

A New Holographic View of Singularities

Gary Horowitz

UC Santa Barbara

with A. Lawrence and E. Silverstein

arXiv:0904.3922

Some papers by Tom and Willy:

- Dualities versus singularities (with Motl, 1998)
- An Holographic cosmology (2001)
- Black crunch (2002)
- Holographic cosmology 3.0 (2003)
- The holographic approach to cosmology (2004)
- Space-like Singularities and Thermalization (2006)

AdS/CFT Correspondence

(Maldacena, 1997)

AdS: Anti de Sitter spacetime

CFT: Ordinary (nongravitational) quantum field theory that is conformally invariant.

The AdS/CFT correspondence states that string theory on spacetimes that asymptotically approach $\text{AdS} \times K$ is completely equivalent to a CFT living on the boundary.

Advantages of using AdS/CFT:

Maps the problem of spacetime singularities into a problem in ordinary field theory

Disadvantages of using AdS/CFT:

- a) The world is not asymptotically AdS
- b) It has been difficult to describe observers falling into a black hole in the CFT

Outline

- I. Simple example:
 - A. Bulk spacetime
 - B. Dual CFT
- II. Implications for singularities
- III. Generalizations
- IV. Conclusions

Simple example

Consider the following static black hole

$$ds^2 = - (r^2 - 1) dt^2 + \frac{dr^2}{r^2 - 1} + r^2 d\sigma^2$$

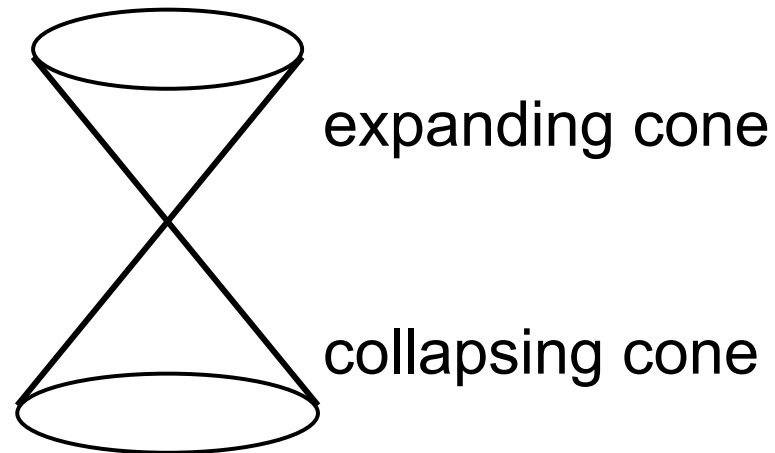
where $d\sigma^2$ is the metric on a unit 3D hyperboloid, compactified to finite volume.

This metric is locally equivalent to AdS_5 . It is a higher dimensional analog of the 3D BTZ black hole.

In Minkowski spacetime, the metric inside the light cone, in Milne coordinates, is:

