Non-Anomalous Family U(1)' symmetry in AMSB

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Anomaly Mediated Supersymmetry Breaking

L. Randall, R. Sundrum, Nucl. Phys. B557 79 (1999);
G.F. Giudice, M. A. Luty, H. Murayama, R. Rattazzi, JHEP 12, (1998) 027;
I. Jack, D. R. T. Jones, Phys. Lett. B 473, 102 (2000).

Soft Mass Term

$$\mathcal{L}_{soft} = -(m^2)^i_j \phi^i \phi^j - \left(\frac{1}{2} b^{ij} \phi^i \phi^j + \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} M_a \lambda_a \lambda_a + h.c. \right)$$

AMSB is one of the promising SUSY breaking mechanisms.

$$M_a = m_{3/2} \beta_{g_a} / g_a,$$

$$h^{ijk} = -m_{3/2} \beta_Y^{ijk},$$

$$(m^2)^i_j = \frac{1}{2} m_{3/2}^2 \mu \frac{d}{d\mu} \gamma^i_j,$$

$$b^{ij} = \kappa m_{3/2} \mu^{ij} - m_{3/2} \beta_\mu^{ij},$$

pros: fewer parameters,
UV insensitive,
very predictive, etc
cons: tachyonic problem

Tachyonic problem

The anomalous dimension:

Gauge Interaction + Yukawa Interaction

$$
\begin{aligned}
16\pi^2\gamma_{H_1} &= 3\lambda_b^2 + \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{H_2} &= 3\lambda_t^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{L} &= \lambda_\tau^2 - \tfrac{3}{2}g_2^2 - \tfrac{3}{10}g_1^2, \\
16\pi^2\gamma_{Q} &= \lambda_b^2 + \lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{3}{2}g_2^2 - \tfrac{1}{30}g_1^2, \\
16\pi^2\gamma_{t^c} &= 2\lambda_t^2 - \tfrac{8}{3}g_3^2 - \tfrac{8}{15}g_1^2, \\
16\pi^2\gamma_{b^c} &= 2\lambda_b^2 - \tfrac{8}{3}g_3^2 - \tfrac{2}{15}g_1^2, \\
16\pi^2\gamma_{\tau^c} &= 2\lambda_\tau^2 - \tfrac{6}{5}g_1^2,
\end{aligned}
$$

$$m_{\tilde{q}}^2 = \frac{m_{3/2}^2}{(16\pi^2)^2}(8g_3^4 + \cdots)$$

$$m_{\tilde{e}_L}^2 = -\frac{m_{3/2}^2}{(16\pi^2)^2}(\frac{3}{2}g_2^4 + \frac{99}{50}g_1^4 + \cdots) < 0$$

$$m_{\tilde{e}_R}^2 = -\frac{m_{3/2}^2}{(16\pi^2)^2}(\frac{198}{25}g_1^4 + \cdots) < 0$$

U(1)' D-term

P. Fayet, J. Iliopoulos, Phys. Lett. B 51, 461 (1974);
I. Jack, D.R.T. Jones, Phus. Lett. B465, 148 (1999);
I. Jack, D. R. T. Jones, Phys. Lett. B 473, 102 (2000).

One way to solve the tachyonic problem

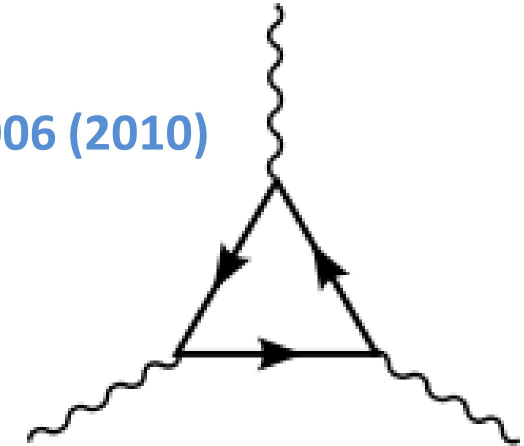
$$
\begin{aligned}
\bar{m}_Q^2 &= m_Q^2 + \zeta q_{Q_i} \delta^i_j \, , \\
\bar{m}_{u^c}^2 &= m_{u^c}^2 + \zeta q_{u_i} \delta^i_j \, , \\
\bar{m}_{d^c}^2 &= m_{d^c}^2 + \zeta q_{d_i} \delta^i_j \, , \\
\bar{m}_L^2 &= m_L^2 + \zeta q_{L_i} \delta^i_j \, , \\
\bar{m}_e^2 &= m_e^2 + \zeta q_{e_i} \delta^i_j \, , \\
\bar{m}_{H_u}^2 &= m_{H_u}^2 + \zeta q_{H_u} \, , \\
\bar{m}_{H_d}^2 &= m_{H_d}^2 + \zeta q_{H_d} \, ,
\end{aligned}
$$

