Problem Set #7

Astro 2: Spring 2012

Solutions

Problem 1

(a) From the redshift alone we can only determine how fast a light source is moving away from us. Hubble's constant relates that speed, the recession speed, to the distance from the observer. Thus to find Hubble's constant, we need to know both distance as well as the speed determined from redshift.

(b) From the graph, you should get on the order of $10^4\ \mathcal{L}_{sun} \approx 4 \times 10^{30}$ J/s .

(c) Rearranging the equation from the inverse square law, we get:

$$d = \sqrt{\frac{\mathcal{L}}{4\pi F}} = \sqrt{\frac{4 \times 10^{30}\ \text{J/s}}{4\pi(3 \times 10^{-20}\ (\text{J/s})/\text{m}^2)}} \approx 100 Mpc \tag{1}$$

(d) From the redshift, we find that:

$$v = \frac{\Delta\lambda}{\lambda} c = 0.024(3.0 \times 10^5\ \text{km/s}) \approx 7.2 \times 10^4\ \text{km/s} \tag{2}$$

(e) Since an object 1 Mpc away is travelling away from us at approximately 70 km/s, then the number of years since we were at the same point is:

$$t = \frac{1\ \text{Mpc}}{70\ \text{km/s}} = 4.4 \times 10^{17}\ \text{sec} = 13.9\ \text{billion years} \tag{3}$$

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Problem 2

(a) Reusing Eq. 2 from the last problem:

$$v = 0.056c = 1.7 \times 10^4 \text{km/sec} \tag{4}$$

(b) Using Hubble's Law:

$$d = \frac{v}{H} = \frac{1.7 \times 10^4 \text{km/sec}}{70 \text{ (km/s)/Mpc}} \approx 240 \text{ Mpc}| \tag{5}$$

(c) Rearranging the inverse square law to solve for luminosity:

$$\mathcal{L} = 4\pi d^2 F = 4\pi (240 \text{ Mpc})^2 (10^{-14} \text{ (J/s)/m}^2) = 6 \times 10^{36} \text{ J/s} \tag{6}$$

(d) Finally, we use the small angle formula to estimate the size of the object:

$$D = d \, \frac{\theta}{206265} = 240 \text{ Mpc} \, \frac{120 \text{ arcsec}}{206265} = 0.14 \text{ Mpc} = 140 \text{ kpc} \tag{7}$$

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Problem 3

Due to the Pauli Exclusion Principle, no two fermions (such as neutrinos) can occupy the same state. If we assume that the neutrino momentum cannot exceed a certain value (see next problem), then that implies there is a finite number of neutrinos in our galaxy. After adding up the masses of all possible neutrino states, we do not calculate a sufficient enough mass to account for the unexplained dark matter.

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Problem 4

Since the WIMPs are noninteracting, there is no reason to assume that they would travel at the same velocity of the baryonic matter around it. The lack of interaction implies they would just pass right through objects in their way, rather than bump into them and slow down. The only real limit on the neutrino velocity is that it cannot exceed the escape velocity for the Milky Way. If it did, the neutrino would just fly off to infinity. Using values obtained from Wikipedia, and the equation for escape velocity:

$$v_e = \sqrt{2\frac{GM_{MWG}}{R_{Halo}}} = \sqrt{2\frac{(6.67 \times 10{-}11 \text{ N (m/kg)}^2)(10^{12} M_{sun})}{10^5 \text{ light years}}} \approx 5 \times 10^5 \text{ m/s} \tag{8}$$

Note that the numbers used in this approximation are very rough, such as using the whole mass of the Milky Way and assuming that the neutrino are on the outer part of the halo.

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