2.1. \( F = m \vec{a} = (4 + 2t - 3t^2) \hat{\imath} \) N

\( m = 5 \text{ kg} \)

\( \vec{V}_o = 0 \)

\( \vec{r}_o = 0 \)

\( \vec{a} = (0.8t^2 \hat{\imath} - 0.6t \hat{j}) \frac{m}{s^2} \)

\( a = \int \vec{a}(t') \, dt' = \left( \frac{4t}{15} + \frac{3t^2}{10} - \frac{3t^3}{10} \right) \frac{m}{s} \)

\( \dot{\vec{V}} = \vec{V}_o + \int \vec{a}(t') \, dt' = \left( \frac{4t}{15} + \frac{3t^2}{10} - \frac{3t^3}{10} \right) \frac{m}{s} \)

\( \vec{r} = \vec{r}_o + \int \vec{V}(t') \, dt' = \left( \frac{1t^4}{15} + \frac{1t^3}{10} + 3t^2 \right) \frac{m}{s} \)

\( \vec{F} \times \vec{V} = (r_y V_z - r_z V_y) \hat{\imath} + (r_z V_x - r_x V_z) \hat{j} + (r_x V_y - r_y V_x) \hat{k} \)

\( = \left( -\frac{1}{50} t^6 + \frac{2}{75} t^5 \right) \hat{k} \frac{m^2}{s} = \frac{16}{150} \hat{k} \frac{m^2}{s} \)

2.2. \[ \begin{bmatrix} M_1 \ \\ M_2 \end{bmatrix} \]

\( M_2 g - T = M_2 \vec{a} \)

\( T = M_1 \vec{a} \)

\( M_2 g - M_1 \vec{a} = M_2 \vec{a} \)

\( \vec{V} = \frac{M_2 g}{M_1 + M_2} t \rightarrow x = \frac{1}{2} \frac{M_2 g t^2}{M_1 + M_2} \)

Check: \( \text{If } M_1 = M_2, \ x = \frac{1}{2} \frac{M_2 g t^2}{2M_2} = \frac{9t^2}{4} \sqrt{ \text{ checks out!} } \)

2.3. \[ \begin{bmatrix} M_1 \ \\ M_2 \end{bmatrix} \]

\( M_1 a = F - F' \)

\( M_2 a = F' \rightarrow a = \frac{F'}{M_2} \)

\( M_1 \frac{F'}{M_2} = F - F' \rightarrow (1 + \frac{M_1}{M_2}) F' = F \rightarrow F' = \frac{F}{1 + \frac{M_1}{M_2}} = \frac{M_2 F}{M_1 + M_2} \)

\( F' = \left( \frac{1 \text{ kg} \cdot 3 \text{ N}}{2 \text{ kg} + 1 \text{ kg}} \right) = 1 \text{ N} \)
\[ F_1 + F_2 = 0 \]
\[ F_1 = m\ddot{r}_i = m\left[ (\ddot{r}_i - r_i \dot{\theta}_i^2) \hat{r}_i + (r_i \ddot{\theta}_i + 2 \dot{r}_i \dot{\theta}_i) \hat{\theta}_i \right] \]
\[ = m[-r_i \omega^2] \hat{r}_i = -mw^2 r_i \hat{r}_i \]
\[ F_2 = M\ddot{r}_2 = -Mw^2 r_2 \hat{r}_2 \]
\[ F = |F_1| = |F_2| = mw^2 r_i = Mw^2 \Rightarrow r_2 = Mw^2 (R - r_i) \]
\[ r_i = \frac{E}{mw^2}, \quad R - r_i = \frac{E}{Mw^2} \Rightarrow \frac{R}{Mw^2 + mw^2} = \frac{E}{w^2 (m + M)} \]

2.5.

