Problem Set #5

Astro 2: Spring 2012

Solutions

Problem 1

MAssive Compact Halo Objects (MACHO) are a type of dark matter. They are astronomical bodies which emit very little to no light, yet have enough mass to create nontrivial gravitational effects around them. If a MACHO passes in front of a star that we are observing, the MACHO's gravity will bend more of the light toward us, making the star appear temporarily brighter. This is known as 'gravitational microlensing.'

The MACHO Project specifically looked for microlensing events in front of the Large Magellanic Cloud (LMC). They hoped to show that a significant fraction of the dark matter in the halo of the Milky Way is made up of MACHOs.

In the end, they determined that MACHOs with mass $\approx 5M_{sun}$ make up in total 10-20% of the dark matter in our own Milky Way.

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Problem 2

(a) Spiral Galaxies:

1. Most spiral galaxies show flat rotation curves (Fig. 23-18, pg. 619), which suggests the presence of considerably more total mass than luminous mass.
2. MACHO-related gravitational lensing. (see previous problem)

(b) Elliptical Galaxies

1. The gravitational lensing for some elliptical galaxies is too strong to be totally explained by the luminous mass.
2. There is a relation between the total mass of an elliptical galaxy and the velocity distribution of stars in it. A wider distribution is measured for more massive galaxies, and so when elliptical galaxies have distributions wider than the amount that the luminous mass would indicate, we can assume the presence of dark matter.

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Problem 3

With only the apparent brightness, we do *not* have enough energy to extrapolate all the way to a distance. If we had the absolute brightness, we could, but such information was not available for various reasons (see the link provided with the original question.)
All was not lost without this information however, as the conversion between apparent and absolute brightness needed a single data point to fix a conversion between the two. This was eventually found.

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Problem 4

For a given luminosity, the relation between flux and distance is given by $F = \mathcal{L}/(4\pi d^2)$. Thus, for a star to have measured flux F_0 or greater, it must be within a distance:

$$d_{min} = \left(\frac{\mathcal{L}}{4\pi F_0}\right)^{1/2} \tag{1}$$

Since we assume a uniform distribution of stars, the number of stars within a given distance is:

$$N_* = \rho_* \frac{4}{3}\pi d^3 \tag{2}$$

where ρ_* is density of stars (assumed to be the same everywhere). Combining the two previous equations, we find that the number of stars with measured flux F_0 or greater is obtained by plugging Eq. 1 into Eq. 2, giving us:

$$N_* = \rho_* \frac{4}{3}\pi \left(\frac{\mathcal{L}}{4\pi F_0}\right)^{3/2} \propto F_0^{-3/2} \tag{3}$$

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Problem 5

If we assume that the spiral arms of our galaxy are assemblages of stars and interstellar matter that travel around the Galaxy together, then the spirals should blend together (Fig. 23-31 in textbook). However, this is not what is observed.

It is resolved with the density-wave model, where the spirals are understood as a bunching together of stars, a celestial traffic jam (Fig. 23-22). Instead of being made of the same stars for an entire rotation around the center of the galaxy, the spiral arms consists of material passing through a dense region, triggering star formation.

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Problem 6

Population I	Population II
luminous, hot, young contains heavier elements located primarily in disks of spiral galaxies	older, colder, and less bright fewer heavy elements found near the center and in the halo of spiral galaxies, and are much more common in elliptical galaxies

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Problem 7

The air pressure at sea level is 1 atm $= 1.01 \times 10^5$ N/m^2 where the unit N/m^2 is commonly called a Pascal. Noting the original units, this means that the air exerts 1.01×10^5 N of force on 1 m^2 of Earth. If we assume that this comes from the Earth pulling down on the aforementioned column of air, then we get the equation:

$$\frac{GM_E\, m_{air}}{R_E^2} = P_{air}(1 \text{ m}^2) \tag{4}$$

where m_{air} is the mass of the column of air. We then solve for the mass of the Earth and get:

$$\begin{aligned} M_E &= P_{air}(1 \text{ m}^2)\frac{R_E^2}{Gm_{air}} = (1.01 \times 10^5 \text{ N})\frac{(6.3 \times 10^6 \text{ m})^2}{(6.67 \times 10^{-11} \text{ N (m/kg)}^2)(1.03 \times 10^4 \text{ kg})} \\ &= 6.0 \times 10^{24} \text{ kg} \end{aligned} \tag{5}$$

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