Problem Set #4

Astro 2: Spring 2012

Solutions

Problem 1

The equivalence principle is what allows us to jump from special relativity to general relativity. It says that we cannot distinguish (in a closed space ship) between:

1. an accelerating reference frame (coordinate system), no other forces present
2. a non-accelerating reference frame in the presence of a gravitational field

Using this, it is possible to deduce that light must bend in a gravitational field, as well as well as gravitational red and blue shifts.

See this video here: http://www.youtube.com/watch?v=lNiKH0BDnZw

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Problem 2

Instead light energy being distributed evenly across the shell of a sphere, in Flatland it is distributed evenly across the circumference of a circle (see Fig. 1). Thus, the flux in 2D is given by:

$$F_{2D} = \frac{\mathcal{L}}{2\pi d} \quad (1)$$

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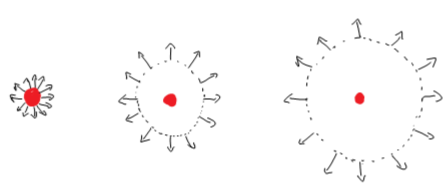
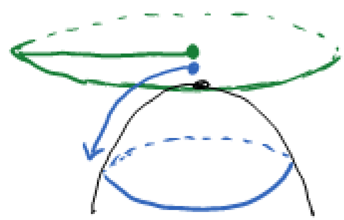


Figure 1: As the light (arrows) move farther away from the source, in 2D they distribute themselves on the outer boundary of a circle. This leads to the brightness in 2D being given by Eq. 1.

Problem 3

As discussed in the previous problem, the energy from a point source on 2D distributes itself over the outer boundary of a circle. As shown here



the circle is smaller after moving some distance down the sphere (blue), in comparison to when moving away on flat space (green). Since the circle is smaller, that means the density of light would be greater there, and thus would correspond to a greater flux.

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Problem 4

The Eddington Limit is given on pg. 682, and simply requires plugging in $M = 60M_{sun}$

$$\begin{aligned} L_{Edd} &= 30000\left(\frac{60M_{sun}}{M_{sun}}\right)L_{sun} \\ &= 1.8\times 10^6 L_{sun} = 6.9\times 10^{32} J/s \end{aligned} \tag{2}$$

The more massive a star, the more intense nuclear fusion is at its core. The increased rate of nuclear fusion leads to greater outer pressure. If the mass of the star is too great, then the outward pressure is too great and the star blows off outer layers, until the mass/pressure is reduced to the Eddington Limit.

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Problem 5

The radius of each disk is given by:

$$R = 10R_{Sch} = 20\frac{GM}{c^2} \tag{3}$$

Since the luminosity per square meter of the accretion disk is given by σT^4, the total luminosity is given by:

$$\begin{aligned} \mathcal{L} &= \sigma T^4 A_{Disk} = \sigma T^4 (2\pi R^2) = \sigma T^4 (2\pi \left(20\frac{GM}{c^2}\right)^2) \\ &= 800\pi\ \sigma T^4 \left(\frac{GM}{c^2}\right)^2 \end{aligned} \tag{4}$$

If each black hole is radiating at the Eddington Limit, $\mathcal{L} = \mathcal{L}_{Edd}$, and we can solve for the temperature:

$$T = \left(\frac{c^4 \mathcal{L}_{Edd}}{800\pi\ \sigma G^2 M^2}\right)^{1/4} \tag{5}$$

(a) Using the process of Problem 4 (Eq. 2), we find that the Eddington Limit of a $10M_{sun}$ blackhole is $\mathcal{L}_{Edd} = 1.17 \times 10^{32}$ J/s. Plugging this into Eq. 5 with $M = 10_{Sun}$, we get:

$$\begin{aligned} T &= \left(\frac{c^4 (1.17 \times 10^{32}\ \text{J/s})}{800\pi\ \sigma G^2 (10M_{sun})^2}\right)^{1/4} \\ &= 7.8 \times 10^6\ \text{K} \end{aligned} \tag{6}$$

(b) We follow the same steps as in (a). The Eddington Limit for a $10^9 M_{Sun}$ object is $\mathcal{L}_{Edd} = 1.17 \times 10^{40}$ J/s. This leads to:

$$\begin{aligned} T &= \left(\frac{c^4 (1.17 \times 10^{40}\ \text{J/s})}{800\pi\ \sigma G^2 (10^9 M_{sun})^2}\right)^{1/4} \\ &= 7.8 \times 10^4\ \text{K} \end{aligned} \tag{7}$$

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