Neutrino Mixing with the Tetrahedral Group

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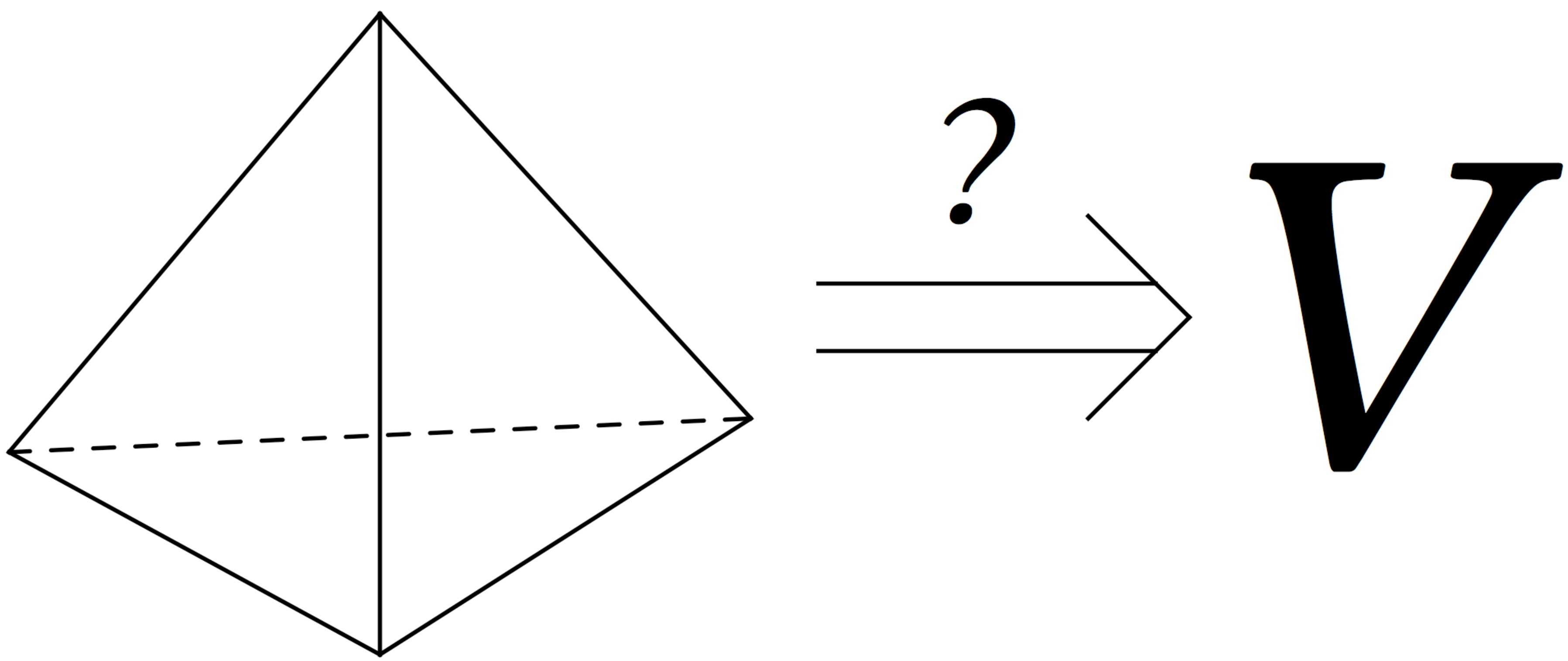

1 Neutrino Oscillations

1.1 Theory

2-flavor oscillations:

$$P_{2\nu}(\nu_e \to \nu_e) = 1 - \sin^2(2\theta)\sin^2[\frac{L}{4E}(m_2^2 - m_1^2)]$$
$$\neq 1$$

3-flavor oscillations:

$$e, \mu, \tau \text{ mass basis} \to \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{U_L^{-1} U_\nu}_{\equiv V} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \leftarrow \nu \text{ mass basis}$$

1.2 Data

(Gonzalez-Garcia, Maltoni 0704.1800)

$$|V_{\text{exp}}| \approx \begin{pmatrix} 0.77\text{–}0.86 & 0.50\text{–}0.63 & 0.00\text{–}0.22 \\ 0.22\text{–}0.56 & 0.44\text{–}0.73 & 0.57\text{–}0.80 \\ 0.21\text{–}0.55 & 0.40\text{–}0.71 & 0.59\text{–}0.82 \end{pmatrix}$$

$$m_2^2 - m_1^2 = 7.59 \begin{pmatrix} +0.61 \\ -0.69 \end{pmatrix} \times 10^{-5}\ \text{eV}^2\ , \quad \text{and}$$

$$m_3^2 - m_1^2 = \begin{cases} +2.46 \pm 0.37 \times 10^{-3}\ \text{eV}^2 & (\text{“normal hierarchy”}) \\ -2.36 \pm 0.37 \times 10^{-3}\ \text{eV}^2 & (\text{“inverted hierarchy”}) \end{cases}$$

$$\Longrightarrow\ R \equiv \frac{m_3^2 - m_1^2}{m_2^2 - m_1^2} = \begin{cases} +25.5\ \text{ to }\ +41.0 & (\text{“normal” hierarchy}) \\ -39.6\ \text{ to }\ -24.3 & (\text{“inverted” hierarchy}) \end{cases}$$

