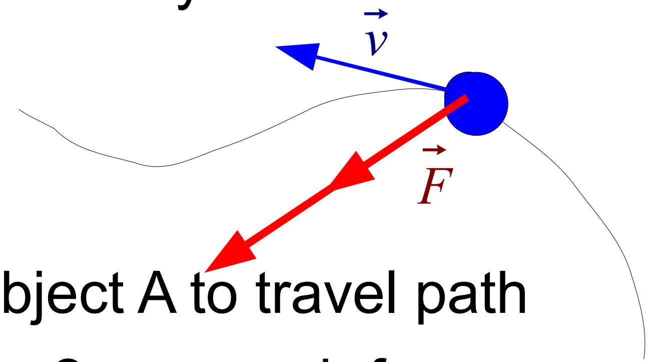
NEWTON'S LAWS

Kinematics vs. Dynamics

- Kinematics → describe motion paths mathematically
  - No description of the matter that may travel along motion path

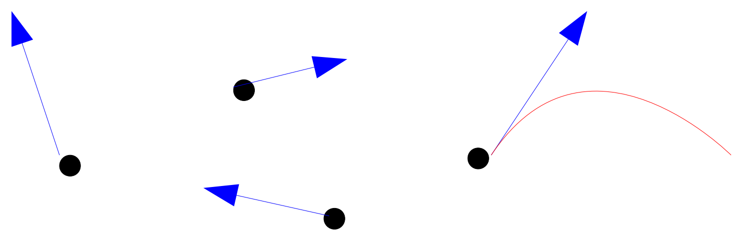
- Dynamics → prediction of motion path(s) of matter
  - Requires explanation of how matter interacts → "forces"
  - Properties of forces are determined by experiments
  - How does force affect motion? → determined by "mass"



- For a given motion path:
  - Particular amount of force required for Object A to travel path
  - Object B (with 2x as much mass) requires 2x as much force

Mathematical Description of Mass

"Point particles"

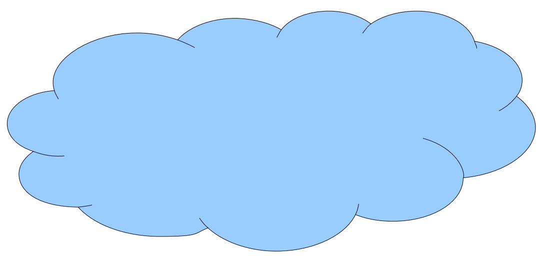


- Infinitesimally small – each particle traces a motion path
- Particle "i" has:

$$mass = m_i$$
$$position = \vec{r}_i$$
$$velocity = \vec{v}_i$$
$$acceleration = \vec{a}_i$$

- Larger objects exist due to forces holding particles together

"Mass distribution"



- Mass is a "field" – a function which maps every point to the density at that point: $\rho(x, y, z)$

$$m_{total} = \int \rho(x, y, z)\, dx\, dy\, dz$$

- Infinite # of points/events → there is no single motion path
- Requires different type of math

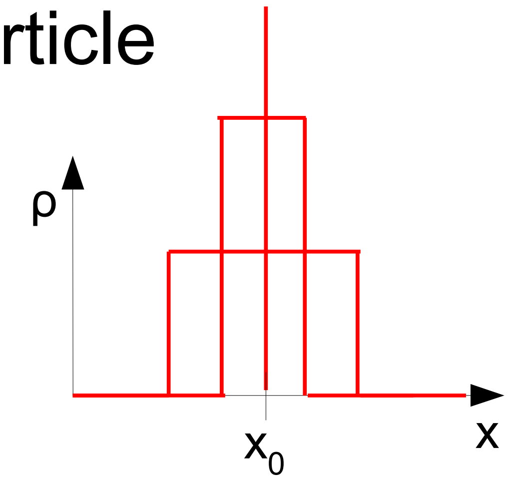
Dirac Delta-Function

- Consider a mass distribution with total mass M
  - In the shape of a line of length L → 1-Dimensional distribution

$$M = \int_{x_0 - L/2}^{x_0 + L/2} \rho(x)\, dx$$

- Imagine shrinking line down to a point particle
  - Without changing its total mass
  - Eventually get "Dirac Delta-Function":

$$\rho(x) = M\,\delta(x - x_0) \equiv M \begin{cases} \infty & at\ x = x_0 \\ 0 & everywhere\ else \end{cases}$$