$$ds^2 = - dt^2 + t^2 d\sigma^2$$

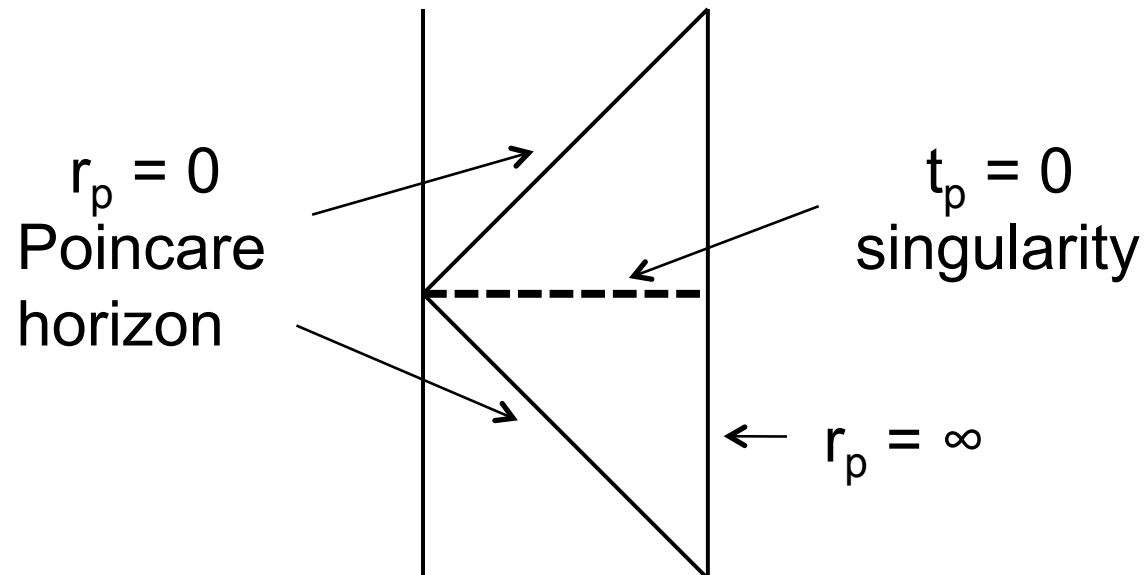
One can identify points so that $d\sigma^2$ becomes compact.



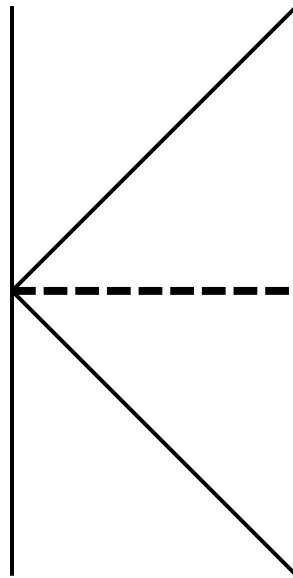
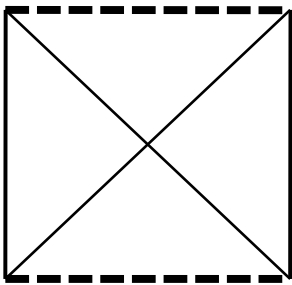
AdS can be written in Poincare coordinates

$$ds^2 = r_p^2(-dt_p^2 + t_p^2 d\sigma^2) + \frac{dr_p^2}{r_p^2}$$

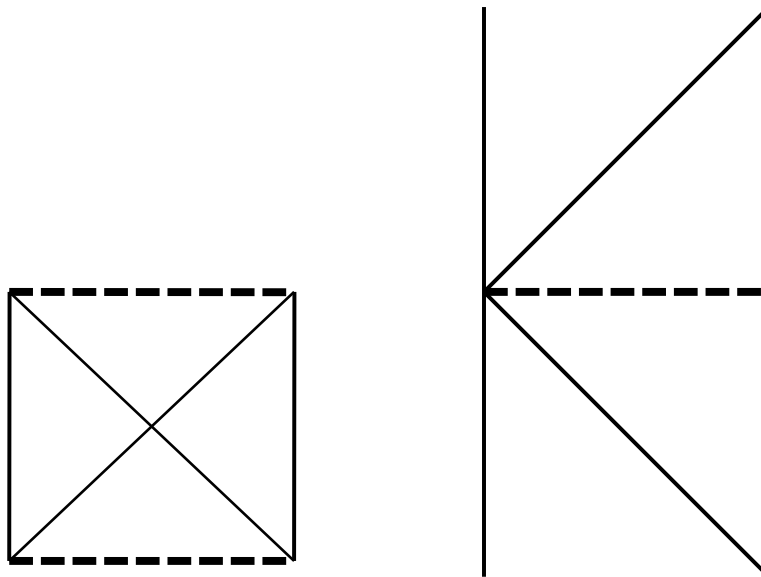
One can make a similar identification on each Minkowski slice.



