The Tortoise and the Hare

A Causality Puzzle in AdS/CFT

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Based on [2011.08934] with Prof. D. Berenstein (UCSB).

Invitation

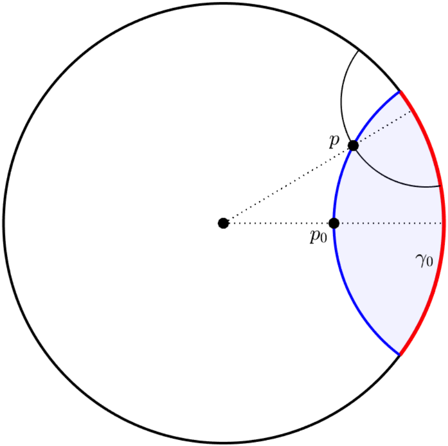
Encoding Intervals in AdS_3

In $\mathrm{AdS}_3/\mathrm{CFT}_2$, bulk fields are encoded by nonlocal operators smeared over the boundary.

Time slices of AdS_3 are Poincaré disks D^2. Spacelike geodesics in D^2 are circular arcs $\perp$ to ∂D^2.

Drop a spacelike geodesic from p subtending γ. Then γ encodes all points inside the geodesic.

Thus identify $p \in D^2$ with its **encoding interval** $\gamma \subset \partial D^2$.



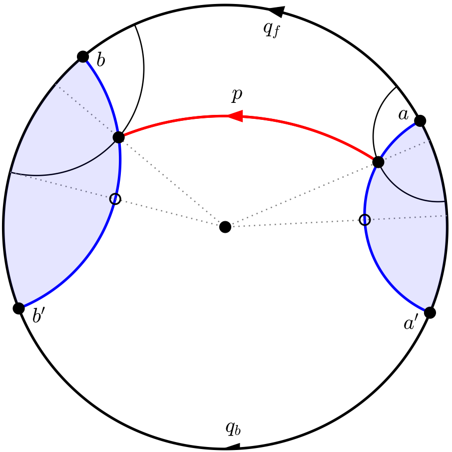
A Holographic Causality Puzzle

If p is a null path, its encoding interval moves faster than light!

Non-minimal "swinging" encodings make things worse.

Send a signal along ∂D^2 from within p's encoding interval; ask it to stay inside as p moves.

Thus p (hare) races q (tortoise). Causality is preserved if the tortoise wins: $\Delta t = t_p - t_q \geq 0$.



Teaser: Minimal Encoding

Teaser: Non-Minimal Encoding

The Tortoise Coordinate

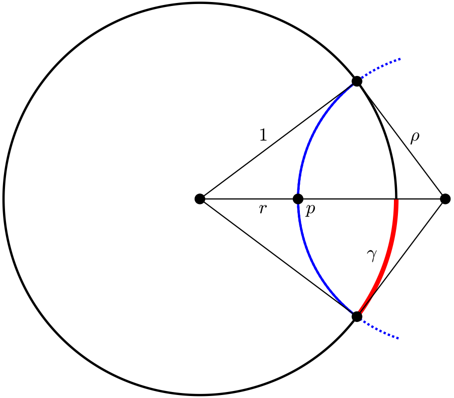
Parametrize the radial position of $p \in D^2$ by the length γ of its **minimal** encoding interval.

In "sausage" coordinates (γ, θ, t), the AdS_3 metric is

$$\boxed{\mathrm{d}s^2 = \frac{-\mathrm{d}t^2 + \mathrm{d}\gamma^2 + \cos^2\gamma\,\mathrm{d}\theta^2}{\sin^2\gamma}.}$$

Radial $(\mathrm{d}\theta = 0)$, null $(\mathrm{d}s^2 = 0)$ infall satisfies $\mathrm{d}t = \pm\mathrm{d}\gamma$; this makes γ a *tortoise coordinate*.

Thus γ measures depth in the bulk and encoding nonlocality.



Circular Motion: Trade-Off

At fixed γ, circular paths have

$$\mathrm{d}s_p^2 = \frac{-\mathrm{d}t^2 + \cos^2\gamma\,\mathrm{d}\theta^2}{\sin^2\gamma} = 0.$$

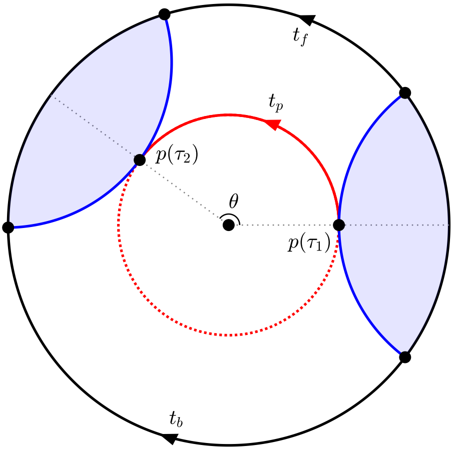
Then $\mathrm{d}t = \pm\cos\gamma\,\mathrm{d}\theta$, so the angular velocity of p is

$$\omega_p = \left|\frac{\mathrm{d}\theta}{\mathrm{d}t}\right| = \frac{1}{\cos\gamma} \geq 1.$$

Bulk and boundary travel times:

$$t_p = \frac{\theta}{\omega} = \theta\cos\gamma,$$
$$t_f = \theta - 2\gamma,$$
$$t_b = 2\pi - \theta - 2\gamma.$$

For $\theta < \pi$, we have $t_f < t_p$.
For $\theta > \pi$, we have $t_b < t_p$.