- Properties:

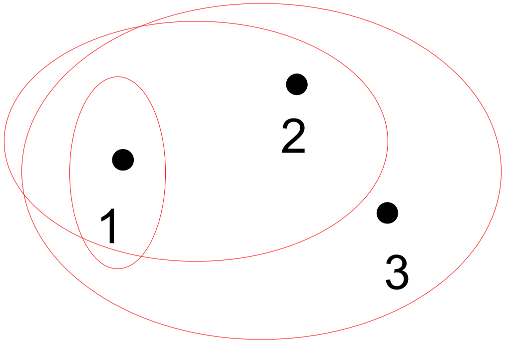
$$\int_{-\infty}^{\infty} \delta(x - x_0)\, dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\,\delta(x - x_0)\, dx = f(x_0)$$

  - Mathematical "bridge" between point particles and distributions

Physical "Systems"

- First step in studying forces:
  - Identify the matter the forces act upon
  - Called a physical "system"



- Choice of system is arbitrary
  - Could describe forces on particle 1 (exerted by 2 & 3)
  - Or forces on 1 & 2 (exerted by 3) – F_{12} is now an internal force
  - Or forces on 1, 2, & 3 (exerted by some outside source)

- Physics → must work for any choice of system
  - External forces on system determine motion of system

Center of Mass

- What are the attributes of a physical system?
  - Mass? Yes! $M_{total} = \sum m_i$
  - Volume? No! (systems only include particles, not space)
  - Position? Sort of! (many particles can have many positions)

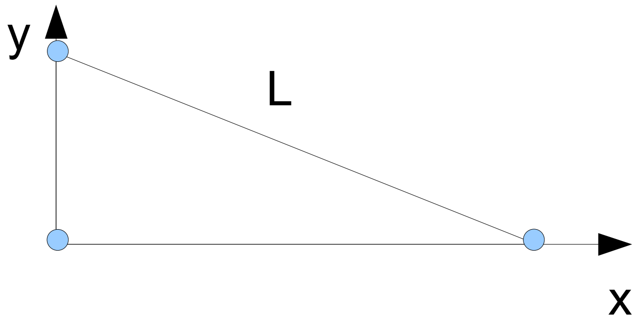
- Center of Mass (CM)
  - The average position of the mass in a system
  - External forces on a system → determine the motion of CM

$$\vec{r}_{CM} \equiv \frac{\sum m_i \vec{r}_i}{M_{total}} \longrightarrow \vec{v}_{CM} \equiv \frac{\sum m_i \vec{v}_i}{M_{total}} \longrightarrow \vec{a}_{CM} \equiv \frac{\sum m_i \vec{a}_i}{M_{total}}$$

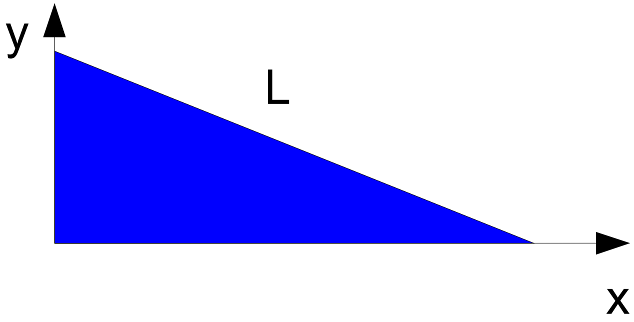
- For mass distributions: $$\vec{r}_{CM} \equiv \frac{\int \vec{r}\, dm}{M_{total}} = \frac{\int \vec{r}\, \rho\, dx\, dy\, dz}{M_{total}}$$

Center of Mass Example

- Right triangle with a 30º angle and hypotenuse = L
  - With 3 identical point particles at the vertices
  - Find the position vector of the CM