2 Tetrahedral Group: $T \simeq A_4 \subset SO(3)$

(E. Ma, A. Zee, X.G. He, ...)

Generators: I, $\{r_1, r_2, r_3\}$, $\{c, r_1 c r_1, r_2 c r_2, r_3 c r_3\}$, $\{a, r_1 a r_1, r_2 a r_2, r_3 a r_3\}$
Four irreps: $1,\ 1',\ 1'',\ 3$
Clebsch-Gordan coefficients for $(vw) \sim 3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_A \oplus 3_S$:

$$
\begin{aligned}
(vw)_1 &= v_1 w_1 + v_2 w_2 + v_3 w_3 \\
(vw)_{1'} &= v_1 w_1 + \omega\, v_2 w_2 + \omega^* v_3 w_3 \\
(vw)_{1''} &= v_1 w_1 + \omega^* v_2 w_2 + \omega\, v_3 w_3
\end{aligned}
$$

$$
(vw)_{3_A} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix} \ , \quad (vw)_{3_S} = \begin{pmatrix} v_2 w_3 + v_3 w_2 \\ v_3 w_1 + v_1 w_3 \\ v_1 w_2 + v_2 w_1 \end{pmatrix}
$$

$$
(\omega \equiv e^{i2\pi/3} = -\tfrac{1}{2} + i\tfrac{\sqrt{3}}{2} \quad \Longrightarrow \quad \omega^2 = \omega^* \ , \ \ 1 + \omega + \omega^2 = 0)
$$

3 An example using $T \simeq A_4$

(He, Keum, Volkas 0601001)

$$\text{Leptons: } \ell \equiv \begin{pmatrix} \nu \\ e \end{pmatrix} \sim 3\ , \quad \nu^c \sim 3\ , \quad e^c \sim 1 \oplus 1' \oplus 1''$$

$$\text{Higgs doublets: } \varphi \equiv \begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix} \sim 3\ , \quad \phi \equiv \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \sim 1$$

$$\text{SM-invariant scalar: } \chi \sim 3$$

Charged leptons:

$$\langle \varphi^0 \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \implies M_\ell = U_L^* \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} U_R$$

$$U_R = I \ , \quad U_L^* = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \leftarrow \text{independent of masses!}$$

Neutrinos:

$$\langle \chi^0 \rangle \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \ , \quad \langle \phi^0 \rangle \neq 0 \implies M_\nu = U_\nu^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_\nu^\dagger$$

$$U_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix} \leftarrow \text{independent of masses!}$$

Mixing matrix (independent of masses):

$$V \equiv U_L^\dagger U_\nu = (\text{phases}) \begin{pmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \end{pmatrix} (\text{phases})$$

Compare:

$$\text{Prediction: } |V| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \approx \begin{pmatrix} 0.82 & 0.58 & 0 \\ 0.41 & 0.58 & 0.71 \\ 0.41 & 0.58 & 0.71 \end{pmatrix}$$

$$\text{Data: } |V_{\text{exp}}| \approx \begin{pmatrix} 0.77\text{–}0.86 & 0.50\text{–}0.63 & 0.00\text{–}0.22 \\ 0.22\text{–}0.56 & 0.44\text{–}0.73 & 0.57\text{–}0.80 \\ 0.21\text{–}0.55 & 0.40\text{–}0.71 & 0.59\text{–}0.82 \end{pmatrix}$$

Compatible $\checkmark$

Disclaimers:

1) $G = A_4 \otimes X$ where X forbids $\varphi \sim 3$ from contributing to M_ν

2) "Sequestering problem": $\mathbb{Z}_3 = \{I, c, a\}$ vs. $\mathbb{Z}_2 = \{I, r_2\}$

4 Double Tetrahedral Group: $T' \subset SU(2)$

(Frampton and Kephart, Chen and Mahanthappa, ...)

$SU(2)$ double covers $SO(3)$, so construct double cover of $T \simeq A_4$.

7 irreps: $1, 1', 1'', 3, 2, 2', 2''$.

