Symmetry and Aesthetics in Contemporary Physics

CS-10, Spring, 2016

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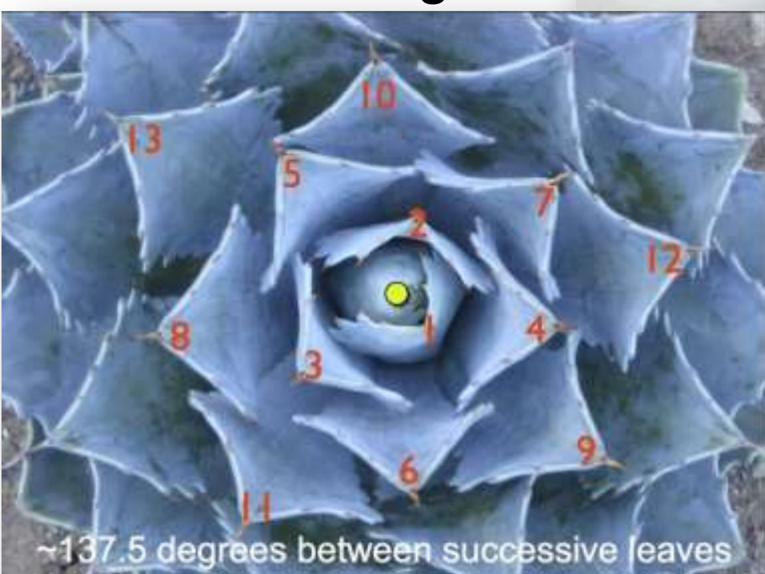
CLASS 3:
SYMMETRY IN PHYSICAL LAWS

FYI:

3-D Fibonacci aniforms –

you can make on the 3-D printer in the Physics Department.

put on turn table borrow strobe light



http://www.instructables.com/id/Blooming-Zoetrope-Sculptures/step2/How-these-were-made/

http://vimeo.com/116582567

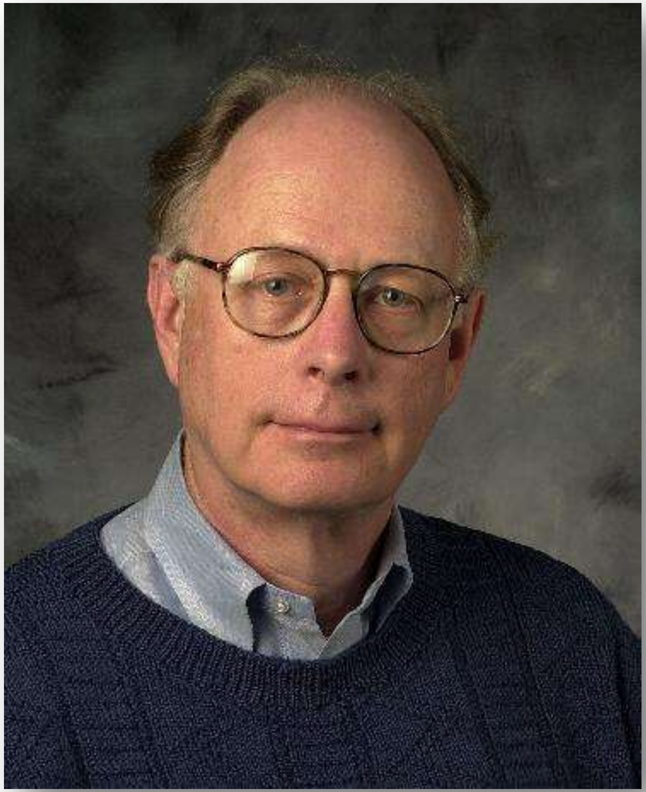
Jim Hartle:
Theories of Everything and Hawking's Wave Function of the Universe

1. General comments about the article?

2. What are fundamental laws of nature?

3. Why does he say there are two parts to fundamental laws?

4. Do you agree that initial conditions are fundamental laws?



Professor Jim Hartle, UCSB

Hartle on fundamental physical laws

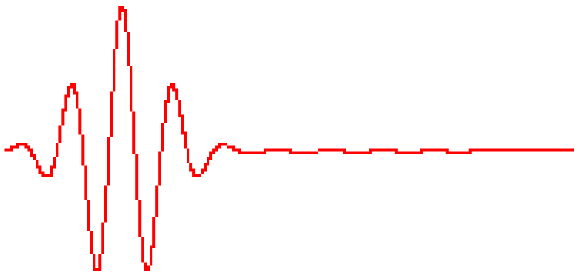
Dynamical laws predict regularities in time

Initial conditions predict regularities in space

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

where H is a function that describes the sum of kinetic and potential energies of a system over time