Define positive acceleration as \( M \) going down and \( m \) going up.

\[ F_m = ma = T - mg \Rightarrow T = m(a + g) \]
\[ F_m = Ma = Mg - T = Mg - ma - mg \]
\[ Ma + ma = Mg - mg \]
\[ a = \frac{M-m}{M+m} g \]

Check: If \( M = 2m \), \( a = \frac{2m-m}{2m+M} g = \frac{m}{3m} g = \frac{1}{3} g \) √

\[ T = m(a + g) = m\left[ \left( \frac{m}{M+m} \right) g + g \right] = m\left[ \frac{M-m}{M+m} g + \frac{M+m}{M+m} g \right] \]
\[ T = \frac{2Mmg}{M+m} \]

Check: If \( M = 2m \), \( T = \frac{4m^2 g}{3m} = \frac{4mg}{3} = \frac{2Mg}{3} \) √
2.6. Ingredients will stick to the wall if the gravitational force isn't higher than the centripetal force needed for circular motion. We don't know their masses, so we'll talk about acceleration instead.

\[ \vec{a}_c = (\vec{r} - \vec{r}_0) \hat{r} + (\dot{\vec{r}} + 2\vec{r}_0) \hat{r} = -RW^2 \hat{r} \]

So the ingredients stick if \( g \leq RW \), or \( w \geq \sqrt{g/R} \)

Check: \( g = 32 \text{ ft/s}^2 \), \( R = 2 \text{ ft} \), \( \omega_{\text{max}} = \sqrt{\frac{32(1/4)}{2(1/2)}} = 4 \frac{\text{ft}}{\text{s}} \)

2.7. a.

\[ F \]

\[ M_1: \]
\[ N_1 \]
\[ N_2 \]
\[ M_2: \]
\[ N_{1+M_2} \]
\[ \vec{F} \]

\[ M_1 = M_1 g \]
\[ N_2 = (M_1 + M_2) g \]

When we are not slipping, \( \alpha_1 = \alpha_2 \).

\[ M_1 \alpha_1 = F - MN_1 = F - \mu M_1 g \]
\[ M_2 a_2 = \mu N_1 = \mu M_1 g \]

\[ \frac{F - \mu M_1 g}{M_1} = \frac{\mu M_1 g}{M_2} \Rightarrow \frac{F}{M_1} = M_1 g \left(1 + \frac{M_1}{M_2}\right) \Rightarrow F = \mu M_1 g \left(1 + \frac{M_1}{M_2}\right) \]

b.

\[ F \]

\[ M_1: \]
\[ N_1 \]
\[ N_{1+M_2} \]
\[ M_2: \]
\[ N_2 \]
\[ \vec{N} \]

\[ M_1 = M_1 g \]
\[ N_2 = (M_1 + M_2) g \]

\[ \alpha_1 = \alpha_2 \text{, as before.} \]
\[ M_1 \alpha_1 = MN_1 = \mu M_1 g \Rightarrow \alpha_1 = Mg \]
\[ M_2 a_2 = F - MN_1 = F - \mu M_1 g \]

\[ \frac{F - \mu M_1 g}{M_2} = Mg \Rightarrow F - \mu M_1 g = M_2 Mg \Rightarrow [F = \mu (M_1 + M_2) g] \]
The table is smooth (frictionless).
The surface between the boxes is rough (coefficient of friction $\mu$).

**Force diagrams**

![Force diagrams]

$f$ is the force due to friction.

For $m_1$: $f = m_1 a$ \[\Rightarrow a = \frac{f}{m_1}\]

For $m_2$: $F - f = m_2 a$

$\therefore F - f = m_2 \frac{f}{m_1}$

$f \left( \frac{m_2}{m_1} + 1 \right) = F$

$f = \frac{F m_1}{m_1 + m_2}$
The two boxes move together as long as
\[ f \leq \mu s N_1 \]

The critical force for which the boxes slip is:
\[ f_c = \mu s m_1 g = \frac{F m_1}{m_1 + m_2} \]

\[ \Rightarrow \mu s = \frac{F}{(m_1 + m_2)g} \]

Now consider a force acting on the upper block.

