Introduction to Special Relativity

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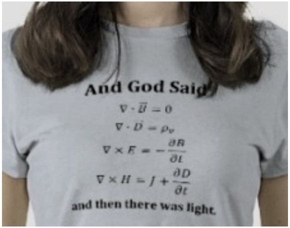
SIMS 2016 Physics 20

August 24th, 2016

Maxwell's Equations

- Maxwell's Equations deduced in 19th century:



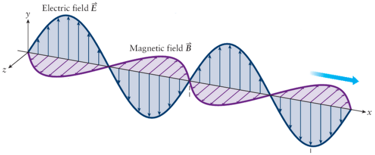
- Tell us the behavior of electric and magnetic fields in response to currents and charges
- In addition, Lorentz force law tells us force on charge:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Electromagnetic Waves

- When no charges or currents, we find a wave equation

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0\,, \quad \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0\,,$$



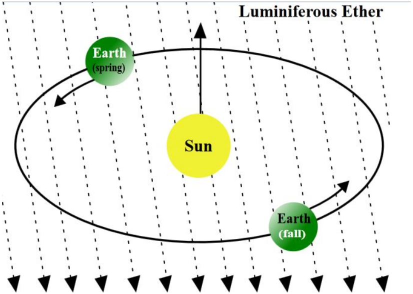
- The value of c is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \,\mathrm{m\ s^{-1}}$$

- The nature of light answered at last!
- Predicted by Maxwell, verified by Hertz

The Luminiferous Aether

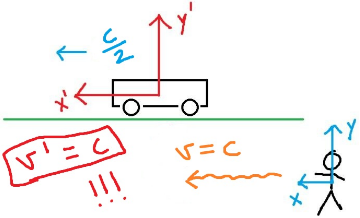
- But what is this c with respect to?



- Michelson Morley experiment in 1887 sets out to measure motion with respect to Aether
- Motion through Aether should change velocity of light, similar to a boat riding next to a water wave
- Of course, their conclusion is...

Invariance of Speed of Light

- There is no change!



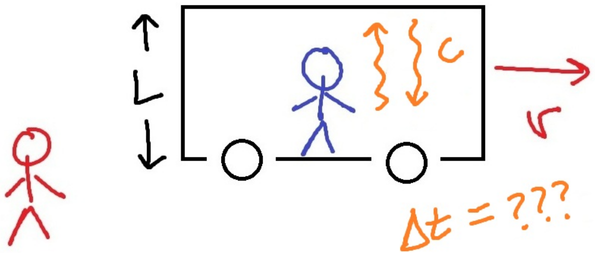
- Verified today to high level of accuracy that c is strictly same for every observer!
- You can never "ride" a wave of light

Special Relativity

- Despite efforts by many, speed of light always seen to be same
- This idea was ultimately embraced by Einstein, who noticed a fundamental problem with our understanding of space and time
- In 1905, he introduced the theory of Special Relativity
- The postulates:
  - Any two inertial frames with constant relative velocity are equally valid
  - The speed of light in vacuum is a universal constant, independent of the motion of the emitting body
- What are the consequences of Special Relativity?

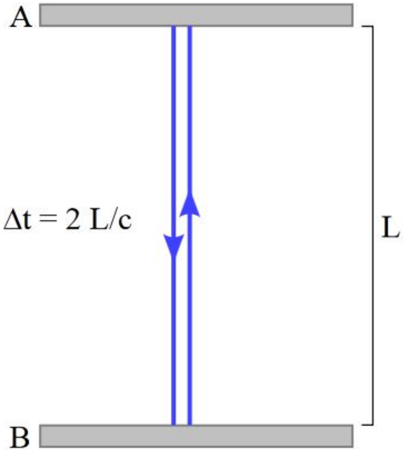
Time Dilation

- Observer on ground watches observer on train pass by
- Observer on train shines light beam from ground to ceiling, and then it reflects back
- This serves as a primitive time-keeping device
- How long does it take for this to happen?



