(2.53) \( (1) \) \[ m \frac{d\vec{v}}{dt} = \vec{F} = q \left( \vec{E} + \vec{v} \times \vec{B} \right) \]

Let \( \vec{E} = E \hat{\mathbf{e}} \), \( \vec{B} = B \hat{\mathbf{e}} \)

Only the 1st term contributes to the z component of the force:

\[ m \frac{d\nu_z}{dt} = q E \]

The 1st term does not contribute to the x-y components, so these are the same as in section (2.5):

\[ m \frac{d\nu_x}{dt} = q B \nu_y \], \[ m \frac{d\nu_y}{dt} = -q B \nu_x \]

The solution to the 1st eq. is

\[ \nu_z = \frac{q E}{m} t + \nu_{z0} \]

\[ \Rightarrow z(t) = \frac{q E}{2m} t^2 + \nu_{z0} t + z_0 \]

The solution for \( x(t) \) and \( y(t) \) is the same as (2.7):

\[ x + iy = (x_0 + iy_0) e^{-i\omega t} \]

with \( \omega = \frac{q B}{m} \)

The particle moves in a helix about the z axis, with increasing distance between cycles due to the acceleration in the z direction.
\[ m \dot{V} = q (\dot{E} + \dot{V} \times \dot{B}) \text{ with } \dot{E} = E \dot{y}, \dot{B} = B \dot{z} \]

(a) The $z$ component of the force is zero.
So $m \dot{v}_z = 0 \Rightarrow v_z = \text{const}$
Initially, $z = 0, v_z = 0$, so $z = 0$ for all $t$.
The motion stays in a plane.

\[ \begin{align*}
\dot{v}_x &= q B v_y, \\
\dot{v}_y &= q E - q B v_x
\end{align*} \]

(b) The initial motion in $x$ direction is undeflected if the $y$ component of the force is zero:
\[ q E - q B v_x = 0 \Rightarrow v_x = \frac{E}{B} = V_{dr} \]
This is the drift speed at which $\dot{v}_y = 0$, so if $v_y = 0$ initially, it stays zero and hence $\dot{v}_x = 0$.

(c) Let $u_x = v_x - V_{dr}$, $u_y = v_y$. Then
\[ \begin{align*}
\dot{u}_x &= q B u_y, \\
\dot{u}_y &= -q B u_x
\end{align*} \]
These are just the eqs solved in sec. (2.5).
Since the initial velocity is in $\hat{x}$ direction:
\[ u_x = A \cos \omega t, \quad u_y = -A \sin \omega t \]

\[ \Rightarrow \begin{align*}
\dot{v}_x &= \dot{V}_{dr} + A \cos \omega t, \\
\dot{v}_y &= -A \sin \omega t
\end{align*} \]
with initial cond:
\[ V_{xo} = V_{dr} + A \Rightarrow A = V_{xo} - V_{dr} \]
(3.12) (3) (a) If mass of fuel is \(0.6 \, m_0\), the final mass is \(m = 0.4 \, m_0\). From eq. (3.8)

\[ v = v_0 + \frac{V}{V_{ex}} \ln \left( \frac{m_0}{m} \right) \]

Starting at rest, \(v_0 = 0\), the final speed is

\[ v = \frac{V}{V_{ex}} \ln \left( \frac{1}{0.4} \right) \]

\[ V = 0.916 \, V_{ex} \]

(b) Speed after 1st stage:

\[ v_1 = \frac{V}{V_{ex}} \ln \left( \frac{1}{0.7} \right) = 0.357 \, V_{ex} \]

Mass at beginning of 2nd stage: \(m_1 = 0.6 \, m_0\)

" end " " " \(m = 0.73 \, m_0\)

So

\[ v = v_1 + \frac{V}{V_{ex}} \ln \left( \frac{0.6}{0.73} \right) \]

\[ = \left( 0.357 + 0.693 \right) V_{ex} \]

\[ V = 1.05 \, V_{ex} \]

which is greater than part (a).
(3.19) (4) (a) From eq. (3.12), the center of mass is only affected by external forces. Even after the explosion, it continues to follow a parabola.

Since the pieces land at the same time, the explosion added momentum $p_x^{(1)}$ to first piece and $p_x^{(2)}$ to second piece. Conservation of momentum implies $p_x^{(2)} = -p_x^{(1)}$. There are no forces in x-direction.

The extra distance traveled by the first piece is

$$\frac{p_x^{(1)}}{m_1} t_0 = 100 \text{m}$$

So the extra distance traveled by second piece is

$$\frac{p_x^{(2)}}{m_2} t_0 = -100 \text{m} \quad \text{since} \quad m_2 = m_1$$

Since center of mass was at x=100m, the second piece lands back at the starting point!

(c) This would not be true if they landed at different times, since then $t_0$ would be replaced by $t_1$ or $t_2 \neq t_0$. 
3.27) (5)(a) The angular momentum is \( \vec{\ell} = \vec{r} \times \vec{p} = m \vec{r} \times \vec{v} \)

In polar coordinates:
\[
\vec{V} = \dot{r} \hat{r} + r \dot{\phi} \hat{\phi}
\]
\[\vec{r} = r \hat{r} \quad \text{and} \quad \hat{r} \times \hat{r} = 0, \quad \hat{r} \times \hat{\phi} = \hat{\phi}, \quad \text{so} \]
\[\vec{\ell} = mr^2 \dot{\phi} \hat{\phi}
\]
Since \( \dot{\phi} = \omega \), the magnitude of the angular momentum is
\[|\vec{\ell}| = mr^2 \omega
\]

(b) In a short time \( dt \), the angle \( \phi \) changes by \( d\phi \)
The area increases by \( dA = \frac{1}{2} \text{(base) x height) } = \frac{1}{2} (r)(r d\phi) \)

Note:
\[
\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\phi}{dt} = \frac{1}{2} r^2 \omega
\]

Note:
The height is really \( (r-dr) d\phi \)
But in the limit of very small time intervals, the correction term goes to zero.

Finally \( \frac{dA}{dt} = \frac{l}{2m} \)

Since \( \ell \) is conserved, planets sweep out equal areas in equal times.