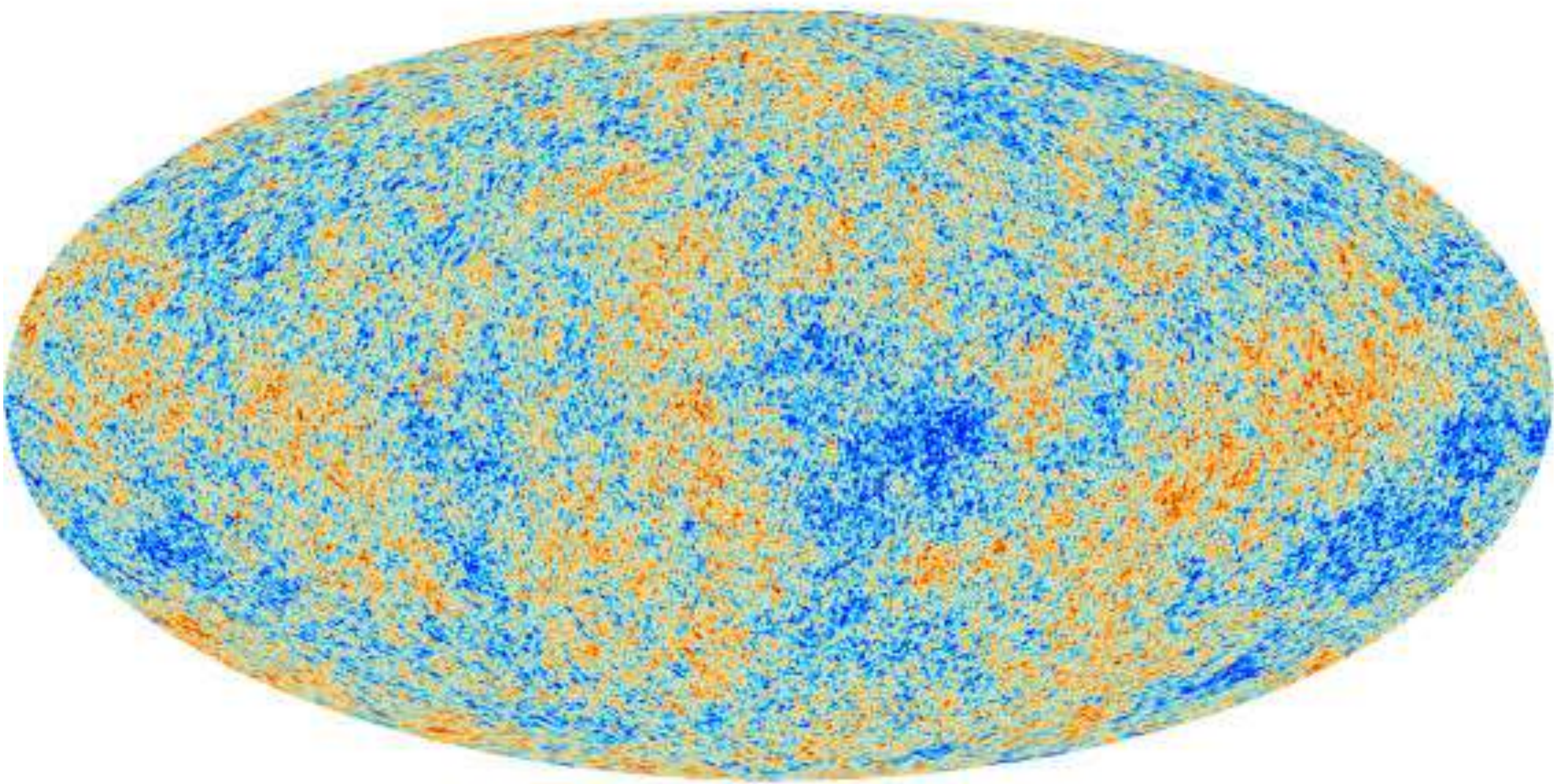
$$\Psi = \int \delta g\, \delta\phi\, e^{-I(g,\phi)}$$