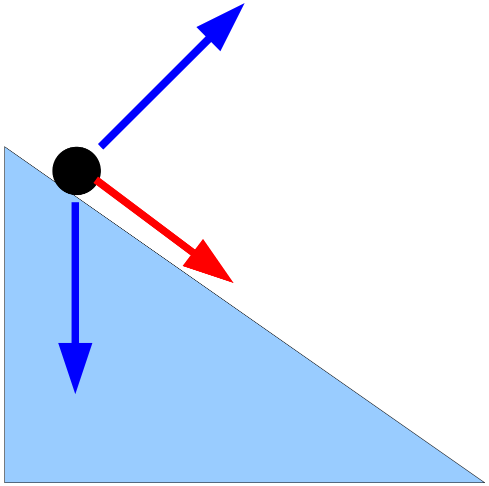


- Repeat for a <u>mass</u> <u>distribution</u>:
  - Which is uniform along the triangle's surface



Force and Net Force

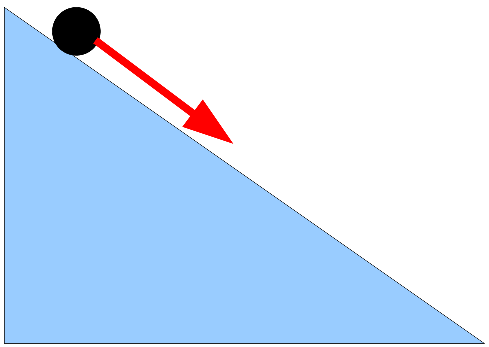
- Observation → forces affect CM motion of systems
  - Example: drop a rock – force of gravity "pulls" it down
  - With some magnitude and direction → force is a vector
- More than one force can act on a system at one time
  - Example: rock on a ramp – moves down and to the side
  - Ramp must also be exerting a force!



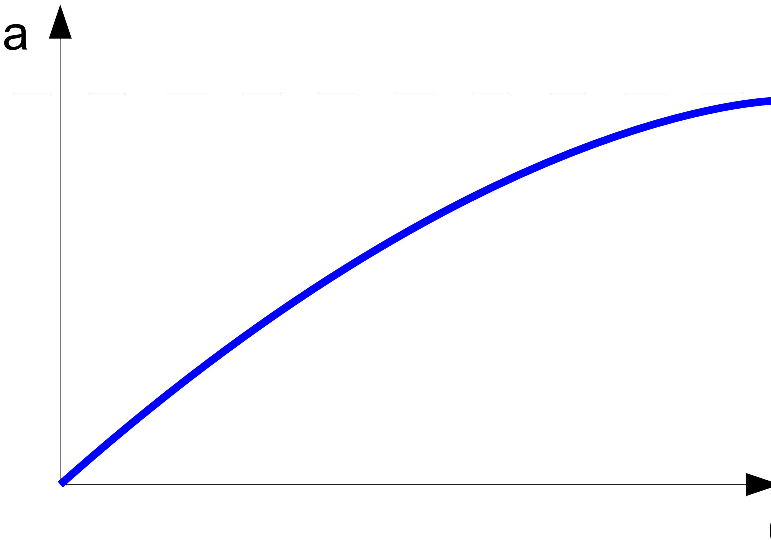
- Units of force: "Newtons" (N)
- When more than one force acts:
  - $\mathbf{F}_{net}$ = vector sum of all the forces ("superposition")

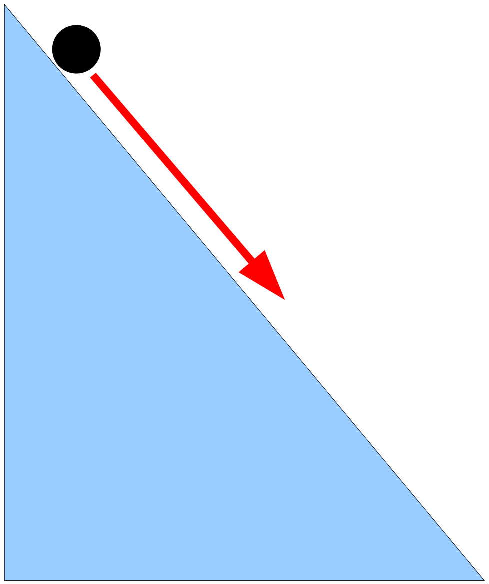
Inclined Plane Experiments

- Using inclined planes (ramps), Galileo could:
  - Measure the position of an object at particular times
  - Control the net force on the object



