1 Ch. 14, §9, 2

\[ w = \frac{z + 1}{2i} = \frac{x + iy + 1}{2i} = \frac{y}{2} - i\left(\frac{x}{2} - 1\right) \]  

(1.1)

Lines of constant \( u \) and \( v \) are plotted on the next page.

2 Ch. 14, §9, 8

\[ w = \cosh(z) = \frac{e^x e^{iy} + e^{-x} e^{-iy}}{2} = \frac{e^x \cos(y) + e^{-x} \cos(y) + ie^x \sin(y) - ie^{-x} \sin(y)}{2} = \cosh(x) \cos(y) + i \sinh(x) \sin(y) \]  

(2.1)

Lines of constant \( u \) and \( v \) are plotted on the next page.

3 Ch. 14, §9, 11

The Riemann surface for \( \log z \) is an infinite sheeted spiral, like an infinite rotelli noodle. Going from one sheet to the one above (or below) it corresponds to increasing (or decreasing) \( \Im(\log z) \) by \( 2\pi \).

4 Ch. 14, §10, 4

Under the map \( w = \log z \), the quarter disk with \( r \in (0, 1), \theta \in (0, \pi/2) \) is mapped to the rectangular region \( u \in (-\infty, 0), v \in (0, \pi/2) \) in the \( u + iv \) complex plane. Thus, we have to solve the heat equation with insulating boundary conditions at \( u = -\infty, 0, T = 0 \) at \( v = 0 \) and \( T = 100 \) at \( v = \pi/2 \). The solution is given by \( T = 200v/\pi \). Thus, \( T = 200\theta/\pi = 200 \arctan(y/x)/\pi \). The isotherms are given by \( y = \tan(\pi T/200)x \), which are lines going radially outward from the origin, as you might have expected.
(*Section 9, Problem 2*)
(*Lines of constant u*)

\texttt{ContourPlot[(y/2), \{x, -1, 1\}, \{y, -1, 1\}]}
ContourPlot[(-(x + 1)/2), {x, -1, 1}, {y, -1, 1}]
(* Lines of constant v *)
(*Section 9, Problem 8*)
(*Lines of constant u*)

\texttt{ContourPlot[(\text{Cos}[y] \text{Cosh}[x]), \{x, -1, 1\}, \{y, -1, 1\}]}

\begin{center}
\includegraphics[width=\textwidth]{hw6.nb}
\end{center}
(* Lines of constant v *)
ContourPlot[Sin[y] Sinh[x], {x, -1, 1}, {y, -1, 1}]