RG Invariant

U(1)' D-term

Anomaly Cancellation Conditions

M.-C. Chen, J. Huang (2010), Phys. Rev. D 82, 075006 (2010)



$$[SU(3)]^2 U(1)'_{\mathrm{NAF}} \ : \ \sum_i [2q_{Q_i} - (-q_{u_i}) - (-q_{d_i})] = 0 \ ,$$

$$[SU(2)_L]^2 U(1)'_{\mathrm{NAF}} \ : \ \sum_i [q_{L_i} + 3q_{Q_i}] = 0 \ ,$$

$$[U(1)_Y]^2 \, U(1)'_{\mathrm{NAF}} \ : \ \sum_i [2 \times 3 \times \left(\frac{1}{6}\right)^2 q_{Q_i} - 3 \times \left(\frac{2}{3}\right)^2 (-q_{u_i}) - 3 \times \left(-\frac{1}{3}\right)^2 (-q_{d_i}) + 2 \times \left(-\frac{1}{2}\right)^2 q_{L_i} - (-1)^2 (-q_{e_i})] = 0 \ ,$$

$$[U(1)'_{\mathrm{NAF}}]^2 \, U(1)_Y \ : \ \sum_i [2 \times 3 \times \left(\frac{1}{6}\right) q_{Q_i}^2 - 3 \times \left(\frac{2}{3}\right) \times (-q_{u_i})^2 - 3 \times \left(-\frac{1}{3}\right) (-q_{d_i})^2 + 2 \times \left(-\frac{1}{2}\right) (q_{L_i})^2 - (-1)(-q_{e_i})^2] = 0 \ ,$$

Most Difficult One!

$$U(1)'_{\mathrm{NAF}} - \text{gravity} \ : \ \sum_i [6q_{Q_i} + 3q_{u_i} + 3q_{d_i} + 2q_{L_i} + q_{e_i} + q_{N_i}] = 0 \ ,$$

$$[U(1)'_{\mathrm{NAF}}]^3 \ : \ \sum [3(2(q_{Q_i})^3 - (-q_{u_i})^3 - (-q_{d_i})^3) + 2(q_{L_i})^3 - (-q_{e_i})^3 - (-q_{N_i})^3] = 0$$

Parametrization

M.-C. Chen, A. de Gouvea, B. A. Dobrescu (2007); M.-C. Chen, J. Huang (2010); M.-C. Chen, D. R. T. Jones, A. Rajaraman, H. B. Yu (2008)

$[SU(3)]^2\, U(1)'_{NAF}$, $[SU(2)_L]^2\, U(1)'_{NAF}$, $[U(1)_y]^2\, U(1)'_{NAF}$ will be satisfied by this parametrization!

16 free parameters

$$
\begin{aligned}
q_{Q_1} &= -\frac{1}{3}q_{L_1} - 2a\,, & q_{d_1} &= \frac{4}{3}q_{L_1} + q_{e_1} - 2c\,,\\
q_{Q_2} &= -\frac{1}{3}q_{L_2} + a + a'\,, & q_{d_2} &= \frac{4}{3}q_{L_2} + q_{e_2} + c + c'\,,\\
q_{Q_3} &= -\frac{1}{3}q_{L_3} + a - a'\,, & q_{d_3} &= \frac{4}{3}q_{L_3} + q_{e_3} + c - c'\,,\\
q_{u_1} &= -\frac{2}{3}q_{L_1} - q_{e_1} - 2b\,, & q_{N_1} &= -2q_{L_1} - q_{e_1} - 2d\,,\\
q_{u_2} &= -\frac{2}{3}q_{L_2} - q_{e_2} + b + b'\,, & q_{N_2} &= -2q_{L_2} - q_{e_2} + d + d'\,,\\
q_{u_3} &= -\frac{2}{3}q_{L_3} - q_{e_3} + b - b'\,, & q_{N_3} &= -2q_{L_3} - q_{e_3} + d - d'\,.
\end{aligned}
$$

