Quantum treatment

- Excitations - lower the spin once

\[
|i\rangle = \frac{S_i^-}{\sqrt{2S}} \prod_j |S_j^z = S\rangle = |S_i^z = S - 1\rangle \prod_{j \neq i} |S_j^z = S\rangle
\]

e.g.

\[S = 1/2\]
Quantum treatment

• Hamiltonian

\[ H = -J \sum_{\langle ij \rangle} \left( S_i^z S_j^z + \frac{1}{2} S_i^+ S_j^- + \frac{1}{2} S_i^- S_j^+ \right) \equiv H_z + H_\pm \]

• zz terms

\[ H_z |i\rangle = \left( E_0 - Jz[S(S - 1) - S^2] \right) |i\rangle = \left( E_0 + JSz \right) |i\rangle \]
Quantum treatment

• +- terms

\[ H_{\pm} = -\frac{J}{2} \sum_{\langle jk \rangle} (S_j^+ S_k^- + S_j^- S_k^+) \]

\[ H_{\pm} |i\rangle = -\frac{J}{2} \sum_{\langle ji \rangle} S_j^- S_i^+ |i\rangle \]

\[ = -\frac{J}{2} 2S \sum_{\langle ji \rangle} |j\rangle = -JS \sum_{\mu} |i + \mu\rangle \]
Quantum treatment

- All together

\[ H|i\rangle = (E_0 + JSz)|i\rangle - JS\sum_{\mu}|i + \mu\rangle \]

- Looks like a hopping Hamiltonian

- Bloch:

\[ |k\rangle = \frac{1}{\sqrt{N}} \sum_i e^{ik \cdot r_i} |i\rangle \]

\[ H|k\rangle = \left( E_0 + JSz - JS\sum_{\mu} e^{ik \cdot e_\mu} \right) |k\rangle \]

\[ E_0 + \epsilon(k) \]
Quantum treatment

- These are “spin waves” or “magnons”

\[ \epsilon(k) = JSz - JS \sum_{\mu \lambda} e^{i k \cdot e_\mu} \]

\[ = 2JS \left( d - \sum_{\alpha=1}^{\lambda} \cos k_\alpha a \right) \]

\[ \approx JSa^2 k^2 \quad \text{quadratic for small } k \]

- Magnons are gapless
Magnons

- Magnons are gapless
- Energy vanishes as $k \to 0$
- This is because
  $$|k = 0\rangle \propto S^-_{TOT}|0\rangle$$
- This is an example of a “Goldstone mode”
- “Goldstone’s theorem”: if $H$ has a continuous symmetry that is “broken” by the ground state, there will be a gapless mode
Magnons

• Spin wave/magnon can be regarded as a quantized precession wave of slightly tilted spins

• $\varepsilon \sim k^2$ behavior, a general property for ferromagnets, can be understood this way
Magnons

• Think of effective field due to other spins
  \[ h(\mathbf{r}) \approx c_0 \mathbf{m} + c_1 \nabla^2 \mathbf{m} + \cdots \]

• Local spins precess in this field
  \[ \partial_t \mathbf{m} = h \times \mathbf{m} \approx c_1 \nabla^2 \mathbf{m} \times \mathbf{m} \]
Magnons

\[ \partial_t \mathbf{m} = \hbar \times \mathbf{m} \approx c_1 \nabla^2 \mathbf{m} \times \mathbf{m} \]

\[ \mathbf{m} = (m_x, m_y, \sqrt{m_0^2 - m_x^2 - m_y^2}) \approx (m_x, m_y, m_0) \]

\[ \partial_t \begin{pmatrix} m_x \\ m_y \end{pmatrix} = c_1 m_0 \begin{pmatrix} 0 & -k^2 \\ k^2 & 0 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \end{pmatrix} \]

\[ \omega = \pm c_1 m_0 k^2 \]
Neutron scattering

- Neutron has a $S=1/2$ similar to an electron, and its own dipole moment, which interacts with magnetic dipoles in materials

$$H_{d-d} = -\frac{\mu_0}{4\pi r^3} \left[ 3(m \cdot r)(m' \cdot r) - m \cdot m' \right]$$

- Consequently, a neutron can exchange energy and momentum with electronic spins

$$k = k_i - k_f, \omega = \omega_i - \omega_f$$
Neutron scattering

- Integrate over all energies: get total scattering
- Tool for detecting magnetic ordering in solids
- Resolve both momentum and energy of neutrons: measure spin waves

Example of spin waves in Yb$_2$Ti$_2$O$_7$ (2012)

First measurement in an insulating FM: CrBr$_3$, 1971
Antiferromagnets

• Actually it is much more common to have interactions that favor anti-aligned spins

\[ H = +|J| \sum_{\langle ij \rangle} S_i \cdot S_j \]

• This is trickier, because even classically spins take on different orientations

Néel state

“unfrustrated” lattice: can minimize all pairs of interactions simultaneously
Antiferromagnets

- Actually it is much more common to have interactions that favor anti-aligned spins
  \[ H = +|J| \sum_{\langle ij \rangle} S_i \cdot S_j \]
- This is trickier, because even classically spins take on different orientations

“frustrated” lattice: spins must compromise
Unfrustrated case

• For example, square lattice

\[ n = \langle S_i \rangle (-1)^{x_i+y_i} \]

• In MFT, find \textit{staggered magnetization} \(n\) behaves the same as \(m\) for a ferromagnet

• But quantum situation is different!
Unfrustrated case

• Guess the ground state?

\[ |\Psi\rangle = \prod_i |S^z_i = (-1)^{x_i+y_i} S\rangle \]

• Is it an eigenstate? NO!!

\[ H_{\pm} = \frac{|J|}{2} \sum_{\langle jk \rangle} (S^+_j S^-_k + S^-_j S^+_k) \]

\[ H_{\pm} = J + \ldots \]
Singlets

- For two spins, the ground state is actually a singlet
  \[ |s\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

- This can be viewed as a quantum superposition of two classical Néel states

- Consequently, in a real antiferromagnet, the magnetic order will be reduced or even removed by quantum fluctuations below that of MFT

- However, it turns out that most antiferromagnets still manage to order
A brief history of magnetism

~500BC: Ferromagnetism documented in Greece, India, used in China

1949AD: Antiferromagnetism proven experimentally

sinan, ~200BC
A brief history of magnetism

~500BC: Ferromagnetism documented in Greece, India, used in China

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Why so long???
A debate

\[ H = |J| \sum_{\langle ij \rangle} S_i \cdot S_j \]

Néel

\[ \text{antiferromagnet} \]

Landau

\[ \text{singlets} = \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) \]
The right tool...

- Neutron scattering

![Neutron diffraction patterns for MnO at room temperature and at 80°K.](image)

- Now we know antiferromagnetism is commonplace
Landau’s dream

- People are still looking for the “quantum disordered” antiferromagnet, with a singlet ground state instead of a Néel one