Radial Motion: Criticality

At fixed θ, radial paths have

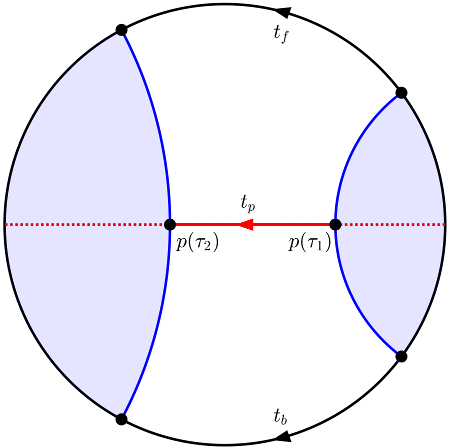
$$\mathrm{d}s_p^2 = \frac{-\mathrm{d}t^2 + \mathrm{d}\gamma^2}{\sin^2 \gamma} = 0.$$

Then p has speed $v_p = \left|\frac{\mathrm{d}\gamma}{\mathrm{d}t}\right| = 1.$

As p crosses the origin, its encoding interval jumps across the boundary with infinite speed.

The size of γ prevents a paradox, since $t_p = t_f = t_b \implies \Delta t = 0.$

Call such bulk encodings **critical**.



Plan of Attack

Is causality *always* preserved? It suffices to consider null geodesics, the fastest bulk curves. These are arcs in D^2, parametrized by $\ell \in [0,1]$.

Goal: show explicitly that $\Delta t = \min\{t_p - t_f,\, t_p - t_b\} \geq 0$.

1. Minimal encodings yield to a convexity argument (cf. circular).
2. There is a unique critical encoding for which $\Delta t(\tau) \equiv 0$ (cf. radial).
3. Deviations from criticality reduce to elementary spherical geometry.

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If p starts and ends on ∂D^2, then causality is preserved by Gao-Wald. If not, then RT gives a prescription to extend Gao-Wald into the bulk.

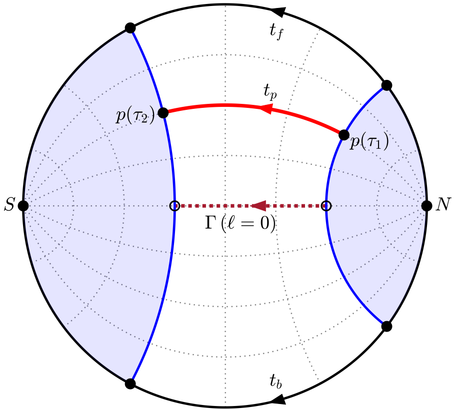
N.B. It is well known that AdS does not violate causality. Our advantages are (1) the tortoise point of view, (2) the elementary nature of the derivation, and (3) the explicitness of our results.

The Critical Encoding

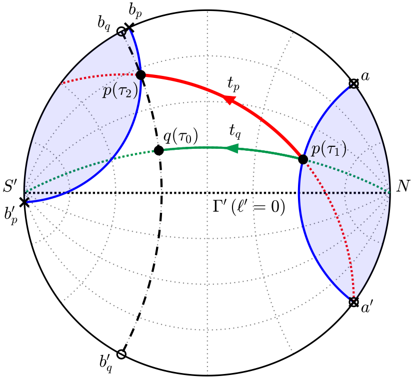
Consider the encoding symmetric about the null geodesic Γ with $\ell = 0$. By symmetry, $t_f = t_b$.

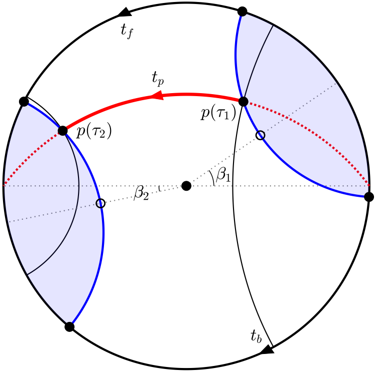
The bulk travel time can be computed by tracking the **gem** of the encoding, which lies on Γ.

But this is just our analysis of radial infall; hence, $\Delta t \equiv 0$.



Deviations from Criticality





Conclusions

There once was a bulk field on AdS_3.

On null paths it roamed the universe free.

    Its CFT dual appeared superluminal:

    The universe, cruel, rejected the noumenal—

But circles are round; the theories agree!