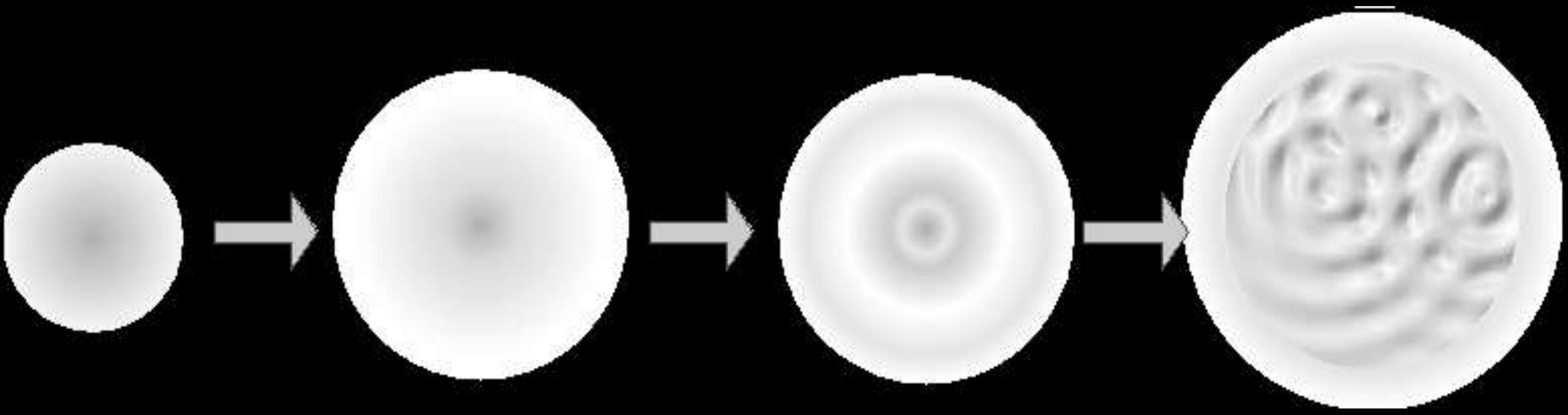
Initial conditions of the universe: "Something" from "nothing???"
–a "frozen quantum accident that produced emergent regularities?" (p.12)

Observing the Initial Conditions

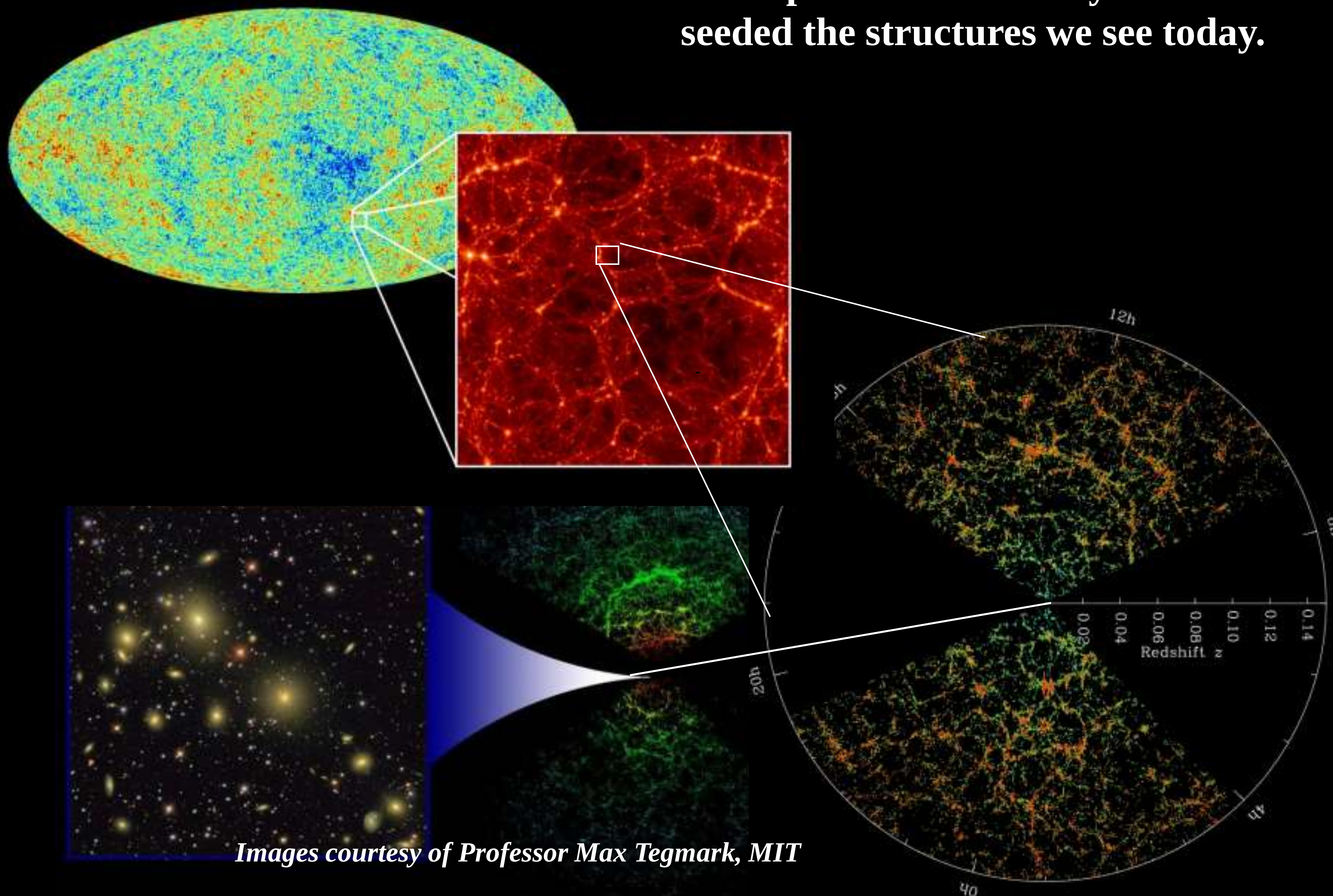
The Cosmic Microwave Background – oldest light we can detect, from 13.8 Gyr ago (almost to the Big Bang)



