Magnetic order

• In a crystal with a periodic lattice of spins, exchange interactions typically induce an ordered state at low temperature

• For example, the ferromagnetic Heisenberg model:

\[ H = -J \sum_{\langle ij \rangle} S_i \cdot S_j \quad J > 0 \]

• Wants every spin parallel to its neighbor, so they choose a global axis

\[ \langle S_i \rangle = m \]
Mean field theory

- When $kT \gtrapprox J$, spins will fluctuate thermally, and $m$ will be reduced.

- We can study this with mean field theory

\[
H = -J \sum_{ij} \langle S_i \cdot S_j \rangle
\]

\[
\rightarrow -J \sum_{ij} \left[ \langle S_i \rangle \cdot S_j + S_i \cdot \langle S_j \rangle - \langle S_i \rangle \cdot \langle S_j \rangle \right]
\]

\[
= -zJ \sum_i m \cdot S_i + \text{const}.
\]
Mean field theory

This reduces the problem to independent spins in an effective “exchange field”

Note: this exchange field can be a thousand times larger than physical laboratory fields!
Mean field theory

- Define $h = z J m$ ($= g \mu_B H_{\text{eff}}$)
- Then we know for a single spin

$$|\langle S_i \rangle| = m = S B_S (\beta h S)$$

$$m = S B_S (\beta z J S m)$$

- For example for $S=1/2$

$$m = \frac{1}{2} \tanh \left[ \frac{z J m}{2kT} \right]$$
Mean field theory

- Non-zero solution for $m$ appears only for $T<T_c$

- Equality of slopes implies $kT_c = zJ/4 = \frac{zJS(S+1)}{3}$
Mean field theory

- Zero field magnetization:

- $T_c$ is called the “Curie point” or critical temperature
Susceptibility

• We may guess that the susceptibility gets large on approaching the Curie point, since the material almost forms a magnetization with no field at all.

• This is indeed true.

• Within MFT, just shift $h \rightarrow h + g \mu_B H$
Susceptibility

\[
m = \frac{1}{2} \tanh \left[ \frac{zJm + g\mu_B H}{2kT} \right] \approx \frac{zJm + g\mu_B H}{4kT} \]

\[
M/N = g\mu_B m \approx \frac{g\mu_B}{1 - \frac{zJ}{4kT}} \frac{g\mu_B H}{4kT} = \frac{(g\mu_B)^2 H}{4kT - zJ}
\]

\[
\chi = \frac{1}{N} \frac{\partial M}{\partial H} = \frac{A}{T - T_c}
\]

“Curie-Weiss law”

Curie law is modified by shift of \(T\) by mean field \(T_c\).
Phase transition

- A lot happens at $T_c$
- Both $m(T)$ and $\chi(T)$ are non-analytic at $T_c$
- This is actually a sign of a phase transition