Exercise 21.11

In an experiment in space, one proton is held fixed and another proton is released from rest a distance of 4.00 mm away.

Initial acceleration of a proton due to another proton 4.00 mm away:

\[ \mathbf{F}_{\text{Coulomb}} = k \frac{q_1 q_2}{r^2} = ma \] (i.e. the acceleration of the proton is caused exclusively by the Coulomb force exerted on it by the other proton.)

So \( a = \frac{k q^2}{m b^2} = \frac{(8.988 \cdot 10^9 \text{ Nm}^2\text{C}^2)(1.602 \cdot 10^{-19} \text{ C})^2}{(1.673 \cdot 10^{-27} \text{ kg})(0.004 \text{ m})^2} \approx 8620 \text{ m/s}^2 \)

Exercise 21.55

A ring-shaped conductor with radius \( a = 2.10 \text{ cm} \) has a total positive charge \( Q = 0.127 \mu \text{C} \) uniformly distributed around it.

What is the magnitude of the electric field at point \( P \), which is on the positive \( x \)-axis at \( x \)?

Magnitude of the electric field at point \( P \)

By symmetry, the \( y \) and \( z \) (axis not labeled) component of the electric field caused by one point on the ring is exactly canceled out by the opposite point on the ring, while the \( x \) components of all points on the ring add up:

Now, rather than calculate the more difficult integral \( E = \int dE \), we can just calculate \( E_x = \int \cos \alpha \, dE \) and then use \( E = E_x x \). Note that according to the diagram, \( \cos \alpha = \frac{x}{r} \) where \( r = \sqrt{x^2 + a^2} \). We will also use \( \frac{dE}{dq} = \frac{k}{r^2} \) (so that \( dE = \frac{k}{r^2} \, dq \)) and \( ds = \frac{Q}{2 \pi a} \) (so that \( dq = \frac{Q}{2 \pi a} \, ds \)).

\[ E_x = \int \cos \alpha \, dE = \int \frac{x}{r} \, dE = \int \frac{x}{r} \left( \frac{k}{r^2} \, dq \right) = \int \frac{kx}{r^3} \, dq = \int \frac{kx}{r^3} \left( \frac{Q}{2 \pi a} \, ds \right) = \frac{Q}{2 \pi a} \frac{kx}{r^3} \, ds \]

Note that the integration variable \( s \) is independent of all other variables, so that

\[ E_x = \frac{Q}{2 \pi a} \frac{kx}{r^3} 2 \pi a = \frac{kQx}{r^3} \quad \text{or} \quad E_x = \frac{kQx}{(x^2 + a^2)^{3/2}} \quad \text{so that} \quad \bar{E}(x) = \frac{kQx}{(x^2 + a^2)^{3/2}} \]

What is the direction of the electric field at point \( P \)?

Electric field lines point away from positive charges. Thus, using our symmetry arguments (or just using that \( Q \) is positive for \( E(x) \) above), we see that \( \bar{E} \) points in the +x direction.
A particle with a charge of \( q \) is placed at the point \( P \) described in part A. What is the magnitude of the force exerted by the particle on the ring? What is the direction of the force exerted by the particle on the ring?

The force exerted on the particle by the ring is:

\[
F = qE_x
\]

By Newton's third law, this is equal and opposite to the force exerted by the particle on the ring.

\[
F_{\text{ring}} = qE_x = \frac{kqQx}{(x^2 + a^2)^{3/2}} \text{ in the } -x \text{ direction.}
\]

Suspending charged particles using electric fields:

What must the charge (sign and magnitude) of a particle of mass \( m \) be for it to remain stationary when placed in a downward-directed electric field of magnitude \( E \)?

We want the force of gravity \( F_G = -mg \) to exactly cancel the electric force \( F_E = q(-E) \). We use + as up and - as down and carefully keep track of signs, while requiring \( E \) and \( q \) to be positive.

\[
F_E = -F_G \quad \text{so} \quad -qE = -(-mg)
\]

That is, \( q = \frac{mg}{E} \)

What is the magnitude of an electric field in which the electric force on a proton is equal in magnitude to its weight?

Use \( 1.67 \times 10^{-27} \text{ kg} \) for the mass of a proton, \( 1.60 \times 10^{-19} \text{ C} \) for the magnitude of the charge on an electron, and \( 9.81 \text{ m/s}^2 \) for the magnitude of the acceleration due to gravity.

To find the magnitude of the required electric field, we just solve for \( E \):

\[
E = \frac{mg}{q} = \frac{m_{\text{proton}}g}{|e|} = \frac{1.67 \times 10^{-27} \text{ kg} \cdot 9.81 \text{ m/s}^2}{1.60 \times 10^{-19}} \approx 1.02 \times 10^{-7} \text{ N/C}
\]
Coulomb's law tutorial

Part A:
The force on particle 0 due to particle 1 is in the $-\hat{y}$ direction, and its magnitude is given by Coulomb's law, so $F = -\frac{kq_0q_1}{d_1^2} \hat{y}$.