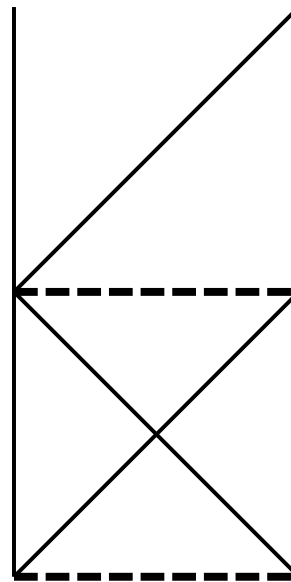
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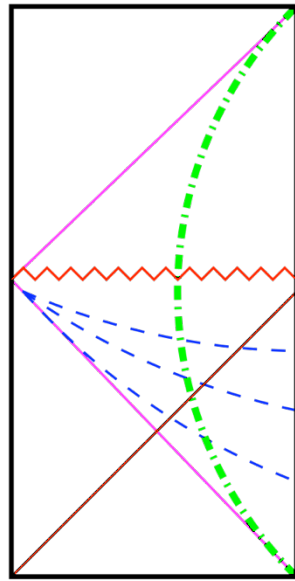


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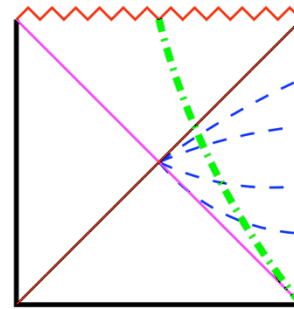


Since we know how to describe physics in the Poincare patch, we can describe physics inside the horizon.

Moreover, we can easily describe infalling observers, since a D-brane stays at constant Poincare radius and this crosses the black hole horizon.



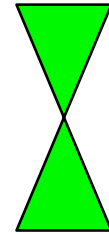
(A)



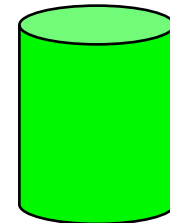
(B)

Green line is motion of a D-brane
Blue lines are
A) Poincare time slices
B) Schwarzschild time slices

The natural metric on the boundary at infinity in the Poincare coordinates is the cone.



The natural metric in the black hole coordinates is a static cylinder.



These are related by a conformal transformation:

$$(1/t^2) [- dt^2 + t^2 d\sigma^2] = - d\eta^2 + d\sigma^2 \quad t = \pm e^{\pm\eta}$$

The collapsing and expanding cone each become an infinite static cylinder.

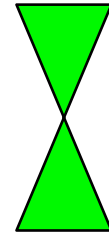
Metric in Poincare coordinates:

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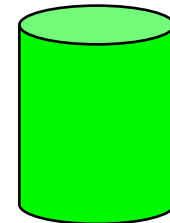
Metric in Schwarzschild coordinates:

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Dual CFT description

If the bulk spacetime is asymptotically $AdS_5 \times S^5$, the dual CFT is $U(N)$ super Yang-Mills.

In the Poincare patch, the SYM naturally lives on the collapsing cone. (Note: This is 4D cone, not 2D cone of matrix big bang by Craps, Sethi, Verlinde.)

This describes physics inside the horizon before the singularity is reached.

The static D-brane in Poincare coordinates is described by setting one of the scalar eigenvalues to a constant $\phi = \phi_0$.

This constant value corresponds to the radial position of the brane: $\phi_0 = r_p$.

Description in terms of SYM on static cylinder:

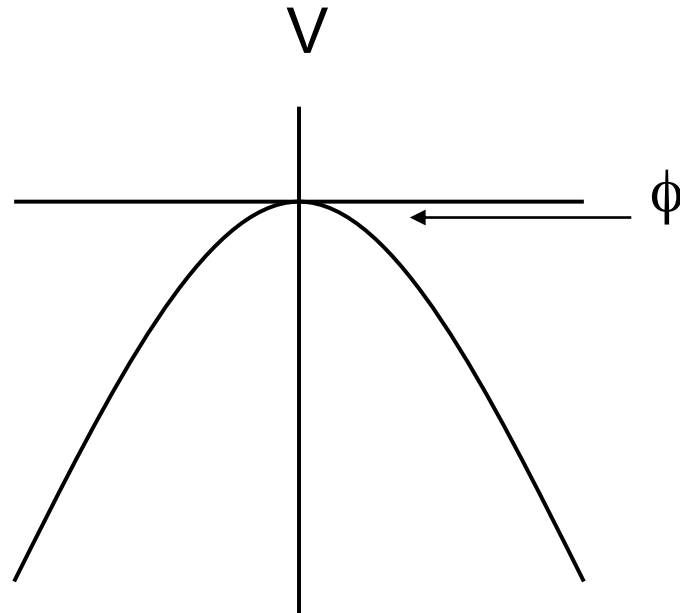
The SYM scalars couple to the curvature of space via $R\phi^2$. The static cylinder:

$$ds^2 = - d\eta^2 + d\sigma^2$$

has negative curvature, so the scalars feel a potential $V(\phi) = -\phi^2$. The solution $\phi = 0$ is unstable.

Note: In some cases, only the zero mode of ϕ is unstable. Inhomogeneous modes have $m_{\text{eff}}^2 > 0$.

Under conformal transformation from cone to static cylinder, $\phi_s = t_p \phi_p$. The solution $\phi = \phi_0$ on the collapsing cone corresponds to $\phi = \phi_0 e^{-\eta}$ on the static cylinder.



Equating the area of the hyperbolic space in Poincare coordinates and black hole coordinates yields $r = t_p r_p$, since

$$ds^2 = r_p^2(-dt_p^2 + t_p^2 d\sigma^2) + \frac{dr_p^2}{r_p^2}$$

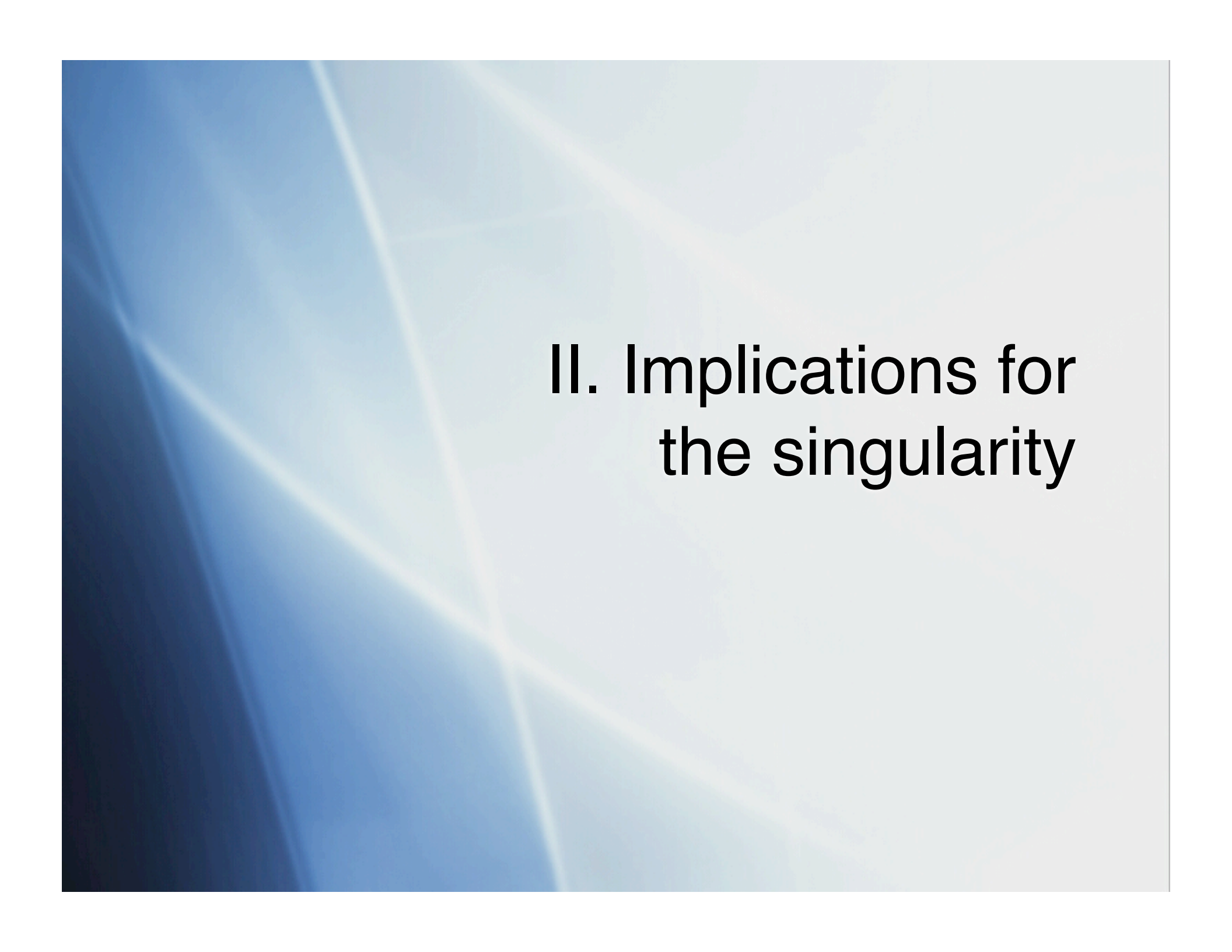
We know $\phi_p = r_p$, so $\phi_s = t_p \phi_p = r$.