Now the sum of the horizontal forces is:

For \( m_1 \):
\[ F' - f' = m_1 a' \]

For \( m_2 \):
\[ f' = m_2 a' \quad \Rightarrow a' = \frac{F}{m_2} \]

\[ \therefore F' - f' = m_1 a' = m_1 \frac{f'}{m_2} \]

\[ f' = \frac{F m_2}{m_1 + m_2} \]
The frictional force is still due to the normal force between the blocks ($N_1$). $N_2$ is the normal force of the table on block 2, but there is no friction on that surface.

\[ f' \leq u_s N_1 \]

\[ f'_c = u_s m_1 g = \frac{F'}{m_1 + m_2} \]

Now solve for $F'$

\[ F' = \frac{u_s m_1 g (m_1 + m_2)}{m_2} \]

\[ = \left( \frac{F}{(m_1 + m_2)g} \right) m_1 g \left( \frac{m_1 + m_2}{m_2} \right) \]

\[ = \left( \frac{m_1}{m_2} \right) F \]

\[ F' = \left( \frac{4 \text{ kg}}{5 \text{ kg}} \right) (27 \text{ N}) \]

\[ F' = 21.6 \text{ N} \]
K & K Problem 2.9

Given $v_0, g, \theta$

To find the radius of the circular path traversed by the particle, balance the forces in the $x$ and $y$ directions.

The particle does not move in the $y$-direction, so the net force is zero.

$$F_y = N_y - mg$$

$$\Rightarrow N_y = mg = N \sin \theta$$

$$\Rightarrow N = \frac{mg}{\sin \theta}$$

In the $x$-direction, the particle moves in a circle, so it has centripetal acceleration (see pg. 36e)

$$a_c = -r \omega^2 \hat{r}$$

$$= -r \left( \frac{v_0^2}{r^2} \right) \hat{r}$$

$$= -\frac{v_0^2}{r} \hat{r}$$
Choose a point in the x-y plane that the particle passes through (as in the diagram). The forces in the x-direction are

\[ F_x = -N_x = ma_x = -\frac{mv_0^2}{r} \]

Therefore,

\[ N \cos \theta = \frac{mv_0^2}{r} \]

\[ \frac{ma}{\sin \theta \cos \theta} = \frac{mv_0^2}{r} \]

Solving for \( r \) gives

\[ r = \frac{mv_0^2}{mg} \frac{\sin \theta}{\cos \theta} \]

\[ r = \frac{v_0^2}{g} \tan \theta \]
K & K Problem 2.10

The period of the orbit is 24 hr:

\[ \omega = \frac{2\pi}{T} = \frac{2\pi}{24 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \]

\[ = 7.3 \cdot 10^{-5} \text{ sec.} \]

The only force on the satellite is Earth's gravity.

\[ F_r = -\frac{G m M_E}{R_s^2} = ma_c = -mw^2 R_s \]

\[ \Rightarrow R_s^3 = \frac{G m M_E}{mw^2} = \frac{G M_E}{w^2} \]

The problem asks us to give the answer in terms of \( R_E \).

\[ R_s^3 = \left( \frac{G M_E}{R_E^2} \right) \frac{R_E^2}{w^2} \]

\[ = \frac{G R_E^2}{w^2} \cdot \frac{1}{w^2 R_E} R_s^3 \]

\[ R_s = \left[ \frac{(9.8 \text{ m/s}^2)}{\left(7.3 \cdot 10^{-5} \text{ sec}^3 \text{m} \right)^2 \left(6.1 \cdot 10^6 \text{ m} \right)^2} \right]^{\frac{1}{3}} R_E \]

\[ = 6.7 R_E \]
There are three forces. The weight due to gravity acts downwards, and the tension in each string acts along the direction of the string.

(b) The mass does not move in the y-direction, so \( F_y = 0 \).

\[
F_y = T_1 \cos 45^\circ - T_2 \cos 45^\circ - mg
\]

\[
= \frac{T_1}{\sqrt{2}} - \frac{T_2}{\sqrt{2}} - mg = 0
\]

\[\Rightarrow \frac{1}{\sqrt{2}} T_1 - \frac{1}{\sqrt{2}} T_2 = mg\]

In the x-direction, the mass moves in a circle so there is centripetal acceleration.

\[
F_x = m a_c = -m l \sin 45^\circ \omega^2
\]

\[
= -\frac{1}{\sqrt{2}} m l \omega^2
\]

where the radius of the circular motion is \( r = l \sin 45^\circ \).
Summing the forces

\[ F_x = -\frac{1}{\sqrt{2}} mlw^2 = -T_1 \sin 45^\circ - T_2 \sin 45^\circ \]

\[ = -\frac{1}{\sqrt{2}} T_1 - \frac{1}{\sqrt{2}} T_2 \]

\[ \Rightarrow \frac{1}{\sqrt{2}} mlw^2 = \frac{1}{\sqrt{2}} T_1 + \frac{1}{\sqrt{2}} T_2 \]  \( \text{(2)} \)