Part B:
The force on particle 0 due to particle 1 and particle 2 is just a linear superposition of each of those forces separately (an important property of Coulomb's law). Thus,

$$\vec{F}_{net} = kq_0 \left(-\frac{q_1}{d_1^2} + \frac{q_2}{d_2^2}\right) \hat{y}$$

Part C:
For no net force on particle 0, $F_{net} = 0$. Thus

$$\vec{F}_{net} = 0 = kq_0 \left(-\frac{q_1}{d_1^2} + \frac{q_2}{d_2^2}\right) \hat{y}$$

which is 0 if $\frac{q_1}{d_1^2} + \frac{q_2}{d_2^2} = 0$

That is, \( \frac{q_2}{d_2^2} = \frac{q_1}{d_1^2} \)

$$\frac{d_2^2}{d_1^2} = \frac{q_1}{q_2} \quad \text{or} \quad \frac{d_1^2}{d_2^2} = \frac{q_2}{q_1}$$

Part D:
The direction of the force is in the $-\frac{1}{2}(\hat{y} + \hat{z})$ direction. Since the distance between 0 and 2 is given by the hypotenuse of a triangle with two legs of length $d_2$,

$$d = \sqrt{d_1^2 + d_2^2} = d_2 \sqrt{2}.$$ 

Thus, the net force is

$$\vec{F} = \frac{kq_0q_3}{(d_2 \sqrt{2})^2} \left(-\frac{1}{2} [\hat{y} + \hat{z}] \right) = -\frac{kq_0q_3}{2d_2} \frac{\sqrt{2}}{2} (\hat{y} + \hat{z})$$
Electric Field due to Multiple Point Charges

Part A: Calculate the electric field at point A.

The x component of the electric field at A, \( E_{Ax} \), is just the sum of the x components of \( E_1 \) and \( E_2 \), given by \( E_1 \sin \theta_1 \) and \( E_2 \sin \theta_2 \), respectively.

From the diagram it is apparent that \( \sin \theta_1 = \frac{d_1}{\sqrt{d_1^2 + d_2^2}} \) and \( \sin \theta_2 = \frac{d_2}{\sqrt{d_1^2 + d_2^2}} \). Also, the magnitudes of \( E_1 \) and \( E_2 \) are \( E_1 = \frac{kq_1}{d_1^2 + d_2^2} \) and \( E_2 = \frac{kq_2}{d_1^2 + d_2^2} \). Thus,

\[
E_{Ax} = -E_1 \sin \theta_1 + E_2 \sin \theta_2 = -\frac{kq_1}{d_1^2 + d_2^2} \frac{d_1}{\sqrt{d_1^2 + d_2^2}} + \frac{kq_2}{d_1^2 + d_2^2} \frac{d_2}{\sqrt{d_1^2 + d_2^2}}
\]

\[
E_{Ax} = -\frac{kq_1 d_1}{(d_1^2 + d_2^2)^{3/2}} + \frac{kq_2 d_2}{(d_1^2 + d_2^2)^{3/2}}
\]

Similarly, \( E_{Ay} = E_1 \cos \theta_1 + E_2 \cos \theta_2 \), where \( \cos \theta_1 = \frac{d_2}{\sqrt{d_1^2 + d_2^2}} \) and \( \cos \theta_2 = \frac{d_2}{\sqrt{d_1^2 + d_2^2}} \). Thus

\[
E_{Ay} = \frac{kq_1 d_2}{(d_1^2 + d_2^2)^{3/2}} + \frac{kq_2 d_2}{(d_1^2 + d_2^2)^{3/2}}
\]

Part B: The numbers in part A are given exactly so that \( E_{Ax} = 0 \). That is, the electric field due to \( q_1 \) and \( q_2 \) point exactly in the y direction. Now we want to figure out the magnitude necessary for a third charge \( q_3 \) to cancel out \( E_{Ay} \) as well. The charge is situated above A at point B given by \((0, d_{0y})\). We want \( |E_{0y}| \) at A to equal \( |E_{Ay}| \).

\[
|E_{0y}| = \frac{kq_3}{(d_{0y} - d_y)^2} = |E_{Ay}|
\]

Thus

\[
q_3 = \frac{(d_{0y} - d_y)^2}{k} E_{Ay}
\]
Since the electric field of the line charge always points radially outward, it is always perpendicular to the lateral surface of the cylinder, so that \( \Phi_E = EA \), with \( E = \frac{\lambda}{2\pi\epsilon_0 r} \) and \( A = 2\pi r l \).

Thus \( \Phi_E = \frac{2\pi r l \lambda}{2\pi\epsilon_0} = \frac{\lambda l}{\epsilon_0} \).

Thus the flux is independent of the radius of the cylinder.

Problem 22.41

The tension on the string in the horizontal direction is due exclusively to the electrical force on the charge. The vertical tension is due exclusively to the force of gravity.

Thus \( \tan(\theta) = \frac{F_E}{F_g} \), or \( \theta = \tan^{-1}\left(\frac{F_E}{F_g}\right) \).

Plugging in \( E = \frac{\sigma}{2\epsilon_0} \) for a charged insulating sheet, and \( F_E = qE \) and \( F_g = mg \):

\( \theta = \tan^{-1}\left(\frac{qE}{mg}\right) = \tan^{-1}\left(\frac{\sigma}{2\epsilon_0 mg}\right) \).