- Experimental results:
  - Constant acceleration for any given ramp
  - Magnitude of acceleration depends on incline



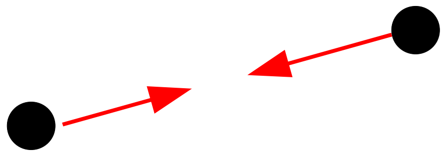


Newton's Laws

- 1st Law: $F_{net} = 0 \rightarrow$ velocity of CM remains constant

- 2nd Law: $\vec{F}_{net} = m\,\vec{a}_{CM}$ Units: $1\ Newton = 1\ kg\ \frac{m}{s^2}$

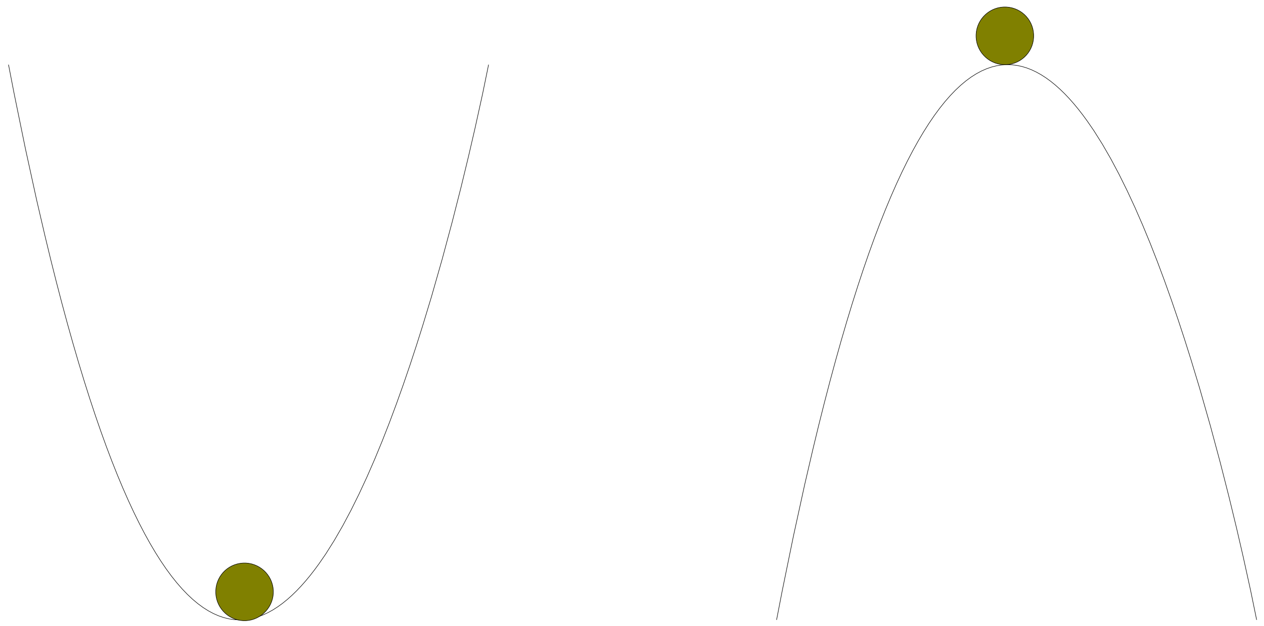
- 3rd Law: If an action force F_{AB} is exerted by system A on system B,
  - There is a reaction force F_{BA} exerted by system B on system A
  - Which is equal in magnitude to F_{AB}
  - And opposite in direction to F_{AB}



- Next: Examples of experimentally measured forces

Equilibrium and Stability

- Equilibrium – configuration of system such that $F_{net} = 0$



- Stability – disturb system from equilibrium
  - Does net force push it back toward equilibrium?
  - Yes – equilibrium is stable; No – equilibrium is unstable