The scalar field again gives the radial position of the D-brane even in Schwarzschild coordinates. The singularity corresponds to $\phi_s = 0$.

Comparison with earlier example of a singularity in AdS

(Hertog, G.H.; Craps, Hertog, Turok)

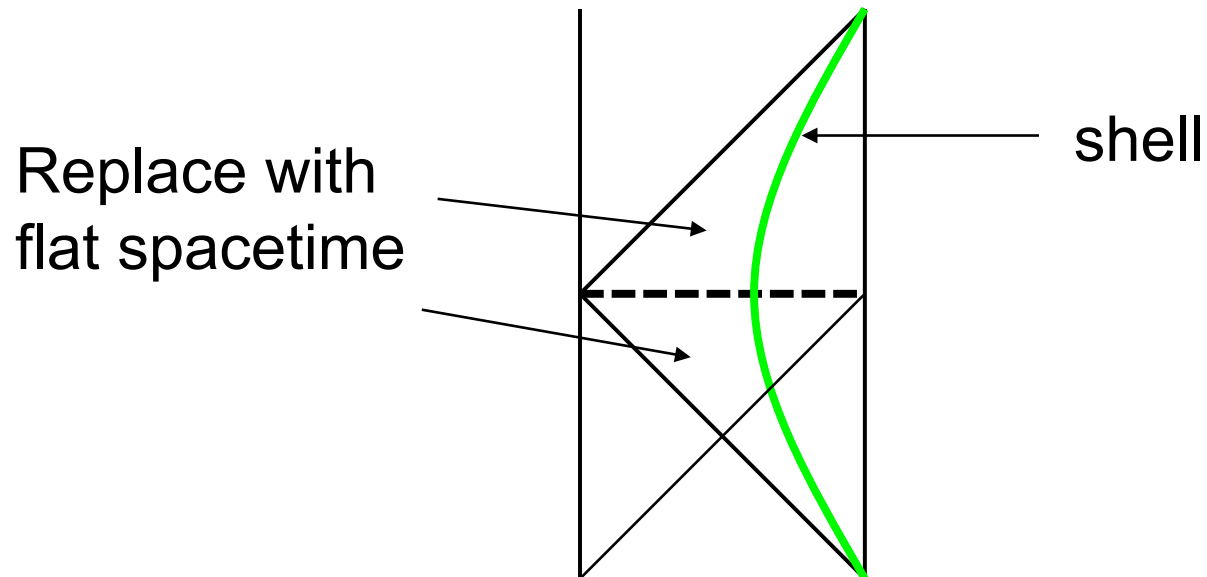
- While both involve potentials unbounded from below, for $V = -\phi^2$ it takes infinite time to roll down. Don't need self adjoint extensions.
- Don't need modified boundary conditions or multitrace operators.
- The physics near the singularity now takes place near the origin of ϕ rather than infinity.
The theory remains perfectly well defined!