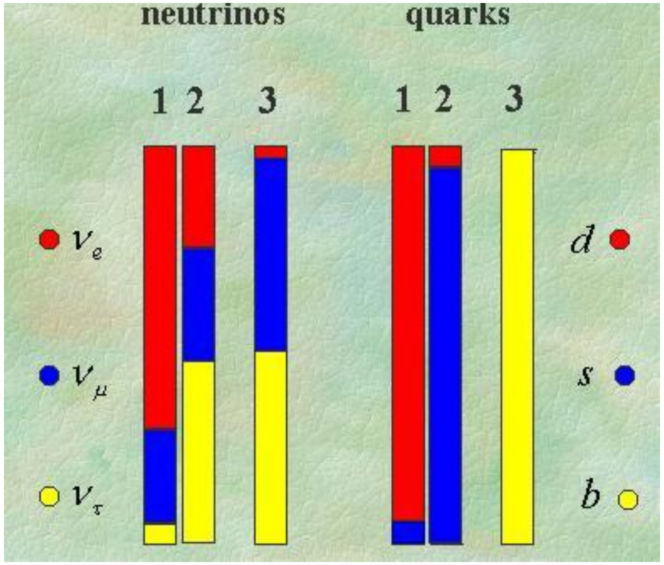
a, a', b, b', c, c', d, d' characterize the charge splitting between different generations.

U(1)' as a Family Symmetry

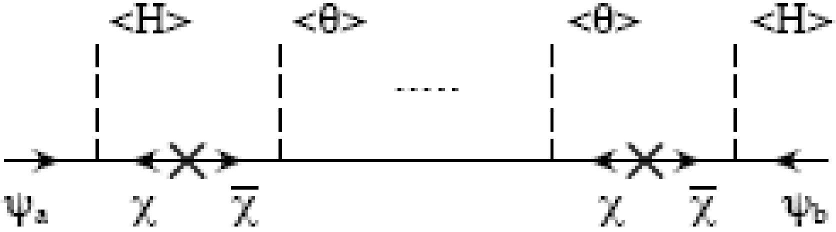
D. D. Froggatt, H. B. Nielsen, Nucl. Phys. B147, 277 (1979)

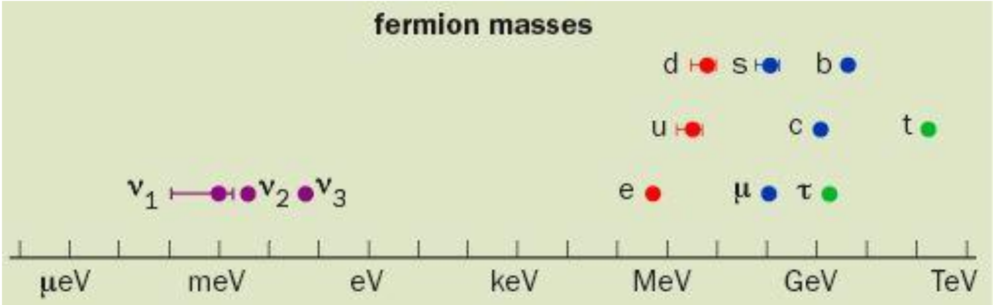
$$W = Y_u H_u Q u^c + Y_d H_d Q d^c + Y_e H_d L e^c + Y_\nu H_u L \nu^c + Y_N \Psi \nu^c \nu^c$$

$$+ \mu H_u H_d + \mu' \Phi \Phi' \, .$$



Froggatt Nielsen Mechanism





$$Y_{ij} \sim \left(y_{ij} \frac{\Phi}{\Lambda} \right)^{3|q_i + q_j + q_H|}$$

$$\lambda = \frac{<\phi>}{\Lambda} \simeq 0.22$$

Effective Yukawa matrices Pattern

$$Y_u \sim \begin{pmatrix} \lambda^{10} & \lambda^{|\frac{7}{2}-\frac{2a'}{5}|} & \lambda^{|\frac{13}{2}+\frac{8a'}{5}|} \\ \lambda^{|\frac{7}{2}+\frac{2a'}{5}|} & \lambda^{|-3|} & \lambda^{|2a'|} \\ \lambda^{|\frac{7}{2}-\frac{8a'}{5}|} & \lambda^{|-3-2a'|} & \lambda^{0} \end{pmatrix}$$

