Problem Set #3

Astro 2: Spring 2012

Solutions

Problem 1 *The Twin Paradox*

We already know your twin has aged 50,000 years since he has been presumably stuck on Earth the entire time, with no time dilation. However, you were traveling fast enough such that time was slowed down by a factor of 1000. As such, our time passed is simply:

$$t = \frac{50,000 \text{ years}}{1000} = 50 \text{ years} \tag{1}$$

These numbers do not agree because the process of turning the ship around breaks the inertial reference frame. There are 2 ways this can be done:

1. The ship decelerated, turned around, then accelerated back up to speed. On the ship, acceleration and deceleration is felt just like when you are stopping or speeding up at a traffic light. As such, that reference frame knows it is undergoing intense acceleration and thus knows time is slowing down for it. Just like with intense gravitational fields, acceleration and deceleration come with a slowing down of time (equivalence).

2. The ship uses the gravitational field of the SMBH to 'slingshot' itself around in the other direction, back at Earth. Here, the ship undergoes intense time dilation caused by the black hole, which breaks the symmetry of the two reference frames.

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Problem 2 *Classical vs. Modern Theories of Gravity*

General relativity fixes this problem by having gravitational effects propogate along spacetime rather than masses directly interacting with each other. As such, masses interact with the time-lagged 'wake' of other masses at its own local position, instead of instaneously 'at a distance.'

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Problem 3 *The Flatlander and You*

(a) Although this is merely an interpretive question and thus anything reasonable put down is correct, it is generally accepted that the presence of mass does not cause bulging into a third spatial dimension. Instead, that bulge is merely a representation for curvature in spacetime which leads to modified trajectories. The bulge is used instead to intuitively indicate to one how the trajectories are modified, playing off our experiences with rolling balls on non-flat surfaces.

(b) To rule out direct interactions, we must use the trajectories of massless and chargless particles to detect curvature. We can do this by using the curvature of some middle object to see multiple images of some further-away object. Here the middle object has bent light rays, originally going around two different sides of the planet, back toward each other, creating the impression that the further away object is on both sides of the middle object.

This does work for 3D. See: http://en.wikipedia.org/wiki/Einstein_Cross.

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Problem 4 *On the Search for Intelligent, Hopefully Nonviolent, Life*

Let us assume that our current technological level has an age of *approx*100 years. We can say it started with the invention of the computer in the middle of the 20^{th} century, and will go on for another 50 years or so, at which point we will enter a new age. (For the doubters, look up: quantum levitation, wireless energy transfer, and graphene- technologies all of which are expected to ripen in the next 50 years. Also robots!)

Now, consider that humans first appeared on this planet approximately 3 million years ago, and could potentially live for millions of more years if we don't manage to annihilate ourselves one way or another. In any case, if we assume that we discover an intelligent alience race at a <u>random</u> point in their evolution, an upper bound for the estimate that they would be in the same technological age is:

$$P = \frac{100 \text{ years}}{3 \times 10^6 \text{ years}} = 0.000033 = 0.0033\% \tag{2}$$

This is a VERY rough estimate, but one should not expect anything greater than this.

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Problem 5 *The Photon Sphere*

(a) The event horizon corresponds to the point at which completely outward pointing light can no longer escape. If light had any tangential component, it would spiral into the black hole. As such, one would expect orbits to need to be farther away than the black hole.

This video may be worth watching:
http://www.youtube.com/watch?v=5feVWB1SY-Y

(b) The orbits for photons moving in the direction of black holes moves closer due to the 'boost' from moving space time. Similarly, the stable orbits for photons moving against the rotation will move farther away.

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