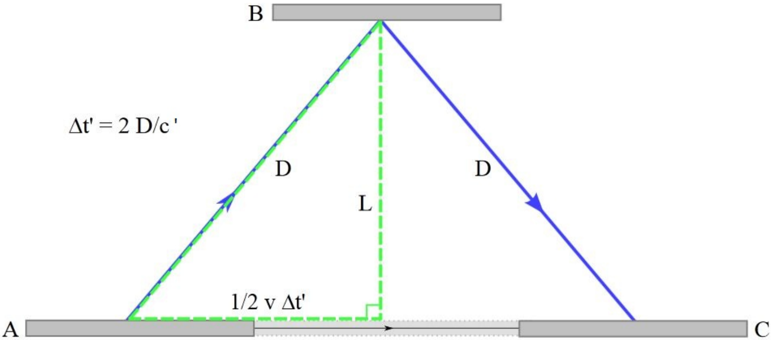
Time Dilation

- Let's ask observer on train:



Time Dilation

- Let's also ask observer on ground:



- By Pythagorean theorem:

$$D = \sqrt{L^2 + \left(\frac{1}{2} v \Delta t'\right)^2}$$

Time Dilation

- Solving for ground time,

$$\Delta t' = \frac{2D}{c'} = \frac{2}{c'}\sqrt{L^2 + \left(\frac{1}{2}v\Delta t'\right)^2} \Rightarrow$$

$$\Delta t' = \frac{(2L/c')}{\sqrt{1 - \frac{v^2}{(c')^2}}}$$

- But c is invariant!

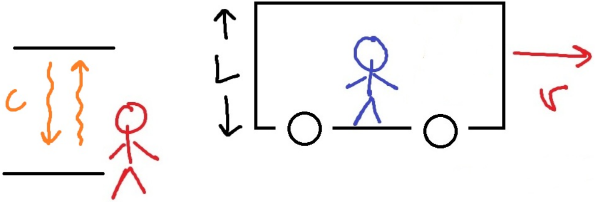
$$\boxed{\Delta t' = \frac{(2L/c)}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv \gamma \Delta t}$$

- The only possible conclusion: the times are different
- **The two observers, making measurements with their clocks, will disagree on the amount of time it takes for this to happen**

Proper Time

- We could repeat this experiment with the roles reversed
- How does the light clock on the ground behave?
- Reversed experiment would **swap the expression**

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv \gamma \Delta t'$$



- Each observer believes that the other one has a clock which is ticking away too slowly
- Neither of them is right or wrong

A Tiny Effect

- Why don't we ever see this?
- We have the Taylor series expansion, for small values of x,

$$\frac{1}{\sqrt{1-x^2}} \approx 1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + \ldots$$

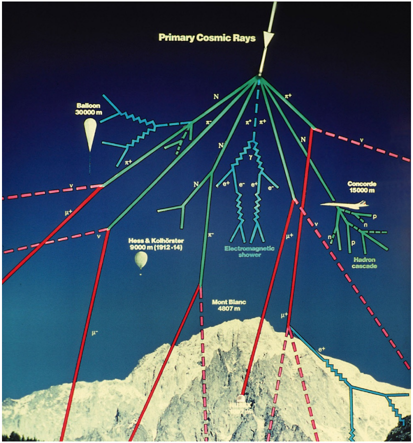
- For small velocities compared with speed of light,

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \approx 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{3}{8}\left(\frac{v}{c}\right)^4 + \ldots$$

- Typical effects are incredibly tiny!

Experimental Verification

- Of course, theory must have some testable effects
- Muons are subatomic (point) particles created in cosmic ray collisions
- Muons mostly decay before reaching ground



Experimental Verification

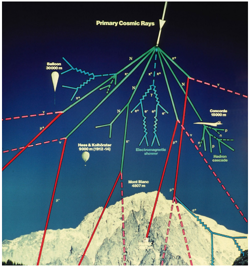
- Muons decay with a characteristic rate

$$N(t) = N_0 e^{-\lambda t}$$

- How many with velocity v make it to ground from an initial detector at height D?

$$t = \frac{D}{v} \Rightarrow N = N_0 e^{-\lambda D/v}$$

- We can do this experiment by setting up detectors at the top and bottom of a mountain



- The actual measured result is...