$$Y_e \sim \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^3 & \lambda^1 \\ \lambda^4 & \lambda^3 & \lambda^1 \end{pmatrix}$$

$$Y_d \sim \begin{pmatrix} \lambda^{5} & \lambda^{|\frac{19}{2}-\frac{2a'}{5}|} & \lambda^{|\frac{15}{2}+\frac{8a'}{5}|} \\ \lambda^{|-\frac{3}{2}+\frac{2a'}{5}|} & \lambda^{3} & \lambda^{|1+2a'|} \\ \lambda^{|-\frac{3}{2}-\frac{8a'}{5}|} & \lambda^{|3-2a'|} & \lambda^{1} \end{pmatrix}$$

$$Y_\nu \sim \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & \lambda^2 \end{pmatrix}$$

$$Y_N \langle\Psi\rangle \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^4 & \lambda^4 \end{pmatrix} \langle\Psi\rangle$$

Mass hierarchies are obtained;
Lepton mixing can be generated;
No quark mixing, good approximation @ LO.

Majorana Neutrinos

Summary of the 14 constraints

✓No/ small suppression on the heavy top quark, bottom quark, tau lepton:

$$q_{Q_3} + q_{u_3} + q_{H_u} = 0, \quad q_{Q_3} + q_{d_3} + q_{H_d} = 1, \quad q_{L_3} + q_{e_3} + q_{H_d} = 1,$$

✓High U(1)' breaking scale: $q_{L_3} + q_{N_3} + q_{H_u} = 2$

14 constraints

✓Lepton mass hierarchy and mixing constraints:

$$q_{L_1} = q_{L_3} + 1, \quad q_{L_2} = q_{L_3}, \quad q_{e_1} = q_{e_3} + 3, \quad q_{e_2} = q_{e_3} + 2 \,.$$

$$d = -\frac{4}{3}\,, \quad d' = 1\,, \quad \Longrightarrow \quad q_{N_1} = q_{N_2} = q_{N_3}\,,$$

✓$[U(1)'_{NAF}]^2$ $U(1)_Y$ Anomaly cancellation condition:

$$b = \frac{364 - 114a' + 18a'^2 - 183b' + 27a'b' + 18b'^2 + 96c' - 27b'c' + 18c'^2}{9(-17 + 3a' + 6b' - 3c')}$$

✓$[U(1)'_{NAF}]^3$ Anomaly cancellation condition: $q_{e3} = f(a', b', c', q_{L3})$

✓Quark mass hierarchy constraints: $c' = -a', \quad b' = -1/2 - a'$

Mass Sum Rules

I. Jack, D.R.T. Jones, Phys. Lett. B 482, 167 (2000)

Anomaly Cancellation Conditions

$$\sum_{i=1}^{3}(\bar{m}^2_{u^c_i} + \bar{m}^2_{d^c_i} + 2\bar{m}^2_{Q_i}) = \sum_{i=1}^{3}(m^2_{u^c_i} + m^2_{d^c_i} + 2m^2_{Q_i})_{\mathrm{AMSB}}\,,$$
$$\sum_{i=1}^{3}(\bar{m}^2_{L_i} + 3\bar{m}^2_{Q_i}) = \sum_{i=1}^{3}(m^2_{L_i} + 3m^2_{Q_i})_{\mathrm{AMSB}}\,,$$
$$\sum_{i=1}^{3}(\bar{m}^2_{u^c_i} + \bar{m}^2_{e^c_i} - 2\bar{m}^2_{Q_i}) = \sum_{i=1}^{3}(m^2_{u^c_i} + m^2_{e^c_i} - 2m^2_{Q_i})_{\mathrm{AMSB}}\,,$$

Physical mass sums

$$m^2_{\tilde{u}_L} + m^2_{\tilde{u}_R} + m^2_{\tilde{d}_L} + m^2_{\tilde{d}_R} + m^2_{\tilde{c}_L} + m^2_{\tilde{c}_R} + m^2_{\tilde{s}_L} + m^2_{\tilde{s}_R} + m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2} + m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2}$$
$$= 2\sum_{i=1}^{3}(2m^2_{\tilde{Q}_i} + m_{\tilde{u}^c_i} + m_{\tilde{d}^c_i})_{\mathrm{AMSB}} + 2\sum_{i=1}^{3}(m^2_{u_i} + m^2_{d_i})\,,$$
$$m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R} + m^2_{\tilde{\mu}_L} + m^2_{\tilde{\mu}_R} + m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2} + m^2_{\tilde{u}_L} + m^2_{\tilde{u}_R} + m^2_{\tilde{c}_L} + m^2_{\tilde{c}_R} + m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}$$
$$= \sum_{i=1}^{3}(m^2_{\tilde{L}_i} + m^2_{\tilde{e}^c_i} + m^2_{\tilde{Q}_i} + m^2_{\tilde{u}^c_i})_{\mathrm{AMSB}} + 2\sum_{i=1}^{3}(m^2_{e_i} + m^2_{u_i})\,.$$

Mass Sum Rules (Cont.)

