c,0}(\nu, Q^2)$$

- W_i^p from form factors
- W_i^c from DIS

- What about $W_1(0, Q^2)$?

Real Part

Re  +  = ?

- Can calculate in two limits:

- $Q^2 \ll m_p^2$
  The photon sees the proton "almost" like an elementary particle
  Use NRQED to calculate $W_1(0, Q^2)$ upto $\mathcal{O}(Q^2)$ (including)

$$W_1(0, Q^2) = 2(c_F^2 - 1) + 2\frac{Q^2}{4m_p^2}\left(c_{A_1} + c_F^2 - 2c_F c_{W1} + 2c_M\right)$$

- $Q^2 \gg m_p^2$
  The photon sees the quarks inside the proton
  Use OPE to find $W_1(0, Q^2) \sim 1/Q^2$ for large Q^2

- In between you will have to model!
  Current calculation **pretends** that there is no model dependence
  How big is the model dependence?

Bound states energies

- Convent to talk about:
  proton W_i^p, Continuum W_i^c, $W_1(0, Q^2)$

1) Proton W_i^p: using dipole form factor

$$E(2p) - E(2s) = -0.016 \text{ meV}$$

2) Continuum [Carlson, Vanderhaeghen arXiv:1101.5965]

$$E(2p) - E(2s) = 0.0127(5) \text{ meV}$$

3) What about $W_1(0, Q^2)$?
   "Sticking In Form Factors" (SIFF) model

$$W_1^{\mathrm{SIFF}}(0, Q^2) = 2F_2(2F_1 + F_2) \quad F_i \equiv F_i(Q^2)$$

SIFF

- "Sticking In Form Factors" (SIFF) model

$$W_1^{\mathrm{SIFF}}(0, Q^2) = 2F_2(2F_1 + F_2) \quad F_i \equiv F_i(Q^2)$$

  Notice that for large Q^2, $W_1^{\mathrm{SIFF}}(0, Q^2) \propto 1/Q^8$
  In contradiction to OPE

- There is **no** local Lagrangian that has a Feynman rule

$$\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q_\nu$$

- Numerically using the dipole form factor

$$\Delta E^{\mathrm{SIFF}} = 0.034 \text{ meV}$$

Model Dependence

- How big is the model dependence?

$$0.018\,\mathrm{meV} = \underset{\substack{\uparrow \\ \text{Model independent}}}{-0.016\,\mathrm{meV}} + \underset{\substack{\uparrow \\ \text{Model dependent}}}{0.034\,\mathrm{meV}}$$

- The model dependent piece is the dominant one!
- Experimental discrepancy $\sim$ 0.3 meV
- Can we find a model that explains (or reduces) the discrepancy?

New Physics?

- It is possible that the discrepancy is due to New Physics...
- New particle that couples to nucleons and μ (but not e or τ)
  Barger, Chiang, Keung, Marfatia [arXiv:1011.3519]
  Assuming same coupling to Υ, η, π rules this out
- New MeV particle that couples to protons (g_p) and muons (g_μ)
  Tucker-Smith, Yavin [arXiv:1011.4922]
  Can explain r_E^p and muon $g-2$ but $g_p \approx g_n$ is problematic
- New $U(1)$ that couples only to right-handed muons
  Batell, McKeen, Pospelov [arXiv:1103.0721]

Conclusions

- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen

Conclusions

- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen
- From model independent extraction of the charge radius from $e-p$ scattering data using the z expansion
- $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$ adding $\pi\pi$ data
- Previous extractions have underestimated the error
- Results are compatible with CODATA value of $r_E^p = 0.8768(69)$ fm

Conclusions

- Analyzed Proton structure effects in hydrogenic bound states Using NRQED
- Isolated model-**dependent** assumptions in previous analyses: $W_1(0, Q^2)$ was calculated by "Sticking In Form Factors" model
- Model **independent** calculation of $W_1(0, Q^2)$: low Q^2 via NRQED, high Q^2 via OPE In between one has to model
- Possibility for a significant new effects in the two-photon amplitude
- NRQED predicts a universal shift for spin-independent energy splittings in muonic hydrogen.

Future Directions

- Applying z expansion to the magnetic and axial-vector form-factors
- Analyze spin dependent effects
- Application to deuterium
- Resolution of the discrepancy?

