K&K Problem 4.1

If the box pushes against the track with a force \( mg \), the track pushes back with a normal force \( mg \).

\[
F_r = -w - F_N = -mg - mg = -2mg
\]

\[
= ma_c = -\frac{mv^2}{R}
\]

\[\Rightarrow v_{\text{top}}^2 = 2gR\]

Conservation of Energy

\[
(K.E. + P.E.)_{\text{initial}} = (K.E. + P.E.)_{\text{top of track}}
\]

\[
o + mgZ = \frac{1}{2}mv^2 + mg(2R)
\]

\[
mgZ = \frac{1}{2}m(2gR) + 2mgR
\]

\[
mgZ = mgR + 2mgR
\]

\[Z = 3R\]
K & K Problem 4.2

Let $v_f$ and $x_f$ be the velocity and position when it first is at rest. Thus $v_f = 0$.

The force is

$$F(x) = F_{\text{spring}} + F_{\text{friction}} = -kx - \frac{1}{M} \frac{d}{dx}$$

$$= -kx - b x M g$$

$$\Delta (\text{K.E.}) = \int_0^{x_f} F(x) \, dx$$

$$\frac{1}{2} M \frac{v_f^2}{x_f} - \frac{1}{2} M v_0^2 = \int_0^{x_f} -kx - b x M g$$

$$- \frac{1}{2} M v_0^2 = -\frac{1}{2} (k + b M g) x_f^2$$

$$x_f^2 = \frac{M v_0^2}{k + b M g}$$
Initially there is only kinetic energy

\[ E_0 = \frac{1}{2} Mv_0^2 \]

When it is at rest there is only potential

\[ E_f = \frac{1}{2} kx_f^2 \]

Thus

\[ E_f - E_0 = \frac{1}{2} \left( Mv_0^2 - \frac{kMv_0^2}{(k + bMg)} \right) \]

\[ = \frac{1}{2} Mv_0^2 \left( 1 - \frac{kM}{k + bMg} \right) \]

\[ = \frac{1}{2} Mv_0^2 \left( \frac{k + bMg}{k + bMg} \right) \]

\[ \Delta E = \frac{1}{2} \frac{bM^2v_0^2}{k + bMg} \]
K & K Problem 4.3

a) Conservation of momentum

\[ mV = (M+m)V' \]

\[ V' = \frac{m}{M+m}V \]

b) Conservation of energy

\( (K.E. + P.E.)_{\text{after collision}} = (K.E. + P.E.)_{\text{at angle } \phi} \)

\[ \frac{1}{2}(M+m)V'^2 + 0 = 0 + (M+m)gh \]

\( (v')^2 = 2gh \)

The height is

\[ h = l - l \cos \phi \]

\[ v' = V' \left( \frac{M+m}{m} \right) \]

\[ V = \frac{(M+m)}{m} \sqrt{2gl(1-\cos \phi)} \]
K&K Problem 4.4

Initially both blocks are at rest, so \( P_i = 0 \).
Call \( V \) the velocity of \( M \).

Conservation of momentum:

\[
P_i = P_f
\]

\[
0 = mv - MV
\]

\[\Rightarrow V = \frac{mv}{M}\]

Conservation of energy:

\[
E_i = E_f
\]

\[
mgr = \frac{1}{2}mv^2 + \frac{1}{2}MV^2
\]

\[
= \frac{1}{2}mv^2 + \frac{1}{2}M \left( \frac{mv}{M} \right)^2
\]

\[
= \frac{1}{2}mv^2 \left( 1 + \frac{m}{M} \right)
\]

\[\Rightarrow V = \sqrt{\frac{2gr}{\sqrt{1 + \frac{m}{M}}}}\]
Initially the mass has velocity $v_1$ at radius $l_1$. Then the rope is pulled with a force $F$ until it is moving at $v_2$ with radius $l_2$.

The acceleration is:

$$a = a_r \hat{r} + a_\theta \hat{\theta} = (\ddot{r} - r \dot{\theta}^2) \hat{r} + (\ddot{\theta} + 2r \dot{r} \dot{\theta}) \hat{\theta}$$

with $a_\theta = 0$. The force is:

$$\vec{F} = -ma_r \hat{r}$$

$$F_r = -m(\ddot{r} - r \dot{\theta}^2)$$

If the string is pulled really slowly then $\ddot{r} = 0$.

$$F_r = +mr \dot{\theta}^2$$

If we call $v_r$ the velocity at radius $r$, then

$$\omega = \frac{v_r}{r}$$

and

$$F_r = -m \frac{v_r^2}{r}$$
We also have
\[ F_\theta = m (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \]
\[ = m \frac{1}{r} \frac{d}{dt}(r^2 \dot{\theta}) = 0 \]

Thus, \( r^2 \dot{\theta} \) is constant in time.

\[ r^2 \omega = r v_r = b_1 v_1 = b_2 v_2 \]
\[ \Rightarrow v_r = \frac{b_1 v_1}{r} \quad v_2 = \frac{v_1 b_1}{b_2} \]

The force is \( F_r = -\frac{mv_1^2 b_1}{r^3} \)