M. Carena, K. Huitu and T. Kobayashi, Nucl. Phys. B 592, 164 (2001)

U(1)' Gauge Invariance →

$$\bar{m}^2_{Q_i} + \bar{m}^2_{u^c_j} + \bar{m}^2_{H_u} = (m^2_{Q_i} + m^2_{u^c_j} + m^2_{H_u})_{\rm AMSB} + (q_{Q_i} + q_{u_j} + q_{H_u})\zeta \ (i,j = 1,2,3)\,,$$
$$\bar{m}^2_{Q_i} + \bar{m}^2_{d^c_j} + \bar{m}^2_{H_d} = (m^2_{Q_i} + m^2_{d^c_j} + m^2_{H_d})_{\rm AMSB} + (q_{Q_i} + q_{d_j} + q_{H_d})\zeta \ (i,j = 1,2,3)\,,$$
$$\bar{m}^2_{L_i} + \bar{m}^2_{e^c_j} + \bar{m}^2_{H_d} = (m^2_{L_i} + m^2_{e^c_i} + m^2_{H_d})_{\rm AMSB} + (q_{L_i} + q_{e_j} + q_{H_d})\zeta \ (i,j = 1,2,3)\,.$$

Mass Square Splitting →

$$m^2_{\tilde{e}_L} - m^2_{\tilde{\mu}_L} = (q_{L_1} - q_{L_2})\zeta = \zeta\,,$$
$$m^2_{\tilde{e}_R} - m^2_{\tilde{\mu}_R} = (q_{e_1} - q_{e_2})\zeta = \zeta\,,$$
$$m^2_{\tilde{u}_L} - m^2_{\tilde{c}_L} = (q_{Q_1} - q_{Q_2})\zeta = (\frac{13}{2} - \frac{2}{5}a')\zeta\,,$$
$$m^2_{\tilde{u}_R} - m^2_{\tilde{c}_R} = (q_{u_1} - q_{u_2})\zeta = (\frac{13}{2} + \frac{2}{5}a')\zeta\,,$$
$$m^2_{\tilde{d}_L} - m^2_{\tilde{s}_L} = (q_{Q_1} - q_{Q_2})\zeta = (\frac{13}{2} - \frac{2}{5}a')\zeta\,,$$
$$m^2_{\tilde{d}_R} - m^2_{\tilde{s}_R} = (q_{d_1} - q_{d_2})\zeta = (-\frac{9}{2} + \frac{2}{5}a')\zeta\,,$$

Numerical Example

Anomaly Cancellation Conditions + Mass Hierarchy and Mixing

→ **Two free parameters, a', q_{L3} are left**

Field	$U(1)'_{\rm NAF}$ charge	Field	$U(1)'_{\rm NAF}$ charge
L_1	$q_{L_1} = 3/2$	Q_1	$q_{Q_1} = 853/450$
L_2	$q_{L_2} = 1/2$	Q_2	$q_{Q_2} = -1522/225$
L_3	$q_{L_3} = 1/2$	Q_3	$q_{Q_3} = 908/225$
e_1^c	$q_{e_1} = 31228381/1586700$	u_1^c	$q_{u_1} = -21278009/1586700$
e_2^c	$q_{e_2} = 29641681/1586700$	u_2^c	$q_{u_2} = -28164287/1586700$
e_3^c	$q_{e_3} = 26468281/1586700$	u_3^c	$q_{u_3} = -40540547/1586700$
N_1	$q_{N_1} = -31757281/1586700$	d_1^c	$q_{d_1} = 10200251/528900$
N_2	$q_{N_2} = -31757281/1586700$	d_2^c	$q_{d_2} = 548909/21156$
N_3	$q_{N_3} = -31757281/1586700$	d_3^c	$q_{d_3} = 1390561/105780$
H_u	$q_{H_u} = 34137331/1586700$	Φ	$q_\Phi = -1/3$
H_d	$q_{H_d} = -25674931/1586700$	Ψ	$q_\Psi = 28583881/793350$

$$a' = -27/5$$

$$q_{L_3} = 1/2$$

Overall shift to the U(1)'$_{\rm NAF}$ charges are complex!

