# Problem Set #2

Astro 2: Spring 2012

Solutions

## Problem 1 *Limits on Faintness of an Object that can be Detected by a Telescope*

(a) Let us assume that all of the light coming from the sun is yellow (500 nm). The energy of each photon is then given by:

$$E_\gamma = \frac{hc}{500 \text{ nm}} = 4 \times 10^{-19} \text{ J} \tag{1}$$

Using this, the total number of photons detected in a single hour is:

$$\begin{aligned} N_\gamma &= \frac{\text{Energy Flux}}{\text{Energy per Photon}}(\text{Efficiency})(\text{Area of Lens})(\text{Time}) && (2)\\ &= \frac{1400 \text{ J/m}^2\text{/s}}{4 \times 10^{-19} \text{ J per photon}}(0.2)(\pi(5 \text{ m})^2)(3600 \text{ sec}) && (3)\\ &= 2 \times 10^{26} \text{ photons} && (4) \end{aligned}$$

(b) Apparent luminosity (or flux) varies as $\frac{1}{R^2}$. So, to reduce our photon counts by a factor of $2 \times 10^{24}$, we need to go a factor $\sqrt{2 \times 10^{24}} = 1.4 \times 10^{12}$ farther away. So, we will measure 100 photons in a given hour if the sun is moved to a distance

$$R_{\gamma=100} = 1.4 \times 10^{12} \text{ AU} = 2.1 \times 10^{23} \text{ m} = 6.8 Mpc \tag{5}$$

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## Problem 2 *Limits on Angular Resolution of Telescopes*

(a) The diameter of a human pupil is approximately 2 mm. Assuming we are looking at light with wavelength 500 nm, we then get a resolution of about:

$$R = \frac{500 \text{ nm}}{2 \text{ mm}} = 0.00025 \text{ radians} = 0.86' \tag{6}$$

(b) The moon is approximately 384,000 km away. Thus we can distinguish two points on the moon if they are $0.00025 \times 384{,}000$ km $= 96$ km away. To be able to resolve an actual body, it would be good to have at least 3 'pixels' worth of information. So 300 km should be large enough for us to be able to resolve the 'dark sea.'

(c) Let us assume that the letters on a stop sign are approximately 25 cm tall, and we need at least 5 pixels to resolve the lettering. As such, we need a spatial resolution (pixel size) of about 5 cm.

$$d = \frac{D}{R} = \frac{5 \text{ cm}}{0.00025} = 200 \text{ m} \tag{7}$$

so in perfect conditions and with 20/20 sight, you should be able to resolve a STOP sign at about the length of two football fields.

(d) Using the information provided:

$$R_{500nm} = \frac{500 \text{ nm}}{2.4 \text{ m}} = 2.1 \times 10^{-7} \text{ radians} = 0.043'' \tag{8}$$

$$R_{2\mu m} = \frac{2\ \mu\text{m}}{2.4 \text{ m}} = 8.4 \times 10^{-7} \text{ radians} = 0.17'' \tag{9}$$

(e) Using the information provided:

$$R_{2\mu m} = \frac{2\ \mu\text{m}}{10 \text{ m}} = 2.0 \times 10^{-7} \text{ radians} = 0.04'' \tag{10}$$

(f) Using the information provided:

$$R_{20cm} = \frac{20 \text{ cm}}{30 \text{ km}} = 6.7 \times 10^{-6} \text{ radians} = 1.37'' \tag{11}$$

(g) The optimal case would involve using two radio telescopes at the poles, with the diameter of Earth between them. According to Google, $D_{Earth} = 12756$ km, which gives us:

$$R_{max} = \frac{20 \text{ cm}}{12,756 \text{ km}} = 1.6 \times 10^{-8} \text{ radians} = 0.003'' \tag{12}$$

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## Problem 3 *Mass to Energy Conversions*

(a) Using Einstein's famous mass-energy equation:

$$E_{1kg} = mc^2 = (1kg)(3 \times 10^8 \text{ m/s})^2 = 9.0 \times 10^{16} \text{ J} \tag{13}$$