The background of the slide is an abstract composition of diagonal lines and gradients. On the left side, there are dark blue and light blue diagonal bands that sweep across the frame. The right side is a lighter, more uniform gradient of blue and white. The overall effect is clean and modern.

II. Implications for the singularity

Consider a static spherical shell of D-branes (in Poincare coordinates).

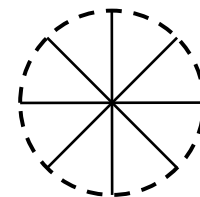
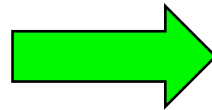
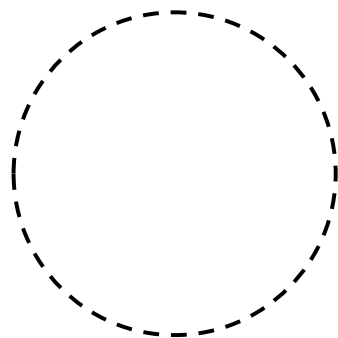
In the black hole interpretation, the shell collapses to form the hyperbolic black hole.



Initially, the SYM scalars are diagonal with eigenvalues coming in from infinity. The off diagonal modes are very massive. As the eigenvalues approach zero, the off diagonal modes become excited. The eigenvalues are trapped near zero. (Kofman et. al., 2004)

Spacetime picture:

Large
shell of
D-branes



Open
strings
excited

Since SYM is strongly coupled, you produce a complicated excited state involving all N^2 degrees of freedom.

Locality probably breaks down:

Away from the singularity, locality can be measured by scalar eigenvalues.

Near the singularity, all of the eigenvalues interact strongly with off-diagonal modes and with each other, the D-brane probes are no longer good definitions of any geometry.

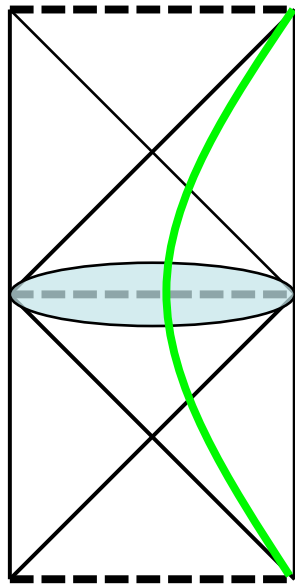
If N is infinite, the eigenvalues will be trapped forever. This describes the formation of a classical black hole.

If N is large but finite, eigenvalues will be trapped for a time $T \sim e^{cN}$.

This is Hawking evaporation of D-branes from the black hole.

(Finite N means quantum gravity important.)

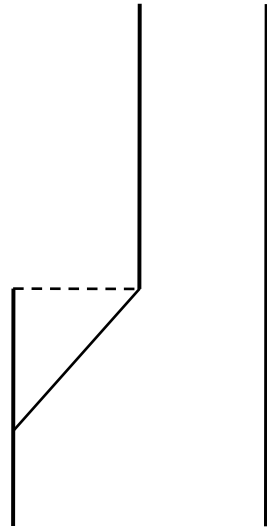
What is final spacetime picture? It is NOT just a smoothing out of the spacetime near the singularity.



not correct

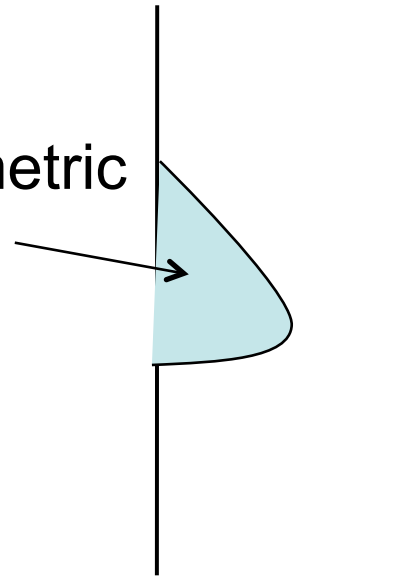
The branes come out in finite time in the SYM on the cylinder.

The branes emerge randomly, not as a coherent shell.



