Problem Set #1

Astro 2: Spring 2012

Solutions

Problem 1

(a) We assume that the Sun gives off light equally in all directions. The total luminosity through a sphere of 1 AU is then:

$$\begin{aligned}\mathcal{L} &= \text{Apparent Luminosity at Earth's} \times 4\pi(1\text{ AU})^2 \\ &= 1400\text{ J/s/m}^2 \times 4\pi(1.5\times10^{11}\text{ m})^2 \\ &= 3.96\times10^{26}\text{ J/s}\end{aligned} \tag{1}$$

(b) We simply use Wein's Law (pg. 107):

$$\lambda_{max} = \frac{0.0029\text{ K m}}{5800\text{ K}} = 500\text{ nm} \tag{2}$$

which is a wavelength corresponding to yellow light in the visible spectrum. (This is why the sun appears yellow.)

(c) Using the small-angle forumla (pg. 9) to get the diameter:

$$D_{sun} = \frac{(1'')(1.5\times10^{11}\text{m})}{206265} = 1.10\times10^9\text{ m} \tag{3}$$

Cutting this in half to get the radius, we end up with $R_{sun} = 6.5\times10^8$ m.

(d) Using information from previous parts of the question:

$$\begin{aligned}\mathcal{L} &= \sigma T^4 \cdot 4\pi R_{sun}^2 \\ &= (5.67\times10^{-8}\text{ W/m}^2/\text{K}^4)(5800\text{ K})^4 4\pi(6.5\times10^8\text{ m})^2 \\ &= 3.4\times10^{26}\text{ J/s}\end{aligned} \tag{4}$$

which is reasonably close to the answer obtained in part (a).

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Problem 2

using the small-angle formula (pg. 9), the distance between the center of Andromeda and a star 1" away from the center is:

$$\begin{aligned} R &= \frac{(1'')(600kpc)}{206265} \\ &= 2.9 \times 10^{-3} \text{ kpc} = 9 \times 10^{13} \text{ km} \end{aligned} \tag{5}$$

Now, since $F = ma$ and $a = v^2/r$ for a rotating body, we obtain the equation:

$$m\frac{v^2}{R} = \frac{GM_{BH}m}{R^2} \tag{6}$$

for a rotating body whose force is given by gravity. The mass of the body cancels out on both sides, and we solve for the mass of the black hole:

$$\begin{aligned} M_{BH} &= \frac{v^2 R}{G} = \frac{(110 \text{ km/s})^2 (9 \times 10^{13} \text{ km})}{6.67 \times 10^{-11} \text{ m}^3/\text{kg/s}^2} \\ &= 1.6 \times 10^{37} \text{ kg} = 8 \times 10^6 M_{sun} \end{aligned} \tag{7}$$

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Problem 3

(a) Our Sun is a particularly average main sequence star. In a bit more detail than required:

1. Gas and dust condense under gravitational attraction, eventually forming a **protostar** (Ch. 18). As the materials become more compact, temperature increases.
2. Nuclear fusion begins once the core temperature reaches 15 million Kelvin, and the star has finally become a **main sequence star**(Section 19-1). Hydrogen fuses to become helium, with a net loss of mass and a net gain in energy.
3. A star of 1 M_{sun} will run out of hydrogen after about 10^{10} years, leaving it with a helium core. Hydrogen fusion now begins outside the core and rapid expansion of the outer shell begins. The surface temperature decreases leading to a change in the color of the peak-emitted wavelength. We now have a **Red Giant**. (19-2)
4. As the nuclear fuel in the core gets used up, the outer layers break off and form a **planetary neubla** with a hot core remaining. This core, a **white dwarf**, slowly loses its remaining energy through blackbody radiation.

(b) As stated above, our Sun will eventually bome a white dwarf. Core temperatures will get hot enough to allow helium to fuse into carbon, and even small amounts of oxygen (but nothing heavier).

(c) Two key points:

- We assume stars in a cluster are all formed at about the same time.
- Their evolution is determined by mass, which we can determine.

By using the reference point of when stars use up their hydrogen and are no longer 'main-sequence' (and by knowing its mass), we can then estimate the age of that star, and thus the cluster that the star is in. (See problem 4 on HW2)

(d) The early stages of heavier stars are relatively similar to main-sequence stars, though they do go through their stages at a quicker rate. Such large stars become much larger than Red Giants; they become **supergiants** (20-5).

In such stars, fusion does not stop at carbon formation, but instead procedes all the up to the formation of iron. Once the core become dense iron, **photodisentrigration** (pg. 537) occurs, ultimately leading to a supernova.

Millions of years of stellar evolution are undone in a violent explosion. In some cases, the core may remain and become a neutron star or a blackhole (if greater than 25 solar masses).

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