Experimental Verification

- More than this - we used the wrong time!
- The muon appears to have a slow clock as compared with ours, so it makes it further before decaying!
- Muon time in terms of earth time is

$$t_m = \frac{t_e}{\gamma} = \frac{D}{v\gamma}$$

- Therefore:

$$N = N_0 e^{-\lambda D/(v\gamma)}.$$

- This result has been verified experimentally in the Frisch-Smith experiment, and many others since
- Notice the universality of the result - no material-dependent properties
- Modern particle accelerations verify this effect all the time

Length Contraction

- But wait - what about equivalence of all inertial frames?
- Muon believes IT is at rest and sees Earth approach it
- How do we reconcile this?
- Inevitable conclusion - lengths are contracted

$$D_m = \frac{D}{\gamma}$$

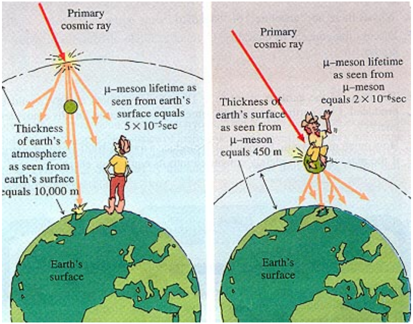
- Muon believes it lives for normal time, while traveling shorter distance than measured on Earth
- From muon perspective

$$N = N_0 e^{-\lambda t_m} = N_0 e^{-\lambda D_m / v} = N_0 e^{-\lambda D/(v\gamma)}$$

- Same result!

Length Contraction

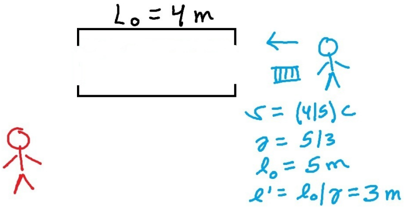
- Muon and Laboratory observers have different opinions



- Either picture is valid! There are no paradoxes here
- **The physically measurable quantity is the number of muons, and all observers agree on this number**

The Barn and Ladder Paradox

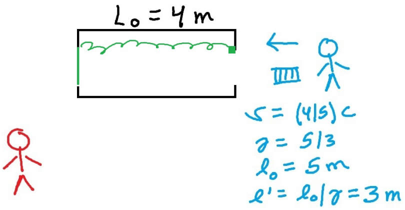
- Imagine running relativistically at open barn with ladder
- Ground observer watches
- Is ladder ever fully contained in barn?



- Ground observer says yes - ladder contracts enough
- Runner says no - BARN contracts, becoming even smaller
- Conclusion: we can no longer maintain simultaneity

Causality

- What if now back door is closed
- Front door has sensor opening back door when ladder fully enters
- Does ladder smash into back door?



- Ground observer says no - ladder contracts enough to open door
- Runner says yes - BARN contracts, becoming even smaller, so door not opened in time
- Resolution: signal from sensor must move slower than c, thereby resulting in both agreeing on crash

Relativistic Mechanics

- Can we still maintain a coherent picture of mechanics?
- Yes, with slight modifications
- Newton's second law still true for a body, with

$$\vec{p} = \gamma m \vec{v}$$

- Expression for energy of free particle changes

$$E = \gamma mc^2 \approx mc^2 \left(1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{3}{8}\left(\frac{v}{c}\right)^4 + ...\right) =$$

$$mc^2 + \frac{1}{2}mv^2 + \frac{3}{8}mc^2\left(\frac{v}{c}\right)^4 + ...$$

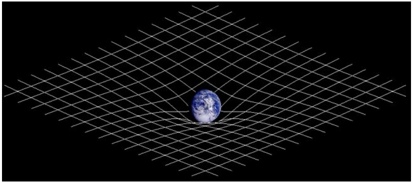
- Mass is energy!
- Colliding particles that stick convert kinetic energy into rest mass!

General Relativity

- One last issue - Newton's law of gravitation violates causality, depending on instantaneous distance

$$F = G\frac{m_1 m_2}{r^2}$$

- Solution: Replace with gravitational field that propagates
- However, gravity is special - gravitational and inertial mass the same



- All bodies move under gravitational field in the SAME way
- Perhaps gravity is a property of space (and time) itself?
- Allows a geometric description of space and time, upon combining Special Relativity with Gravity, known as *General Relativity*

Thanks!

- That's it for the SIMS 2016 Physics 20 lectures
- Thank you for being excellent students this year!
- Good luck on the exam tomorrow!