Standard picture
of evaporating
black hole in AdS

Nongeometric
region



Picture motivated by
dual field theory

Key lesson

Event horizons don't exist in quantum gravity
(cf: Ashtekar and Bojowald)

Event horizons require global causal relations which are not defined in spacetimes with nongeometric regions. (Trapped surfaces and apparent horizons will still exist.)

The singularity is resolved, but this is not an ordinary bounce. It takes a long time for things to pass through the singular region.

Cosmological implication

It is usually assumed that superhorizon size perturbations propagate unaffected through the bounce.

In our case that is unlikely. No causality constraint. We expect off-diagonal excitations to decay into inhomogeneous modes and change the spectrum of perturbations.

III. Generalizations

A hyperbolic black hole can have different masses: (Empanan)

$$ds^2 = - \left(r^2 - 1 - \frac{\mu}{r^2} \right) dt^2 + \frac{dr^2}{r^2 - 1 - \frac{\mu}{r^2}} + r^2 d\sigma^2$$

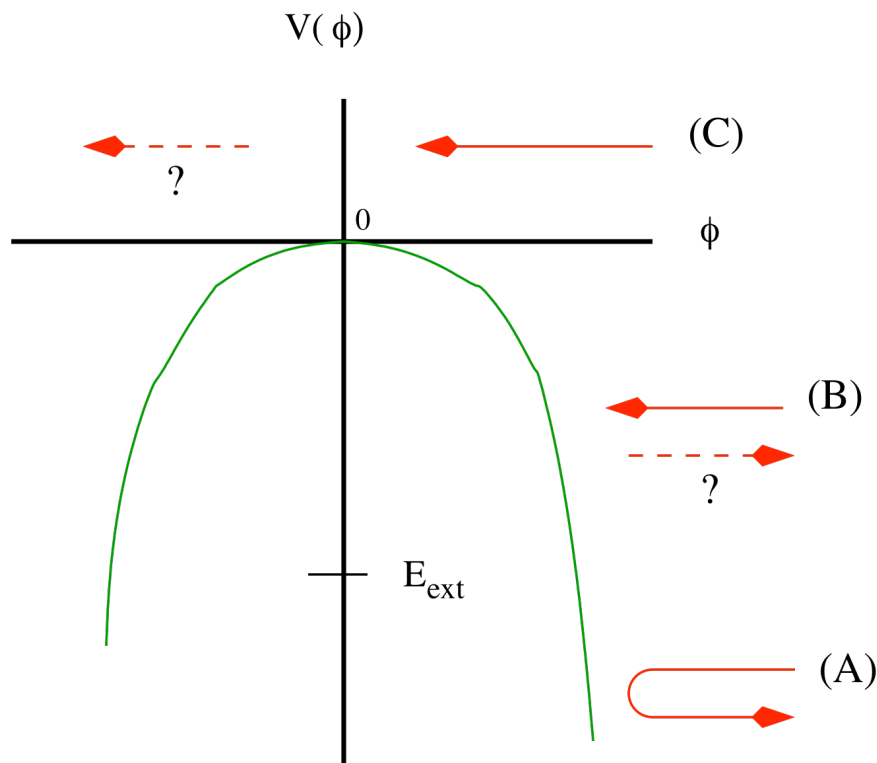
There are three cases:

$\mu > 0$: black hole with spacelike singularity

$-1/4 < \mu < 0$: black hole with timelike singularity

$\mu < -1/4$: naked singularity

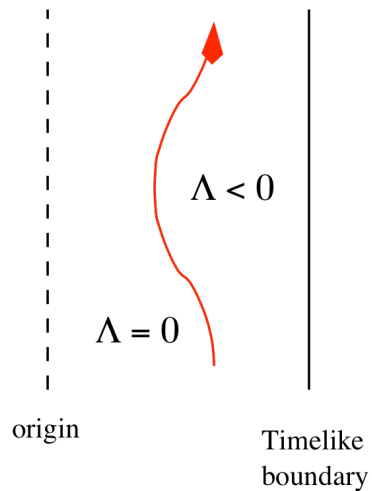
These cases are correlated with the possible motion of the scalars:



For $E > E_{\text{ext}}$
classical evolution
is modified by
quantum
corrections

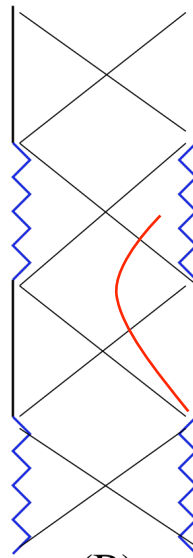
$$E = E_{\text{ext}} \\ \text{corresponds to} \\ \mu = -1/4$$

Motion of shell in classical spacetimes



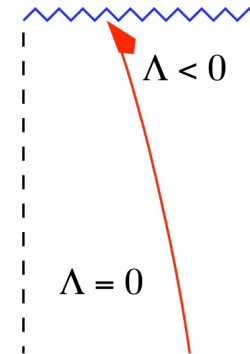
(A)

Can't form a naked singularity since the shell bounces



(B)

This case is currently under study



(C)

General procedure for describing D-branes falling into a black hole

CFT dual of a black hole usually describes physics in Schwarzschild coordinates since action is invariant under $t \rightarrow -t$ and $t \rightarrow t + c$.

In the bulk, to get time slices that cross the horizon, one needs a coordinate change

$$\tilde{t} = t + g(r)$$

The low energy dynamics of D-branes is given by the DBI action. Let us assume D-brane wraps S^3 , T^3 or H^3 so the only dynamics is $r(t)$.

In the super Yang-Mills, the DBI action describes the low energy dynamics of one of the scalar eigenvalues $\phi(t)$.

The analog of the coordinate transformation in the bulk is a

field dependent time reparameterization.

Example: $\mu = 0$ hyperbolic black hole

The DBI action in Schwarzschild coordinates is

$$S = - \int dt \left[r^3 \sqrt{(r^2 - 1) - \frac{\dot{r}^2}{r^2 - 1}} - r^4 \right] ,$$

This breaks down near the horizon. So $\phi_s(t)$ is not a good semi-classical variable near the horizon.

To get Poincare time slices, set

$$\tilde{t} = t + \frac{1}{2} \ln \left(1 - \frac{1}{r^2} \right)$$

The low energy dynamics of D-branes in the coordinates $\tilde{r} = r, \tilde{t}$ is given by the DBI action

$$S = - \int d\tilde{t} \tilde{r}^2 \left\{ \sqrt{\tilde{r}^4 - \left(\frac{d\tilde{r}}{d\tilde{t}} + \tilde{r} \right)^2} - \tilde{r}^2 \right\}$$

which does not break down at the horizon.

To implement this in the field theory:

Introduce a time reparameterization invariance, $t = t(\lambda)$ and then fix the gauge in a way that depends on the scalar field:

$$\lambda = t \quad \text{or} \quad \lambda = t + g(\phi)$$

The Hamiltonian does not change, but the momentum conjugate to the scalar field does change. So the form of $H(\pi, \phi)$ changes.

This is just a field redefinition in the CFT, so the physics doesn't change. But:

It allows one to find the right variables which remain semi-classical through the horizon.

Conclusions

- A simple hyperbolic black hole is equivalent to part of the Poincare patch of AdS
- Can describe formation of a hyperbolic black hole by collapsing a shell of D-branes.
- The physics near the singularity is governed by the SYM with small ϕ . The problem of singularities is no longer that the theory breaks down but simply that it is hard to calculate.

- The event horizon is not well defined in the quantum theory.
- The qualitative behavior of hyperbolic black holes with different mass is correlated with behavior of the scalars in the field theory.
- To change the time slices in the bulk, you need a field dependent time reparameterization in the field theory.

Open questions

- Understand the physics near the singularity better.
- How does the CFT state describing the singularity differ from the thermal state seen by an outside observer?
- For general black holes, extend the description of infalling D-branes to a description of all physics inside.
- Understand the $\mu < 0$ case better in terms of the gauge theory.

**Happy 60th Birthday
Tom and Willy!**