The formation of all structure in the universe began with sound waves reflecting off the edge of the expanding universe and interfering with each other, creating interference patterns which led to small density contrasts in the plasma of the early universe.

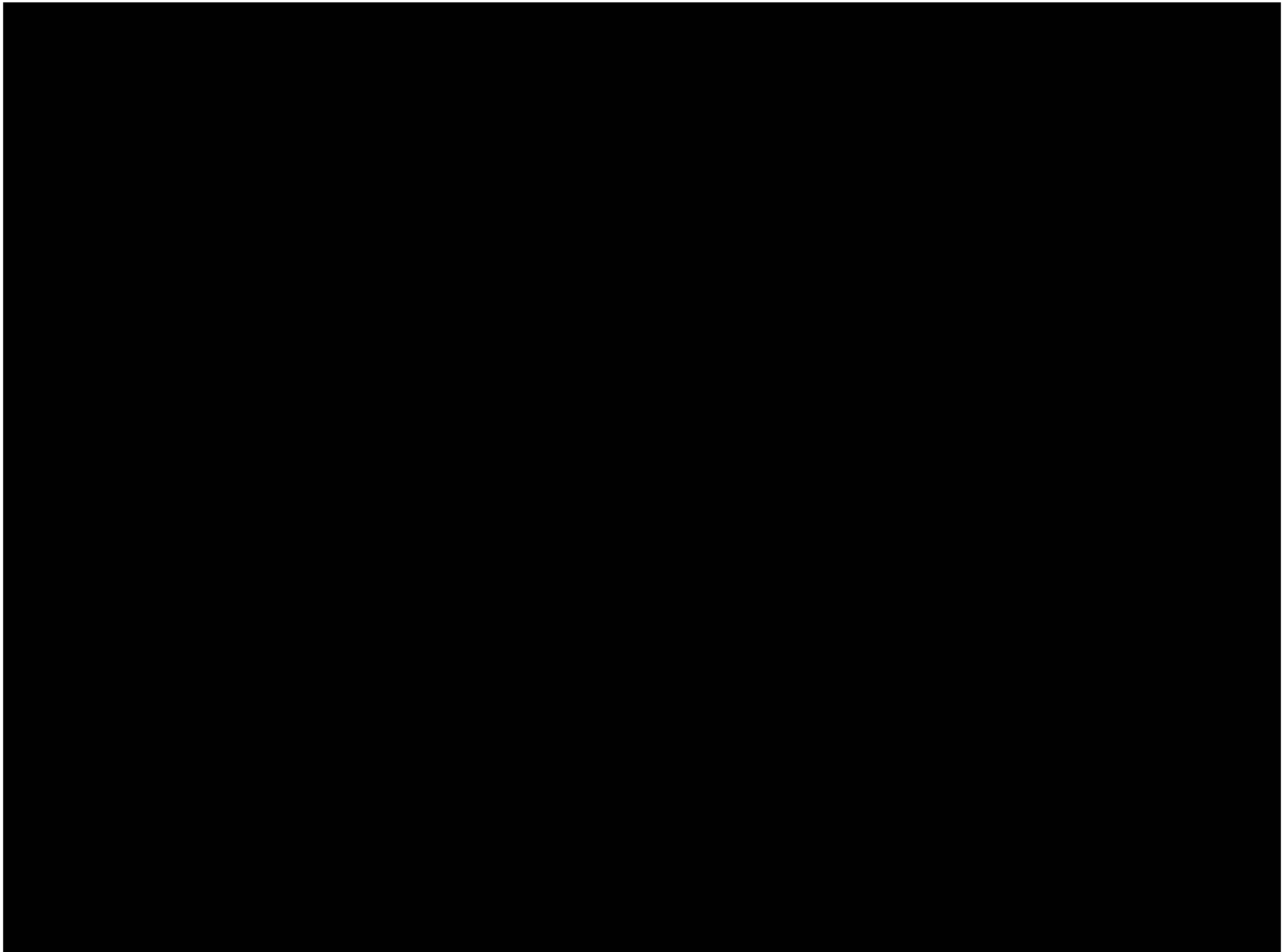


These primordial density contrasts in the plasma of the early universe seeded the structures we see today.