CG for $(\chi\xi) \sim 2 \otimes 2 = 1 \oplus 3$:

$$(\chi\xi)_1 = \chi_1\xi_2 - \chi_2\xi_1 \quad , \qquad (\chi\xi)_3 = \begin{pmatrix} -i(\chi_1\xi_1 + \chi_2\xi_2) \\ -(\chi_1\xi_1 - \chi_2\xi_2) \\ \chi_1\xi_2 + \chi_2\xi_1 \end{pmatrix}$$

CG for $(\chi\phi) \sim 2 \otimes 3 = 2 \oplus 2' \oplus 2''$:

$$(\chi\phi)_2 = \begin{pmatrix} -\sqrt{2}\,\phi_+\chi_2 - i\,\phi_3\chi_1 \\ +\sqrt{2}\,\phi_-\chi_1 + i\,\phi_3\chi_2 \end{pmatrix}$$

$$(\chi\phi)_{2'} = \begin{pmatrix} (+\phi_+ - i2\sqrt{3}\,\phi_-)\chi_2 - i\frac{1}{\sqrt{2}}\phi_3\chi_1 \\ (-\phi_- + i2\sqrt{3}\,\phi_+)\chi_1 + i\frac{1}{\sqrt{2}}\phi_3\chi_2 \end{pmatrix}$$

$$(\chi\phi)_{2''} = \begin{pmatrix} (+\phi_+ + i2\sqrt{3}\,\phi_-)\chi_2 - i\frac{1}{\sqrt{2}}\phi_3\chi_1 \\ (-\phi_- - i2\sqrt{3}\,\phi_+)\chi_1 + i\frac{1}{\sqrt{2}}\phi_3\chi_2 \end{pmatrix}$$

where $\phi_\pm \equiv \frac{1}{\sqrt{2}}(\phi_1 \pm i\,\phi_2)$.

Point: **Complex phases from group theory.**

(Chen and Mahanthappa 0904.1721)

5 An example using $T' \simeq A'_4$

(BenTov and Zee 1101.1987)

$$\text{Leptons:} \quad \begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix} \sim 2\ , \quad \ell_3 \sim 1\ , \quad e_1^c \sim 1\ , \quad e_2^c \sim 1'\ , \quad e_3^c \sim 1''$$

(Neutrino) Higgs doublets: $\phi \sim 3\ , \quad \chi \sim 2$

(Charged Lepton) Higgs doublets:

$$\varphi' \sim 1'\ , \quad \varphi'' \sim 1''\ , \quad \xi \sim 2\ , \quad \xi' \sim 2'\ , \quad \xi'' \sim 2''$$

Tons of Higgs fields – only purpose is that for some parameter values, we get

$$V_{\text{PMNS}} \approx \begin{pmatrix} -0.82 & 0.54 & 0.22\,e^{-i0.72} \\ 0.52\,e^{+i0.14} & 0.61\,e^{-i0.08} & 0.59 \\ 0.24\,e^{-i0.40} & 0.58\,e^{+i0.11} & -0.78 \end{pmatrix}$$

with all couplings and VEVs real.

Phases PURELY from group theory.

For reference:

$$V_{\text{PMNS}} = \begin{pmatrix} -c_2 c_3 & c_2 s_3 & \hat{s}_2^* \\ c_1 s_3 + s_1 \hat{s}_2 c_3 & c_1 c_3 - s_1 \hat{s}_2 s_3 & s_1 c_2 \\ s_1 s_3 - c_1 \hat{s}_2 c_3 & s_1 c_3 + c_1 \hat{s}_2 s_3 & -c_1 c_2 \end{pmatrix}$$

where $c_I \equiv \cos\theta_I$, $s_I \equiv \sin\theta_I$ and $\hat{s}_2 \equiv s_2\, e^{\,i\delta_{\text{CP}}}$.

CP-violating angle $\delta_{\mathbf{CP}} \neq 0$ with "large" $V_{e3} \sim 0.22$.

6 Discussion

- Existence proof: Discrete group $\implies$ V independent of masses.

- Existence proof: Discrete group $\implies$ CP violation.

- Look for FCNC that violate lepton number but preserve, e.g., $\mathbb{Z}_3 = \{I, c, a\} \subset A_4$.

  Example: suppose you see $\tau^+ \to \mu^+\mu^+e^-$ but not $\mu^+ \to e^+e^+e^-$.

  might suggest... $e, e^c \sim 1$; $\mu, \tau^c \sim 1'$; $\tau, \mu^c \sim 1''$

  operator $(\tau^c\mu)(e^c\mu)$ allowed, but $(\mu^c e)(e^c e)$ forbidden.