Introduction to Holographic Duality

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1 The Holographic Principle

General relativity, the most beautiful physical theory ever invented [1], is notoriously difficult to quantize. A host of technical issues and disastrous infinities plague all naïve attempts to do so, but over the past 50 years, tremendous progress has been made in our understanding of quantum gravity. One theme that unifies much of this progress is the study of black holes, which are by their nature both quantum-mechanical and gravitational systems.

In 1974, Stephen Hawking showed [2] that a quantum-mechanical observer in the vicinity of a black hole would feel themselves suffused in a bath of thermal radiation emanating from the black hole. This radiation, known as *Hawking radiation*, makes black holes into thermodynamic objects: they have temperature, produce entropy, and so on. But the production of entropy always implies the loss of information—for instance, sudden amnesia about some of a system's degrees of freedom—and quantum systems are not supposed to leak information.[1] This is the core of Hawking's information paradox; it set off a decades-long debate on the whereabouts and general nature of the missing information.

Twenty years later, 't Hooft and Susskind considered Hawking's formula $S_{\mathrm{BH}} = \frac{k_{\mathrm{B}}c^3}{4G\hbar}A$ for the entropy of a black hole in terms of the area A of its event horizon (and other physical constants). The formula suggested [3, 4] that the information locked away inside the black hole is somehow encoded or stored on its surface at the event horizon, much like a hologram. In 1997, Maldacena proposed a vast generalization of this *holographic principle* known as the AdS/CFT correspondence [5]. In AdS/CFT, almost *any* quantum gravitational system—not just a black hole—bares its true degrees of freedom on the asymptotic boundary of the "bulk" spacetime it lives in; those degrees of freedom then coalesce into a special type of quantum system known as a conformal field theory (CFT) on the boundary [6]. One caveat to these proclamations is that the bulk spacetime must approach an anti-de Sitter (AdS) geometry out at infinity. This describes an open cosmology—a space of constant negative curvature—and is often imagined as a solid cylinder where distances become infinite near the edges. In these terms, one may think of AdS/CFT as a test tube for quantum gravity, or rather for a particular brand of quantum gravity where much can be calculated and understood.

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[1] The von Neumann entropy $S_{\mathrm{vN}}[\rho] = \mathrm{Tr}(\rho \ln \rho)$ of a density matrix ρ is a fine-grained, microscopic account of the information content of a quantum state. Unlike thermal entropy (sometimes called "coarse-grained" entropy), S_{vN} is invariant under the unitary time evolution of quantum mechanics. But Hawking found that in a vicinity of a black hole, $S_{\mathrm{vN}}[\rho]$ increases, in blatant violation of unitarity!

2 Entanglement and Spacetime

The AdS/CFT correspondence is nothing less than a miracle: the boundary CFT, whose definition and formal structure is well understood, could provide a fully nonperturbative definition of quantum gravity, which (on its own) is poorly understood. In particular, every aspect of the bulk theory finds a dual description on the boundary. This holographic dictionary [6, 7] is particularly well developed in the *semiclassical limit*, where quantum effects in the bulk are weak enough for its dynamics to be well approximated by classical general relativity and quantum field theory on a fixed, weakly curved background. In this regime, bulk quantum fields—including the spacetime metric itself—correspond to local CFT operators that define quantum states on the boundary [8]. Empty AdS space is the CFT ground state, black hole geometries are excited thermal states, and so on. It is this semiclassical milieu which recently produced a huge step towards resolving Hawking's information loss paradox [9]. Many physicists believed that the problem would require the full formalism of quantum gravity to resolve, so the surprising results have shown that semiclassical gravity "knows" more than it has a right to, and that it still has much more to teach us.

One of the most unsettling aspects of holography is that bulk information is encoded on the boundary in a highly nonlocal manner: gravitational physics at a single point in the bulk depends, in its dual description, on CFT information spread out over a large subregion of the boundary [10]. In fact, given access to all CFT operators supported on some subregion A of the boundary, one may fully reconstruct the bulk fields that lie in a corresponding region $\mathcal{W}[A]$ of the bulk [11]. This region is called the *entanglement wedge*, and it lies between the boundary subregion A and a special *Ryu-Takayanagi (RT) surface* that cordons off $\mathcal{W}[A]$ from the rest of the bulk. Miraculously, the area of the RT surface is proportional to the entanglement entropy of the CFT's quantum state, as measured by an observer confined to A [12, 13]. The RT surface therefore acts like an event horizon, even in the absence of a black hole inside $\mathcal{W}[A]$! In some sense, this claim returns to and vastly generalizes Hawking's 1974 area law. The RT construction is as mysterious as it is intricate: the quantum state of the CFT determines a bulk geometry, which dictates the shape of the RT surface, and this surface, in turn, measures the degree of entanglement in the very state that produced it.

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