Charge splittings are very simple

Mass Spectrum & Mass Square Splitting

SoftSUSY 3.1

A. Kudo, M. Yamaguchi, Phys. Lett. B516, 151 (2001)
J. L. Feng et al. Phys. Rev. Lett. 83, 1731 (1999).

$\zeta = 1.5 \times (100\ \text{GeV})^2 \quad \tan\beta = 10 \quad \text{sign}(\mu) = -1 \quad m_{3/2} = 40\ \text{TeV}$

Field	h_0	H_0	A_0	H^+	$\tilde{g}$	χ_1	χ_2	χ_3	χ_4	$\chi_1^\pm$	$\chi_2^\pm$
Mass (GeV)	114.81	275.74	275.51	286.93	879.93	133.99	361.94	518.34	525.65	134.15	524.55
Field	$\tilde{u}_L$	$\tilde{u}_R$	$\tilde{d}_L$	$\tilde{d}_R$	$\tilde{c}_L$	$\tilde{c}_R$	$\tilde{s}_L$	$\tilde{s}_R$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{b}_1$
Mass (GeV)	825.53	795.10	829.11	963.65	742.91	753.27	746.89	1014.38	366.87	780.88	745.06
Field	$\tilde{b}_2$	$\tilde{e}_L$	$\tilde{e}_R$	$\tilde{\mu}_L$	$\tilde{\mu}_R$	$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\nu}_{e_L}$	$\tilde{\nu}_{\mu_L}$	$\tilde{\nu}_{\tau_L}$	$\Delta m_{\chi_1^\pm - \chi_1}$
Mass (GeV)	905.41	322.45	250.78	298.35	218.71	120.09	298.56	312.44	287.44	285.58	0.16

LSP

Δm^2	$m^2_{\tilde{e}_L} - m^2_{\tilde{\mu}_L}$	$m^2_{\tilde{e}_R} - m^2_{\tilde{\mu}_R}$	$m^2_{\tilde{u}_L} - m^2_{\tilde{c}_L}$	$m^2_{\tilde{d}_L} - m^2_{\tilde{s}_L}$	$m^2_{\tilde{u}_R} - m^2_{\tilde{c}_R}$	$m^2_{\tilde{d}_R} - m^2_{\tilde{s}_R}$
$\times(100GeV)^2$	1.496	1.506	1.296	1.296	6.477	−10.035

Mass Spectrum (Cont.)

SoftSUSY 3.1

$$\zeta = 1.7 \times (100\ \text{GeV})^2 \quad \tan\beta = 10 \quad \text{sign}(\mu) = -1 \quad m_{3/2} = 40\ \text{TeV}$$

LSP

Field	h_0	H_0	A_0	H^+	$\tilde{g}$	χ_1	χ_2	χ_3	χ_4	$\chi_1^\pm$	$\chi_2^\pm$
Mass (GeV)	114.22	163.05	162.28	180.81	879.85	133.71	360.71	488.91	497.51	133.86	495.61
Field	$\tilde{u}_L$	$\tilde{u}_R$	$\tilde{d}_L$	$\tilde{d}_R$	$\tilde{c}_L$	$\tilde{c}_R$	$\tilde{s}_L$	$\tilde{s}_R$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{b}_1$
Mass (GeV)	825.20	790.01	828.77	978.97	730.85	742.13	734.89	1035.46	321.22	781.79	747.97
Field	$\tilde{b}_2$	$\tilde{e}_L$	$\tilde{e}_R$	$\tilde{\mu}_L$	$\tilde{\mu}_R$	$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\nu}_{e_L}$	$\tilde{\nu}_{\mu_L}$	$\tilde{\nu}_{\tau_L}$	$\Delta m_{\chi_1^\pm - \chi_1}$
Mass (GeV)	914.58	347.57	273.19	322.26	239.96	142.89	322.04	338.27	312.13	310.42	0.15

Conclusion

- AMSB is very predictive, determined by the low energy dynamics.
- Adding additional U(1) symmetry, tachyonic problem can be solved and it can also play the role of family symmetry.
- Mass Squared splitting might be used to test our U(1)' model at the collider