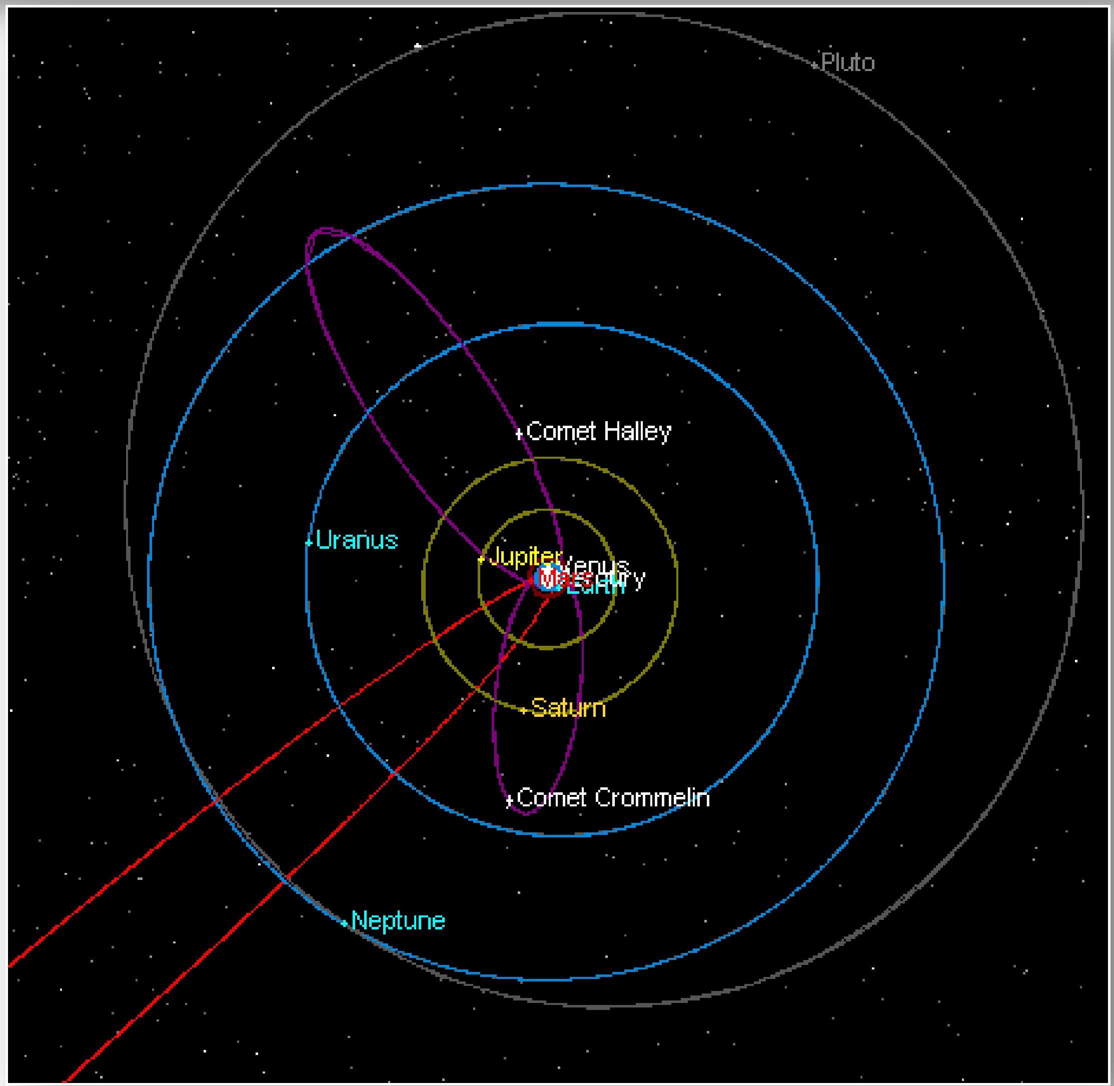


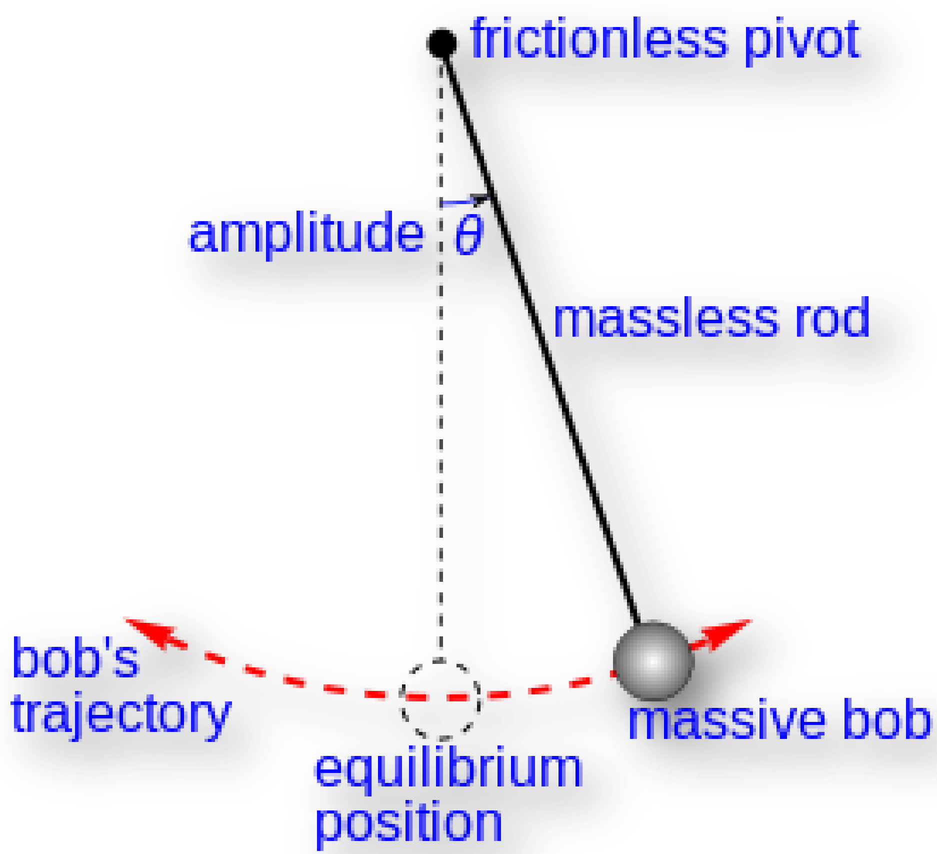
Images courtesy of Professor Max Tegmark, MIT

SDSS: LARGE SCALE DISTRIBUTION OF GALAXIES



The fundamental laws of physics constituting a 'theory of everything' are those which specify the regularities exhibited by every physical system, without exception, without qualification, and without approximation.





Initial conditions:
length, mass, $\theta_{initial}$

Pull the bob aside to some angle θ and let it go.

Dynamical laws of physics predict its motion IF no forces other than gravity act on it.

Lagrangian formulation:

$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

The fundamental laws of physics constituting a 'theory of everything' are those which specify the regularities exhibited by every physical system, without exception, without qualification, and without approximation.

https://www.youtube.com/watch?v=N6cwXkHxLsU&nohtml5=False

A theory of everything is not (and cannot be) a theory of everything in a quantum mechanical universe.

The regularities of human history, personal psychology, economics, biology, geology, etc. are consistent with the fundamental laws of physics, but do not follow from them.

Chandrasekhar on the laws of Nature:

S. Chandrasekhar
Nobel Laureate
1910-1995

- Physical theories are motivated by a sense of aesthetics.
- Physical theories arise out of archetypal patterns of harmony.
- “Beauty is the proper conformity of the parts to one another and to the whole.”
- “There is no excellent beauty that hath not some strangeness in its proportions.”

symmetries predict regularity in physical laws…

1. **The laws of physics are invariant to translations in time**
2. **The laws of physics are invariant to translation in space**
3. **The laws of physics are invariant to rotations in spacetime**