Backup Slides

Charge radius from Classic Lamb shift

- For electronic hydrogen: measured value Lunden and Pipkin '86

$$E_{2s}-E_{2p_{1/2}}=1.057845(9)\,\mathrm{GHz}=0.00437490(4)\,\mathrm{meV}$$

compared to

$$\Delta E_{r_E^p}=0.0000008\,(r_E^p)^2\frac{\mathrm{meV}}{\mathrm{fm}^2}$$

Proton radius effects at a level of 10^{-4}
Experimental uncertainty at a level of 10^{-5}

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Experimental uncertainty at a level of 10^{-5}

- For muonic hydrogen VP from electron loops dominant effect

$$E_{2s} - E_{2p_{1/2}} \approx -205\,\mathrm{meV}$$

compared to

$$\Delta E_{r_E^p} = 5.2\,(r_E^p)^2 \frac{\mathrm{meV}}{\mathrm{fm}^2}$$

Proton radius effects at a level of 2.5%
Experimental uncertainty at a level of 2×10^{-5}

Charge radius from Classic Lamb shift

- For electronic hydrogen: measured value Lunden and Pipkin '86

$$E_{2s} - E_{2p_{1/2}} = 1.057845(9)\,\mathrm{GHz} = 0.00437490(4)\,\mathrm{meV}$$

compared to

$$\Delta E_{r_E^p} = 0.0000008\,(r_E^p)^2\,\frac{\mathrm{meV}}{\mathrm{fm}^2}$$

Proton radius effects at a level of 10^{-4}
Experimental uncertainty at a level of 10^{-5}

- For muonic hydrogen VP from electron loops dominant effect

$$E_{2s} - E_{2p_{1/2}} \approx -205\,\mathrm{meV}$$

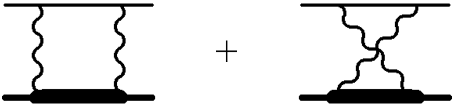
compared to

$$\Delta E_{r_E^p} = 5.2\,(r_E^p)^2\,\frac{\mathrm{meV}}{\mathrm{fm}^2}$$

Proton radius effects at a level of 2.5%
Experimental uncertainty at a level of 2×10^{-5}

- **Muonic hydrogen can give the best measurement of $\mathbf{r_E^p}$!**

Two-photon amplitude: "standard" calculation



- Is insertion of form factors in vertices valid?

- Even if it is, result looks funny
  two-photon amplitude $\Leftrightarrow$ the charge radius
  only for one parameter model for G_E and G_M

- Improvement?
  Treat the two-photon amplitude as a new parameter

- "Zemach" approximation: $m_l, \langle q \rangle \ll m_p$
  [Friar Annals Phys. **122**, 151 (1979),
  Eides et al. Theory of Light Hydrogenic Bound states, Springer]

$$\Delta E = 209.9779(49) - 5.2262 (r_E^p)^2 + 0.00913 \, \langle r^3 \rangle_{(2)} \ \mathrm{meV}$$

Third Zemach Moment

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- The formula for the Lamb shift has two unknowns
  $\Rightarrow$ use the CODATA value of r_E^p and solve for $\langle r^3 \rangle_{(2)}$
- The result [De Rújula PLB **693**, 555 (2010)]

$$[\langle r^3 \rangle_{(2)}]^{1/3} = 3.32 \pm 0.21\ \mathrm{fm} \quad \textbf{muonic hydrogen}$$

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- Looks fine until we compare it to $e - p$ scattering data

$$[\langle r^3 \rangle_{(2)}]^{1/3} = 1.39 \pm 0.02\ \mathrm{fm} \quad \textbf{scattering data}$$

[Sick, Friar PRA **72**, 040502(R) (2005)]

Much more than 5σ... there is still a discrepancy

Zemach?

- "Zemach" approximation: $m_l, \langle q \rangle \ll m_p$
  but for $\Lambda^2 = 0.71\,\mathrm{GeV}^2$
  $\Lambda \approx 0.84$ GeV is not small compared to m_p

- Even worse

- Proton pole term

$$E(2p) - E(2s) = -\,0.016 \text{ meV}$$

- Using the Zemach approximation for proton pole term

$$E(2p) - E(2s) = +\,0.021 \text{ meV}$$

$\Rightarrow$ Thought to be an approximation only because $W_1^{\mathrm{SIFF}}(0, Q^2)$!