The work is the integral of the force:
\[ W = \int_{b_1}^{b_2} \mathbf{F} \cdot d\mathbf{r} = \int_{b_1}^{b_2} F_r dr = -mv_1^2 b_1 \int_{b_1}^{b_2} \frac{1}{r^3} dr \]
\[ = -mv_1^2 b_1 \left[ -\frac{1}{2r^2} \right]_{b_1}^{b_2} \]
\[ = \frac{1}{2} m \left( \frac{v_1^2 b_1^2}{b_2^2} - \frac{1}{2}mv_1^2 \right) \]
\[ = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \Delta(K.E) \]
\[ h = R \cos \theta \]

\[ F_r = -F_{\text{grav}} + F_{\text{normal}} = ma \]
\[ = -mg \cos \theta + F_N = -\frac{mv^2}{R} \]

When the block loses contact with sphere, \( F_N = 0 \), thus
\[ mg \cos \theta = \frac{mv^2}{R} \]

Since there is no friction, energy is conserved.
\[ E_i = E_f \]
\[ mgR = mgR \cos \theta + \frac{1}{2}mv^2 \]

\[ 2mg(1 - \cos \theta) = \frac{mv^2}{R} \]

Using previous result
\[ 2mg(1 - \cos \theta) = mg \cos \theta \]

\[ 2 = 3 \cos \theta \]
\[ \cos \theta = \frac{2}{3} \]
The distance from the top is

\[ x = R - R \cos \theta \]

\[ = R \left( 1 - \frac{2}{3} \right) \]

\[ x = \frac{R}{3} \]
K&K Problem 4.7

The forces on the ring are gravity, tension from thread, and normal force from beads.

\[ F = T - Mg + 2F_N \cos \theta \]

The beads move symmetrically. The normal force \( F_N \) depends on the angle. Conservation of energy gives the velocity of each bead.

\[ mgR = mgR \cos \theta + \frac{1}{2} mv^2 \]

\[ v^2 = 2gR(1 - \cos \theta) \]

\[ \Rightarrow a_c = \frac{v^2}{R} = 2g(1 - \cos \theta) \]

\[ F_r = ma_c = -F_N + mg \cos \theta \]

\[ F_N = -ma_c + mg \cos \theta \]

\[ F_N = -2mg(1 - \cos \theta) + mg \cos \theta \]

\[ = -mg(2 - 3\cos \theta) \]
For the ring to move upward, the force switches from negative to positive, so 
\( F = 0 \) at the threshold point. The thread will go slack so \( T = 0 \) also. 
From the original equation for force

\[-Mg = +2F_N \cos \theta\]

\[-2mg \cos \theta (2-3 \cos \theta)\]

This is a quadratic equation in \( \cos \theta \)

\[0 = 6mg \cos^2 \theta - 4mg \cos \theta + Mg\]

\[\cos \theta = \frac{4mg \pm \sqrt{16m^2g^2 - 24mgMg}}{12mg}\]

\[= \frac{1}{3} \pm \frac{1}{3} \sqrt{1 - \frac{3M}{2m}}\]

For \( \cos \theta \) to be real \( 1 - \frac{3M}{2m} > 0 \), so

\[1 > \frac{3M}{2m}\]

\[m > \frac{3M}{2}\]

This occurs the first time \( \cos \theta \) reaches this value, so we take the positive root.

\[\theta = \cos^{-1} \left( \frac{1}{3} + \frac{1}{3} \sqrt{1 - \frac{3M}{2m}} \right)\]
K & K Problem 4.8

a) Let the amplitude for the first cycle be $A$.
   The work due to friction is force times distance.
   \[ W = -4Af \]

   Remember friction always opposes the motion.
   The work is also the change in Kinetic Energy.
   \[ W = \Delta K = \frac{k(A + dA)^2}{2} - \frac{kA^2}{2} \]

   \[ = \frac{kA^2}{2} + kAdA + \frac{k(dA)^2}{2} - \frac{kA^2}{2} \]

   Since $dA$ is very small, the second term can be ignored.
   \[ W = kAdA = -4Af \]

   \[ \Rightarrow dA = \frac{-4f}{k} \]

   The change in amplitude does not depend on $A$, so it will be the same for each cycle.

b) Each cycle, the amplitude decreases by $dA$.
   The number of cycles before coming to rest is:
   \[ n = \frac{x_0}{|dA|} = \left[ \frac{kx_0}{4f} \right] \]
K & K Problem 4.16

The work energy theorem

\[ P = \frac{dK}{dt} \]

\[ P_{\text{avg}} = \frac{\Delta K}{\Delta t} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \]

\[ v_0 = 0 \]

\[ v_f = \left(60 \text{ mi/h}\right) \left( \frac{1 \text{ hour}}{3600 \text{ sec}} \right) \left( \frac{1609 \text{ m}}{\text{mi}} \right) = 268.8 \text{ m/s} \]

\[ m = \left(1800 \text{ lbs.}\right) \left( 0.454 \frac{\text{kg}}{\text{lbs.}} \right) = 817 \text{ kg} \]

\[ P_{\text{avg}} = \frac{1}{2} \left( 817 \text{ kg} \right) \left( 268.8 \text{ m/s} \right)^2 \]

\[ \frac{\text{sec}}{8} \]

\[ P_{\text{avg}} = 3.67 \cdot 10^4 \text{ W} \]