Add equations \((1) + (2)\)

\[ 2 \frac{1}{\sqrt{2}} T_1 = mg + \frac{1}{\sqrt{2}} mlw^2 \]

\[ T_1 = \frac{1}{\sqrt{2}} mg + \frac{1}{2} mlw^2 \]

Subtract equations \((2) - (1)\)

\[ 2 \frac{1}{\sqrt{2}} T_2 = \frac{1}{\sqrt{2}} mlw^2 - mg \]

\[ T_2 = \frac{1}{2} mlw^2 - \frac{1}{\sqrt{2}} mg \]
When the tablecloth is pulled under the glass, the glass will be accelerated due to the friction of the tablecloth. When the tablecloth is removed, the glass slows down due to the friction of the table.

Since the coefficients of friction for the tablecloth and the table, the distance traveled while speeding up is equal to the distance traveled slowing down. The two time intervals are also equal.

During the first time interval,

During the first time interval, the glass accelerates uniformly due to the force of friction; and it reaches its maximum velocity

\[ a = \frac{v_{\text{max}}}{t_1} = \frac{F_f}{m} \]

\[ \Rightarrow v_{\text{max}} = \frac{umg \cdot t_1}{m} = u1gt_1 \]
If the distance to the edge of the table is \( d \), then during the first interval the glass travels a distance \( \frac{d}{2} \):

\[
x = \frac{1}{2} a t_1^2 = \frac{d}{2}
\]

\[
\Rightarrow \quad t_1^2 = \frac{d}{a}
\]

and we get another expression for velocity

\[
v_{\text{max}} = at_1
\]

\[
v_{\text{max}}^2 = a^2 t_1^2 = a^2 \left( \frac{d}{a} \right) = ad
\]

\[
= \frac{F_c}{m} d = \frac{v_{\text{mg}}}{m} d
\]

\[
\Rightarrow \quad v_{\text{max}} = \sqrt{\frac{F_c d}{m}}
\]

Setting the two expressions equal

\[
v_{\text{max}} = \sqrt{\frac{F_c d}{m}} = \sqrt{\frac{m g t_1^2}{m}}
\]

\[
\Rightarrow \quad t_1 = \frac{\sqrt{F_c d}}{\sqrt{mg}} = \sqrt{\frac{d}{\mu g}}
\]

The total time is twice this:

\[
t = 2t_1 = 2 \sqrt{\frac{d}{\mu g}} = \sqrt{\frac{0.5 \text{ ft}}{0.5 \text{ (32 ft/s)}^2}} = \frac{1}{2\sqrt{2}} \text{ sec.}
\]
It helps to draw force diagrams.

Forces on $M_1$  
\[ T_1 \quad M_1g \]

Forces on $M_2$  
\[ T_2 \quad M_2g \]

Forces on Pulley 2  
\[ T_1 \quad T_2 \]

\[ \begin{align*}
1 \quad M_1a_1 &= T_1 - M_1g \\
2 \quad M_2a_2 &= T_2 - M_2g \\
3 \quad T_1 &= 2T_2
\end{align*} \]

Since $T_1 = 2T_2$, the force on $M_1$ is twice the force on $M_2$. Twice the force means twice the acceleration. (If $M_1$ rises, $M_2$ falls and vice versa.) Thus,

\[ 2a_1 = -a_2 \]

Plugging (3) into (2):

\[ M_2a_2 = \frac{1}{2}T_1 - M_2g \]

\[ M_2(-2a_1) = \frac{1}{2}T_1 - M_2g \]

\[ -4M_2a_1 = T_1 - 2M_2g \]  \(4\)
Subtract equations (1) - (4)

\[ M_1 a_1 + 4 M_2 a_1 = -M_1 g + 2 M_2 g \]

\[ a_1 = \frac{(2M_2 - M_1)}{(M_1 + 4M_2)} g \]