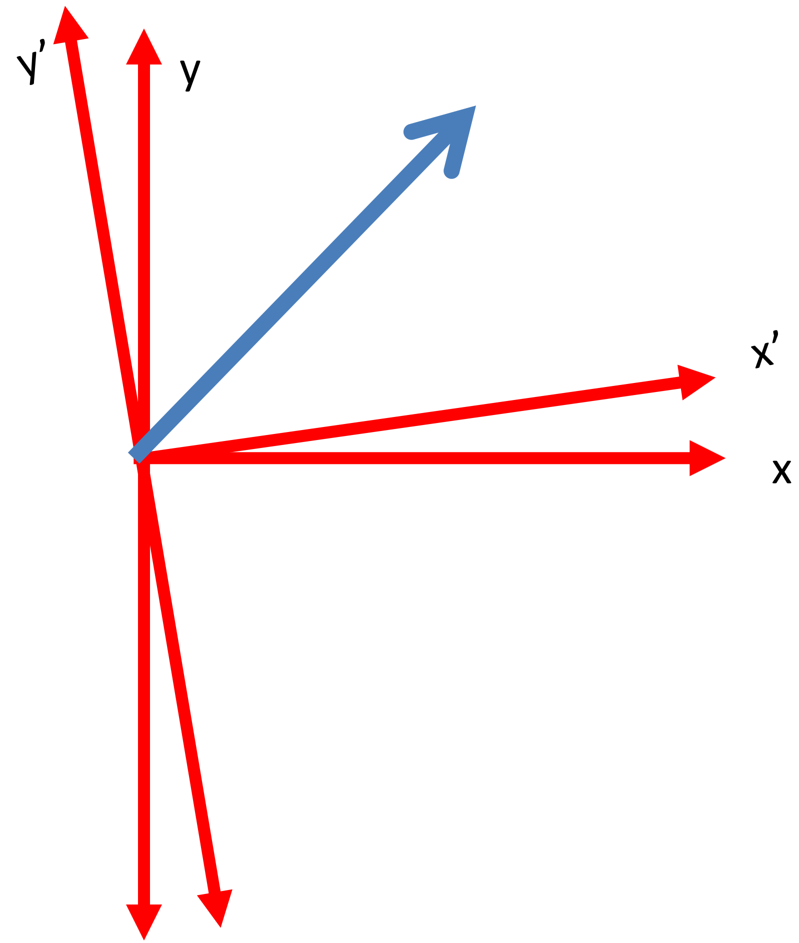
...which are related to conserved quantities in nature

Noether's Theorem:

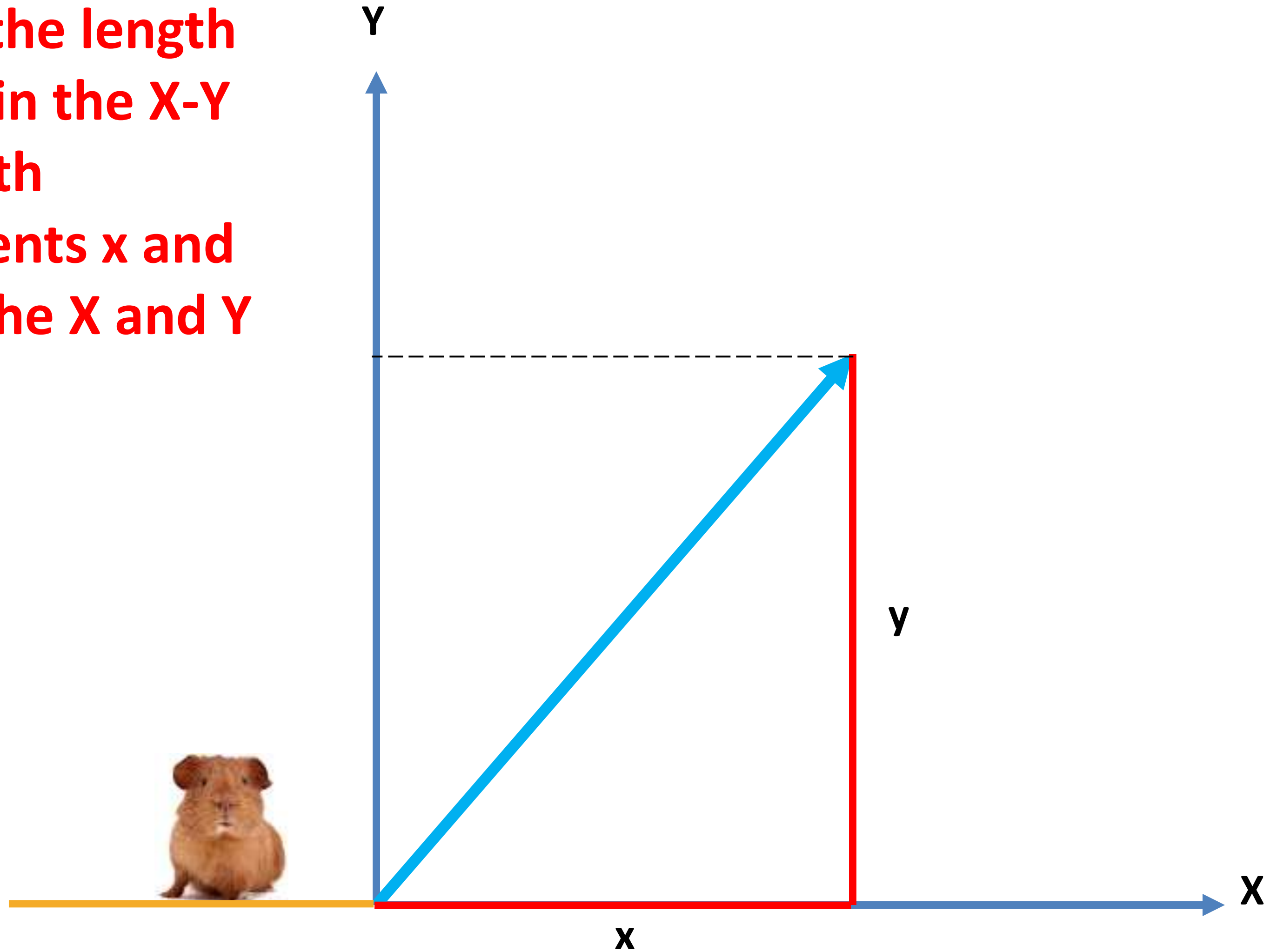
For every symmetry of the *Lagrangian*, there is a conservation law in Nature.

