ELECTRIC POTENTIAL AND ENERGY
Electric Potential Energy

- E fields can do **work** on charged particles
  - So they must contain energy

\[ F = qE \quad \rightarrow \quad \text{Work} = Fd = qEd \]

- \( \text{PE}_{\text{electric}} \) similar to \( \text{PE}_{\text{gravity}} \) (qEd vs. mgd)
  - Except it can be **attractive** or **repulsive**
  - General rule: forces push toward **lowest** possible PE

High PE  |  Low PE  |  Very Low PE
Electric Potential – “Voltage”

- Charge affects PE (which is also called $U$)
  - Measure the effect → one number at each point in space

- Electric potential (or “voltage”)
  - “If I put a test charge here, how much PE will it add?”
  - Measured in Volts (V)

\[ V = \frac{U}{q'} \]

Point A has a large positive voltage
Point B has a large negative voltage
**U and V for Point Charges**

- **U needs reference point → like “floor” in PE**
  - Convention for point charges:
  - Infinite separation is the “zero” of U and V

\[
U = k \frac{qq'}{r}
\]

- U and V can be + or −, depending on charges

\[
V = k \frac{q}{r}
\]

- For more than 2 charges:
  - \( V_{\text{total}} \) = sum of V's from each charge
  - \( U_{\text{total}} \) = sum of U's from each pair of charges
Point Charge Equations

\[ F = k \frac{q_1 q_2}{r^2} \]

\[ U = k \frac{qq'}{r} \]

\[ E = \frac{F}{q} \]

\[ V = \frac{U}{q'} \]

\[ E = k \frac{|q|}{r^2} \]

\[ V = k \frac{q}{r} \]
Storing Electric Energy

- Must separate + and – charges
  - Like stretching an imaginary rubber band (the E field)

- Batteries
  - Place 2 different substances in contact with acid
  - + and – charges separate
  - ΔV is roughly constant for life of battery

- Capacitors
  - 2 metal plates with a gap between them
  - Place + charge on one plate and – charge on the other
  - Good for releasing energy quickly → e.g. camera flash bulb
  - ΔV depends on how much Q is on plates
Capacitors

Capacitance

\[ C = \frac{Q}{V} \]

Units: Farad (F) = 1 C/V

\[ C = \varepsilon_0 \frac{A}{d} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \frac{C}{V \cdot m} \]

\[ E = \frac{V}{d} \]

\[ V = V_b - V_a \]
Energy Storage in Capacitors

• Capacitors store potential energy in the E field
  – 3 ways to express energy:

  \[ U = \frac{1}{2} CV^2 \]

  \[ U = \frac{1}{2} \frac{Q^2}{C} \]

  \[ U = \frac{1}{2} QV \]

• If capacitor is connected to a battery:
  – \( V \) = constant → \( Q \) can change if \( C \) changes

• If capacitor is disconnected:
  – \( Q \) = constant → \( V \) can change if \( C \) changes
Applications of Capacitors

• Quick release of stored energy

• Circuit protection

• Sensors
Dielectrics – Making Capacitors Stronger

• Put an insulator (with “bound electrons”) in an E field
  - e– are not “free” → shape of the electron cloud is affected

• Atom is now “polarized” → energy is stored like a spring
  - Can make capacitors stronger by inserting these “dielectrics”

• Too much polarization → electron separates from nucleus
  - “Dielectric breakdown” – material becomes a conductor
Capacitors with Dielectrics

• Every material has 2 dielectric properties:
  – “Dielectric constant” $K$
  – “Dielectric strength” → E field at which breakdown occurs

• Capacitance with dielectric
  – Is $K$ times bigger

• Increase in $C$ → change in $U$
  – Depends on whether capacitor is connected to battery

\[ C = KC_0 \]
Energy Stored in E Field

- Can calculate amount of energy E field stores
  - Using parallel-plate capacitor as example

\[
U = \frac{1}{2} CV^2 = \frac{1}{2} \left( \frac{\varepsilon_0 A}{d} \right) (Ed)^2 = \frac{1}{2} \varepsilon_0 E^2 (Ad)
\]

\[
\frac{U}{Volume} = \frac{U}{Ad} = \frac{1}{2} \varepsilon_0 E^2 = \text{Energy Density} = u
\]

\[
u = \frac{1}{2} \varepsilon_0 E^2
\]

This turns out to be a general result for all E fields.