(b) Since only 0.7% of the total mass is converted into energy during hydrogen fusion:

$$E_{4H \to He} = 0.007 E_{1kg} = 6.3 \times 10^{14} \text{ J} \tag{14}$$

(c) Inverting the energy-mass equation, we get

$$m = \frac{E}{c^2} = \frac{2 \times 10^{15} \text{ J}}{3 \times 10^8 \text{ m/s}} = 0.022 \text{ kg} \tag{15}$$

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## Problem 4 *Star Cluster Dating*

The luminosity of a $3M_{sun}$ star is 60 $L_{sun}$ (Table 19-1, pg. 500). Let us assume that 10% of the hydrogen in the star is burned during the main sequence phase. This corresponds to f = 0.1 × 0.007 = 0.0007 in Box 19-2 (pg. 501). So

$$t = \frac{fMc^2}{L} = \frac{0.0007(6 \times 10^{30} \text{ kg})(3 \times 10^8 \text{ m/s})^2}{2.3 \times 10^{28} \text{ J/s}} \tag{16}$$

$$= 1.61 \times 10^{16} \text{ sec} = 5 \times 10^8 \text{ years.} \tag{17}$$

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## Problem 5 *White Dwarves on HR Diagram*

Since the size stays constant, a decrease in surface temperature will correspond to a decrease in luminosity. This corresponds to going down and to the right on the HR diagram.

On the other hand, Red Giants get bigger as surface temperature goes down. While each surface area element radiates less light, the overall increase in surface area leads to an overall increase in luminosity for the star as a whole.

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## Problem 6 *The Crab Nebula*

(a) Assuming homogenous expansion, in one second the nebula will expand 1400 km, which to us will appear to be $2.2 \times 10^{-14}$ radians. Thus:

$$d = \frac{D}{\theta} = \frac{1400 \text{ km}}{2.2 \times 10^{-14} \text{ radians}} \tag{18}$$
$$= 6.63 \times 10^{16} \text{ km} = 2059 \text{ pc} \tag{19}$$

would correspond to the distance from Earth.

(b) The Crab Nebula formed as a result of *SN 1054*, which was famously witnessed around the world in 1054 B.C.E. Taking the size of the nebula and dividing it by the rate of expansion, we estimate the age of the nebula to be:

$$t = \frac{6 \times 10^{-4} \text{ radians}}{2.2 \times 10^{-14} \text{ radians/sec}} = 864 \text{ years} \tag{20}$$

which is reasonably close to the time passed since the generating supernova was witnessed.

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## Problem 7 *Surface Temperature and the Implications of Global Warming*

(a) We assume that the Earth is at equilibrium with respect to the power it absorbs and the power it radiates. Since the Earth reflects $\alpha = 0.31$ of the light that hits it,

$$\text{Total power absorbed} = \text{Total power radiated} \tag{21}$$
$$F_e \pi (1-\alpha) R_{Earth}^2 = \sigma T^4 4\pi R_{Earth}^2 \tag{22}$$

where we note that $R_{Earth}$ will cancel out of the equation. Hence the temperature of the planet is (approximately) independent of size. Moving along we solve for T:

$$T = \left(\frac{1-\alpha}{4\sigma} F_e\right)^{1/4} \tag{23}$$
$$= \left(\frac{1-0.31}{4(5.67 \times 10^{-8} \text{ W/m}^{-2}/\text{K}^{-4})}(1400 \text{ W/m}^{-2})\right)^{1/4} \tag{24}$$
$$= 255 \text{ K} \tag{25}$$

(b) Apparently, greenhouse gasses cause an increase in temperature of 45 K. Noting the hint, the increase in temperature caused by the increased $CO_2$ level is:

$$\Delta T_{CO_2} = 0.1 \frac{400 \text{ ppm}}{280 \text{ ppm}} (45 \text{ K}) = 6.4 \text{ K} \tag{26}$$

Subtracting this from the original 4.5 K caused by natural amounts of $CO_2$, we find that the increase from human emissions is approximately 2 K.

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