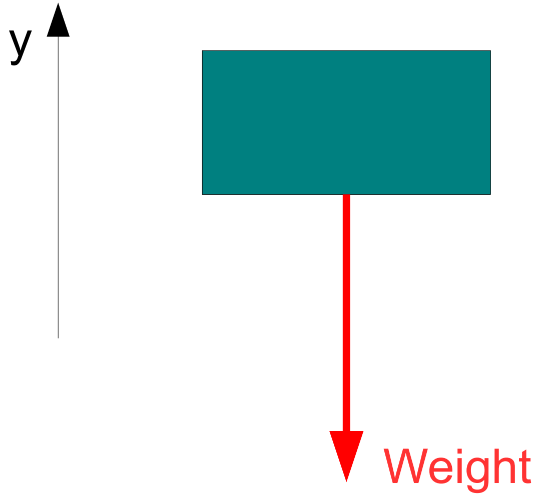
Mass and Weight

- "Free fall" – only force acting on system is gravity
  - Causes acceleration g in the downward direction (experiment)

$$\vec{F}_{net} = m\,\vec{a}_{CM}$$

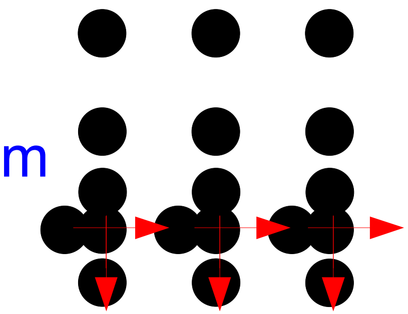
$$\vec{W} = m\left(-g\,\hat{y}\right)$$

$$W = mg$$

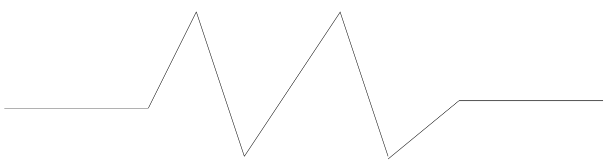


- Mass units: kg – Weight units: Newtons (or pounds)
  - It is technically incorrect to convert kg to lbs...
  - Because g is not perfectly uniform everywhere on Earth

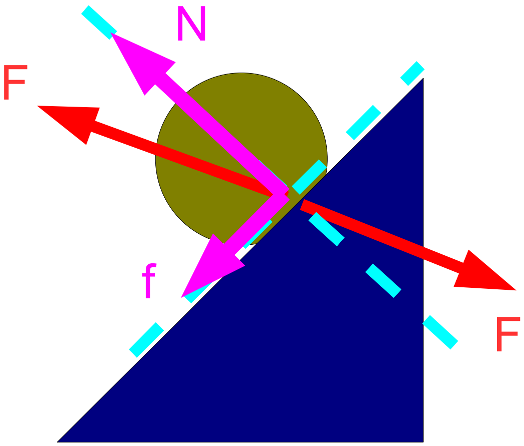
Internal Forces in an Object



- Newtonian view of solid objects:
  - Particles held together by forces in stable equilibrium
- Forces which can occur <u>inside</u> an object:
  - Tension – when object is <u>stretched,</u> it pulls <u>inward</u> on particles
  - Compression – when object is <u>compressed</u>, it pushes <u>outward</u>
  - Shear – when object is <u>sheared</u>, there is a <u>restoring</u> force
- These are called "elastic forces"
  - Under "normal" conditions: $F_x = -k\ x$
  - In Physics – elastic forces are often modeled by a "spring"
  - k is the "spring constant"
  - All elastic forces have limits – eventually objects break



Surface Forces



- When 2 surfaces come in contact:
  - Objects undergo compression & shear
  - Producing an action/reaction pair of forces

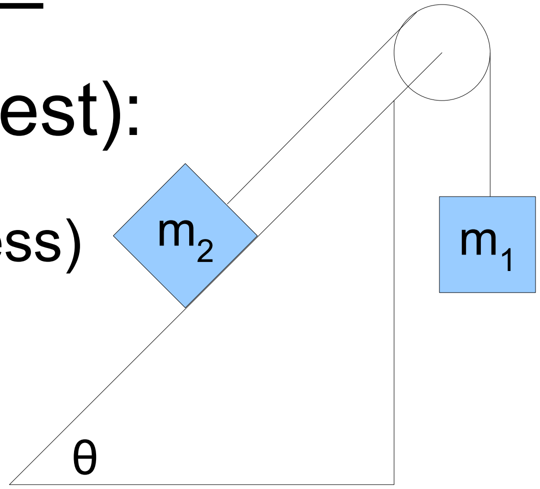
Convention:

  - Component perpendicular to surface is called "normal" force
  - Component parallel to surface is called "friction" force

- Static Friction vs. Kinetic Friction:
  - One mathematical model (fairly well supported by experiment):
  - Static – surfaces do not move relative to each other $F_{s,max} = \mu_s N$
  - Kinetic – surfaces stay in contact, but slide against each other
  - $F_k = \mu_k N$ (independent of sliding speed and surface area)
  - For any 2 materials: $\mu_s \geq \mu_k$ Why must this always be true?

Newton's Law Example

- Given m_1, m_2 and θ (all masses begin at rest):
  - Calculate acceleration of m_1 (ramp is frictionless)



- Now give ramp friction coefficients μ_s and μ_k with m_2
  - Calculate acceleration of m_1 as a function of m_1, m_2, and θ
  - (will need to use a piecewise function)
  - Sketch a graph of acceleration_y vs. m_1 for fixed m_2 and θ

Newton's 2nd Law in Different Frames

- Newton's 2nd Law: $\vec{F}_{net} = m\,\vec{a}_{CM}$
  - To be correct: must be valid in all reference frames...
  - ...with transformation of acceleration vector handled correctly
- Example: Drag race

  - Frames: A = flag dropper, B = race loser, C = race winner
  - Apply Newton's 2nd Law to motion of C in reference frame of:

A (standing at starting line)

$$\vec{F}_{road\ on\ C\ tires} = m_C\,\vec{a}_C$$

$$(6{,}000\ N)\,\hat{i} = (1200\ kg)\left(5\ \frac{m}{s^2}\right)\hat{i}$$

B (in losing car)

$$\vec{F}_{net,\,C}\,' = m_C\,'\,\vec{a}_C\,' \qquad \left(\vec{a}_C\,' = \vec{a}_C - \vec{A}_B\right)$$

Newton's 2nd Law will only work in this frame if:

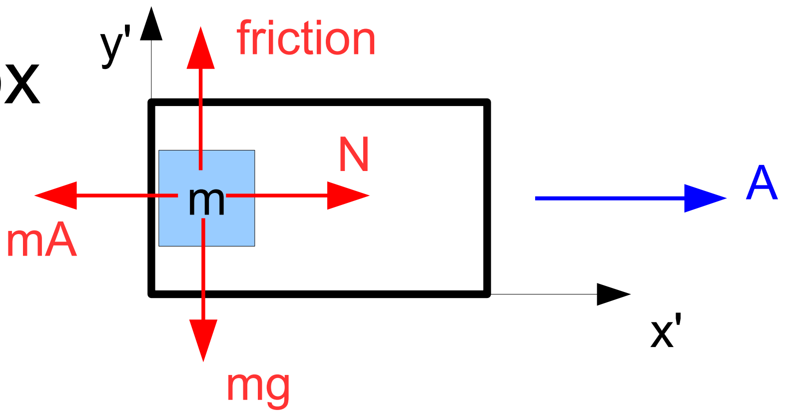
$$\vec{F}_{net,\,C}\,' = \vec{F}_{road\ on\ C\ tires} - m_c\,\vec{A}_B$$

What exerts this force?

"Fictitious" Forces

- In some reference frames:
  - Newton's 2nd Law requires inclusion of "extra" forces
  - In addition to contact forces that particles exert on each other
  - These are called "fictitious" forces:
  - Observers in this ref. frame cannot determine source of force

- Example: Mass in accelerating box
  - In S' → mass stands still
  - So F_{net}' must be zero
  - Given A → calculate minimum μ_s
  - If box has no windows:
  - For which forces can source be determined by measurement?
  - Is gravity a "fictitious" force? → "equivalence principle"

