Physics 20 Homework 0
SIMS 2016

Due: Monday, August 15^{th}

Problem 1

(a) Taking the origin of our coordinate system to be the cannon, with the positive x direction pointing towards the target and the positive y direction pointing up, we have that the initial position of the target is

$$\boldsymbol{r}_{\text{target}}(0) = (d, h)$$

The target starts from rest and falls with a constant downward acceleration g, so its position as a function of time is

$$\boldsymbol{r}_{\text{target}}(t) = \boldsymbol{r}_{\text{target}}(0) + \frac{1}{2}\boldsymbol{a}t^2$$

$$\boxed{\boldsymbol{r}_{\text{target}}(t) = \left(d, h - \frac{1}{2}gt^2\right)}$$

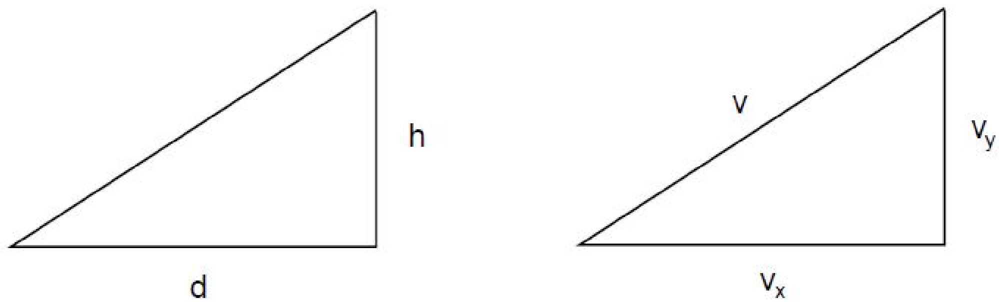
(b) For the bullet, we have

$$\boldsymbol{r}_{\text{bullet}}(t) = \boldsymbol{r}_{\text{bullet}}(0) + \boldsymbol{v}_{\text{bullet}}(0)t + \frac{1}{2}\boldsymbol{a}t^2$$

The initial position of the bullet is

$$\boldsymbol{r}_{\text{bullet}}(0) = (0, 0)$$

To find the initial velocity of the bullet, consider the right triangle formed by the cannon and target, with legs d and h. This triangle is similar to the triangle formed by the x and y components of the initial velocity, as shown:



We can therefore form the relationships

$$\frac{v_x}{v} = \frac{d}{\sqrt{h^2+d^2}}$$
$$\frac{v_y}{v} = \frac{h}{\sqrt{h^2+d^2}}$$

The initial velocity can then be written as

$$\boldsymbol{v}_{\text{bullet}}(0) = \left(\frac{vd}{\sqrt{h^2+d^2}}, \frac{vh}{\sqrt{h^2+d^2}}\right)$$

Like the target, the bullet also accelerates in the negative y direction with magnitude g, so the trajectory of the bullet is

$$\boxed{\boldsymbol{r}_{\text{bullet}}(t) = \left(\frac{vdt}{\sqrt{h^2+d^2}}, \frac{vht}{\sqrt{h^2+d^2}} - \frac{1}{2}gt^2\right)}$$

(c) In order to determine whether or not the bullet hits the target, we need to determine whether or not the trajectories of the bullet and target intersect, i.e., is there a time t_{hit} such that

$$\boldsymbol{r}_{\text{target}}(t_{\text{hit}}) = \boldsymbol{r}_{\text{bullet}}(t_{\text{hit}})?$$

In terms of the trajectories we computed above, the above requirement is

$$\left(d, h - \frac{1}{2}gt_{\text{hit}}^2\right) = \left(\frac{vdt_{\text{hit}}}{\sqrt{h^2+d^2}}, \frac{vht_{\text{hit}}}{\sqrt{h^2+d^2}} - \frac{1}{2}gt_{\text{hit}}^2\right)$$

Equating the x-components, we get

$$d = \frac{vdt_{\text{hit}}}{\sqrt{h^2+d^2}} \Rightarrow t_{\text{hit}} = \frac{\sqrt{h^2+d^2}}{v}$$

Equating the y-components gives

$$h - \frac{1}{2}gt_{\text{hit}}^2 = \frac{vht_{\text{hit}}}{\sqrt{h^2+d^2}} - \frac{1}{2}gt_{\text{hit}}^2 \Rightarrow t_{\text{hit}} = \frac{\sqrt{h^2+d^2}}{v}$$

So indeed, no matter what the values of h, d, g, or v might be, the trajectories will *always* intersect after a time t_{hit}.

This conclusion actually makes an additional assumption, which is that the intersection occurs before the time at which the target hits the ground. By solving for the time that the y component of the target is zero, we see that

$$t_{\text{ground}} = \sqrt{\frac{2h}{g}}.$$

This time is dependent on the value of g, whereas the intersection time is not. Thus, if the bullet is fired slowly enough, the target may hit the ground before the intersection. But, if we ignore the presence of the ground, then the two will always intersect.

Notice that equating the y components was simplified by the fact that the term involving g was the same for both the target and the bullet, and so it canceled from both sides of the equation. This seemingly innocuous statement is actually very powerful, and actually helped lead to the development of General Relativity. We'll talk about this more when we discuss the equivalence principle.