Problem Set #2

Astro 2: Spring 2012

Due: April 17, 2012 (in class)

Problem 1 *Limits on Faintness of an Object that can be Detected by a Telescope*

In principle, if we can detect a single photon from a distant object then we can say that it exists, and we can tell the location on the Celestial Sphere. This turns out to be overly optimistic since the whole sky is brightened from light emission from Earth's atmosphere (except for the Hubble Space Telescope!), and from the Zodiacal Light, and many other interfering factors. These other sources of light make it a lot harder to detect faint objects. As an extremely crude way of taking these other

sources of noise into account, let's say that in order to detect a faint object, we must detect at least 100 photons from it in a single hour. It really depends very greatly on the wavelength of the observation, as well as on many other factors. Nevertheless, I'd like you to assume that we need to count 100 photons from a faint object, in order to be sure we've detected something. Now suppose you are using one of the two big

Keck telescope on the Mauna Kea volcano in Hawaii. The diameter of the mirror is 10 m.

(a) How many photons would the Keck telescope detect if it were looking at the Sun, and you are taking a 1-hour exposure? Assume that 20% of the photons striking the mirror are actually registered by the little imaging chip (similar to the one in digital cameras)? (Answer: $\approx 2 \times 10^{26}$ photons)

(b) Suppose a giant moved the Sun farther and farther away from Earth until if could barely be detected by Keck. That is, suppose that we can collect and register only 100 photons in a one hour exposure. At what distance would we lose sight of the Sun? (Answer: ≈ 7 Mpc)

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Problem 2 *Limits on Angular Resolution of Telescopes*

Ground-based optical telescopes have an angular resolution on the order of 1” (one arcsecond). That means basically that if two stars are 1” apart on the celestial sphere, you can tell there are two stars, but if they are 0” apart, they look pretty much like one star. The limit of 1” is due to blurring (‘seeing’) by Earth’s atmosphere, and getting above the atmosphere to get sharper images was a main motivation behind the Hubble Space Telescope. It’s the reason pictures from that telescope are so awesomely sharp and detailed!

With no atmosphere, and assuming perfect equipment, the angular resolution is set by diffraction, which is ultimately a manifestation of the Heisenberg Uncertainty Principle. The resolution R in radians comes out to be

$$R = \frac{\text{wavelength}}{\text{diameter of telescope mirror}} \tag{1}$$

For an array of telescopes working together as an ‘interferometer,’ we instead have:

$$R = \frac{\text{wavelength}}{\text{size of the array}} \tag{2}$$

(a) Look in the mirror and estimate the diameter of your pupil. Use this to calculate the angular resolution of your eye. (Answer: $\approx 1'$)

(b) How big must a dark ‘sea’ (lava flow) be on the moon, in order for you to see it? Discuss in a sentence or two the relation of your angular resolution to the amount of detail you can see on the full moon. (Answer: $\approx$ 300 km)

(c) From what distance can you read a highway sign, assuming your eyes have good focus and are just limited in sharpness by diffraction? Is this consistent with your driving experience? You have to estimate various quantities here. You won’t be graded on your estimates, but last time I asked this, many students made crazy guesses, so please give yur guesses a little thought. (Answer: Somewhere between 10^2 and 10^3 m)

(d) What is the angular resolution of the Hubble Space Telescope (2.4 m diameter), operating at a wavelength of 500nm, in the visual region? (Answer: $\approx$ 0.05”) How about when it observes light of wavelength 2000 nm = 2 microns? (Answer: $\approx$ 0.2”)

The angular resolution of the Keck 10m telescopes would be something like an arcsec both at 500nm and 2 microns without special technology to cancel the effects of the atmosphere. This technology is in the developmental stage, but it works pretty well already at 2 microns (but not very well at all at optical wavlengths, which are less than 1 micron).

(e) What is the angular resolution of the Keck 10m telescopes at 2 microns observing wavelength? (Answer: ≈ 0.04")

(f) What is the angular resolution of the 'Very Large Array' radio telescope array, in southern New Mexico? (The atmosphere doesn't blur out radio waves as it does optical waves.) The antennas can be spread out in several different ways, and together cover up to 30km. Assume you are observing radio waves of wavelength $\lambda = 20$cm. (Answer: ≈ 1.38")

(g) Often radio telescopes around the world work together to make images. (e.g. http://www.vlba.nrao.edu/) What is the sharpest resolution one can get with an a array of ground-based radio telescopes? (Answer: ≈ 0.003")

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Problem 3 *Mass to Energy Conversions*

(a) How much energy is produced when the sun converts 1 kg of mass into energy? (Answer: 9×10^{16}J)

(b) How much energy is produced when the Sun converts 1 kg of hydrogen into helium? (Answer: 6.3×10^{14}J)

(c) The atomic bombs the U.S. dropped on Japan produced around 20 'kilo-Tons' of TNT equivalent, or 2×10^{15}J of explosive energy. How much matter was converted to energy inside the bomb? (Answer: ≈ 0.022 kg.)

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Problem 4 *Star Cluster Dating*

If stars at the main sequence turnoff on the HR diagrams of a certain cluster have mass of $3M_{sun}$, estimate the age of the cluster. Use the luminosity and the mass burned on the main sequence to obtain this estimation. (Answer: $\approx$ 600 million years)

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Problem 5 *White Dwarves on HR Diagram*

As white dwarf stars cool, they maintain constant radius because of electron degeneracy pressure. Which direction do they move in the HR diagram as they cool? Why?

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Problem 6 *The Crab Nebula*

Using the Doppler Effect to determine the expansion speed of the Crab Nebula supernova remnant, we get an answer of 1400 (km/sec). This is the expansion velocity in the radial direction (i.e., the direction toward Earth). Repeated photographs show expansion with an angular speed of 2.2×10^{-14} radians per second in the direction perpendicular to the line of sight.

(a) How far away is the Crab Nebula? (Answer: $\approx$ 2 kpc.) Hint: Assume that the expansion speed is the same in both directions.

(b) Photographs also show that the angular size of the nebula is 6×10^{-4} radians today. Estimate the time since the supernova occurred. The supernova explosion was seen from Earth. Does the date of the explosion agree with your estimate?

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Problem 7 *Surface Temperature and the Implications of Global Warming*

(a) If Earth had no atmosphere, what would the average surface temperature be? Hint: The sunlight power striking Earth is the apparent brightness of the Sun at Earth $\times \pi R_{Earth}^2$. Also, Earth's albedo is 0.31. (Answer: $\approx$ 255K)

(b) The natural average temperature is about 300 K, which differes from the above value due to greenhouse gasses in the atmosphere. Naturally, carbon dioxide contributes to about 10% of the greenhouse effect, and H_2O contributes most of the rest. However, recent human activities have caused CO_2 concentrations to increase artificially from 280 ppm to $\approx 400ppm$ Make crude estimates of how many degrees Earth's surface will eventually warm if the amount of CO_2 stays at its present value. Hint: If natural CO_2 levels are responsible for a temperature increase of x degrees, then assume doubling CO_2 levels will instead cause an increase of $2x$ degrees. (Answer: $\approx$ 6 K)

More detailed calculations indicated that the rise is less, but your answer will be in the general ballpark. Incidentally, the CO_2 in our atmosphere is not staying the same, but rising rapidly. (see Fig. 9-29)

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