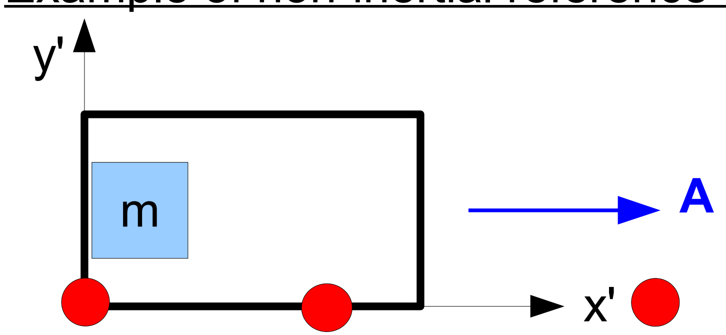


Inertial Reference Frames

- Set of reference frames requiring no “fictitious forces”:
  - These are called inertial reference frames
  - 2 different inertial frames have a relative velocity (translation)
  - But no relative acceleration!
  - And cannot have a relative rotation rate!

- All inertial reference frames are equally valid
  - The universe does not have a “preferred” reference frame

Example of non-inertial reference frame:



Fictitious force: $-m\mathbf{A}$ (on every mass m)

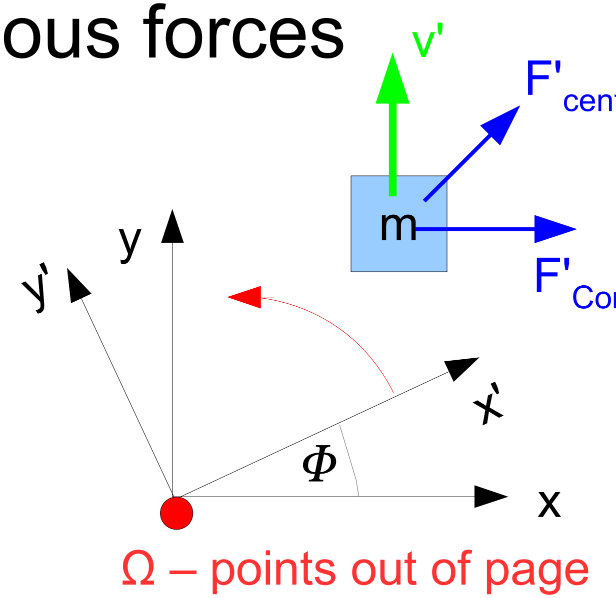
Thought Experiment:
Light bulbs are arranged to flash as left end of box passes them – once per second.

Do light pulses arrive at right end of box once per second?

Rotating Reference Frames

- Angular Velocity vector ($\vec{\Omega}$)
  - Direction: along axis of rotation   Magnitude: rotation rate

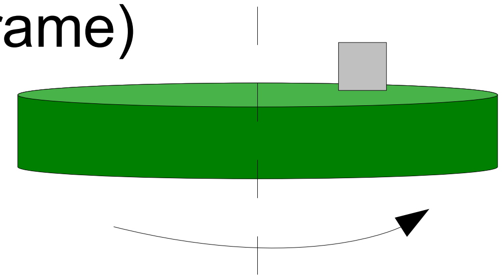
- Transform $\vec{a}$ to rotating frame → fictitious forces
  - Centrifugal force:
  - Direction: Away from rotation axis ($\hat{r}$?)
  - Magnitude: $F'_{cent} = m\,\dot{\Phi}^2\, r_{\perp\, axis}$
  - Coriolis force:
  - $\vec{F}'_{Coriolis} = -2\, m\, \vec{\Omega} \times \vec{v}\,'$
  - Direction: perpendicular to velocity in rotating frame



- F'_{Cor} produces circular motion; F'_{cent} pushes outward

Rotating Reference Frame Examples

- Frictionful disk spinning at constant angular rate ω
  - With mass m stuck on top (at rest in spinning frame)
  - Apply Newton's 2nd Law in these frames:
  - 1) inertial, 2) rotating: $\Omega = \omega$, 3) rotating: $\Omega \neq \omega$



- At some later instant → turn off friction
  - Sketch motion of mass in x'y'-plane of $\Omega = \omega$ frame
  - Identify direction of centrifugal & Coriolis force at a few points
- Rock on surface of Earth (say, in SB → 34° latitude)
  - Does normal force perfectly cancel out weight of rock?
  - Find Coriolis force if you throw rock with speed v:
  - 1) to the North, 2) to the East, 3) to the South, 4) Straight up