Problem Set #8

Astro 2: Spring 2012

Solutions

Problem 1 *The Milky Way before Our Time*

Based on the information given, approximately 4% of the mass of the MWG has undergone hydrogen fusion. Since 0.7% of that mass is converted into light energy, a lower bound on the total energy emitted from the Milky Way Galaxy is:

$$\begin{aligned} E_{H\rightarrow He} &= \epsilon \; M_{H\rightarrow He} \; c^2 = 0.007(0.04\times 10^{11} M_{sun})(3.0\times 10^8 \text{ m/s})^2 \\ &= 5.0\times 10^{54} \text{ J} \end{aligned} \tag{1}$$

Thus, the luminosity (energy emitted per unit time) for the MWG, averaged over its entire history is approximately:

$$\mathcal{L}_{avg} = \frac{5.0\times 10^{54} \text{ J}}{1.2\times 10^{12} \text{ years}} = 1.3\times 10^{37} \text{ J/s} \approx 3\times 10^{10} \mathcal{L}_{sun} \tag{2}$$

Since this number is greater than the present luminosity, it must be the case that the MWG was more luminous in the past than it is presently.

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Problem 2 *On the Eating Habits of SMBHs*

(a) Using the equation for the Eddington Luminosity:

$$\mathcal{L}_{Edd} = 30000(\frac{10^7 M_{sun}}{M_{sun}})\mathcal{L}_{sun} = 3\times 10^{11}\mathcal{L}_{sun} \approx 1.2\times 10^{38} \text{ J/s} \tag{3}$$

(b) Since 0.5 M_{sun} produces $0.05 M_{sun} c^2$ worth of energy, the required mass rate is:

$$\frac{M_{acc}}{1 \text{ sec}} = \mathcal{L}_{Edd}\frac{0.5 M_{sun}}{0.05 M_{sun} c^2} = 1.3\times 10^{22} \text{ kg/s} = 6.6\times 10^{-9} M_{sun}/\text{s} \tag{4}$$

(c) At the rate of mass accretion just calculated, the time it would take to accrete 0.5 M_{sun} would be:

$$t_{0.5} = \frac{0.5 M_{sun}}{6.6 \times 10^{-9} M_{sun}/\mathrm{s}} = 2.42 \text{ years} \tag{5}$$

(d) For a SMBH of $M > 10^8 M_{sun}$, the tidal forces outside of the event horizon are too weak to disrupt a star.

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Problem 3 The Cryogenic Dark Matter Search

(a) The experiments are conducted deep underground in order to hide from the influence of cosmic rays. These are tiny, charged massive particles with a lot of energy, that would otherwise damage the detectors.

(b) A phonon is a tiny vibration within a crystal lattice. Just like a photon, you can think of it as a quantized pulse, but instead of traveling through the vacuum of space, a phonon travels along the bonds between atoms in a lattice. And instead of traveling at the speed of light, phonons travel at the speed of sound of the material they propogate within.

(c) It is impossible to completely eliminate all other outside interactions. Fortunately, the detectors react differently to charged particles than neutral ones such as WIMPs, so charged detections can be sorted out.

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Problem 4 Dark Matter Density

The total mass of the sun and planets is negligably larger than the sun itself. Spreading that mass across the designated sphere, we get an average density:

$$\begin{aligned} \rho_{SS} &= \frac{M_{SS}}{\frac{4}{3}\pi d_{Neptune}^3} \approx \frac{M_{sun}}{\frac{4}{3}\pi(30.1 \text{ AU})^3} \\ &= 5.2 \times 10^{-9} \text{ kg}/m^3 \approx 3 \times 10^{12} m_p/\text{ cm}^3 \end{aligned} \tag{6}$$

Comparing this to $\frac{1}{3} m_p/\text{cm}^3$, we find that the dark matter density is a factor 10^{13} smaller than our reference density.

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