Example:

We will illustrate that the laws of physics are invariant under rotation by looking at the conservation of the length of a line under rotation of coordinate axes.



Let L be the length of a line in the X-Y plane with components x and y along the X and Y axes.

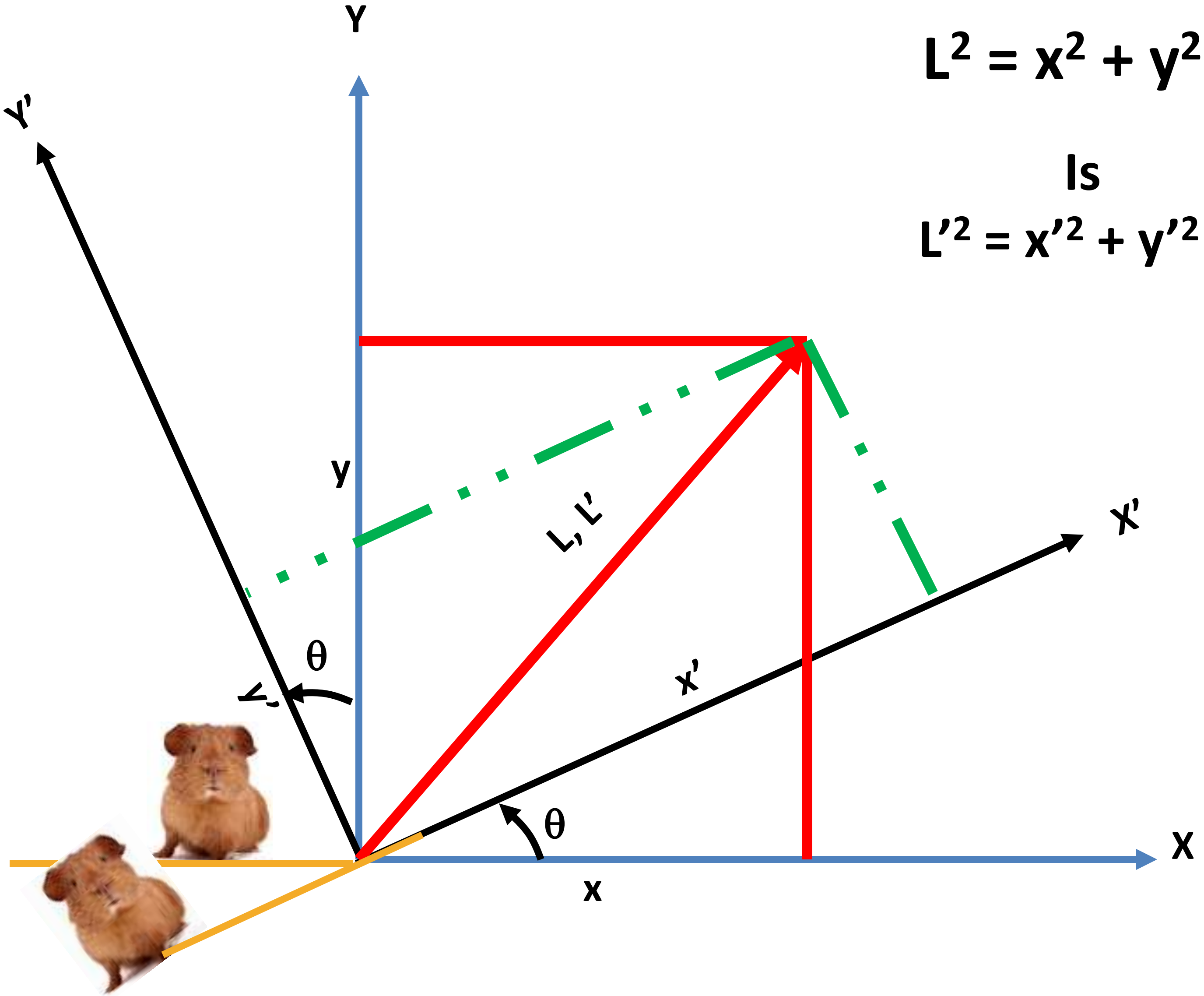


We ask: does L change if we rotate our viewpoint?

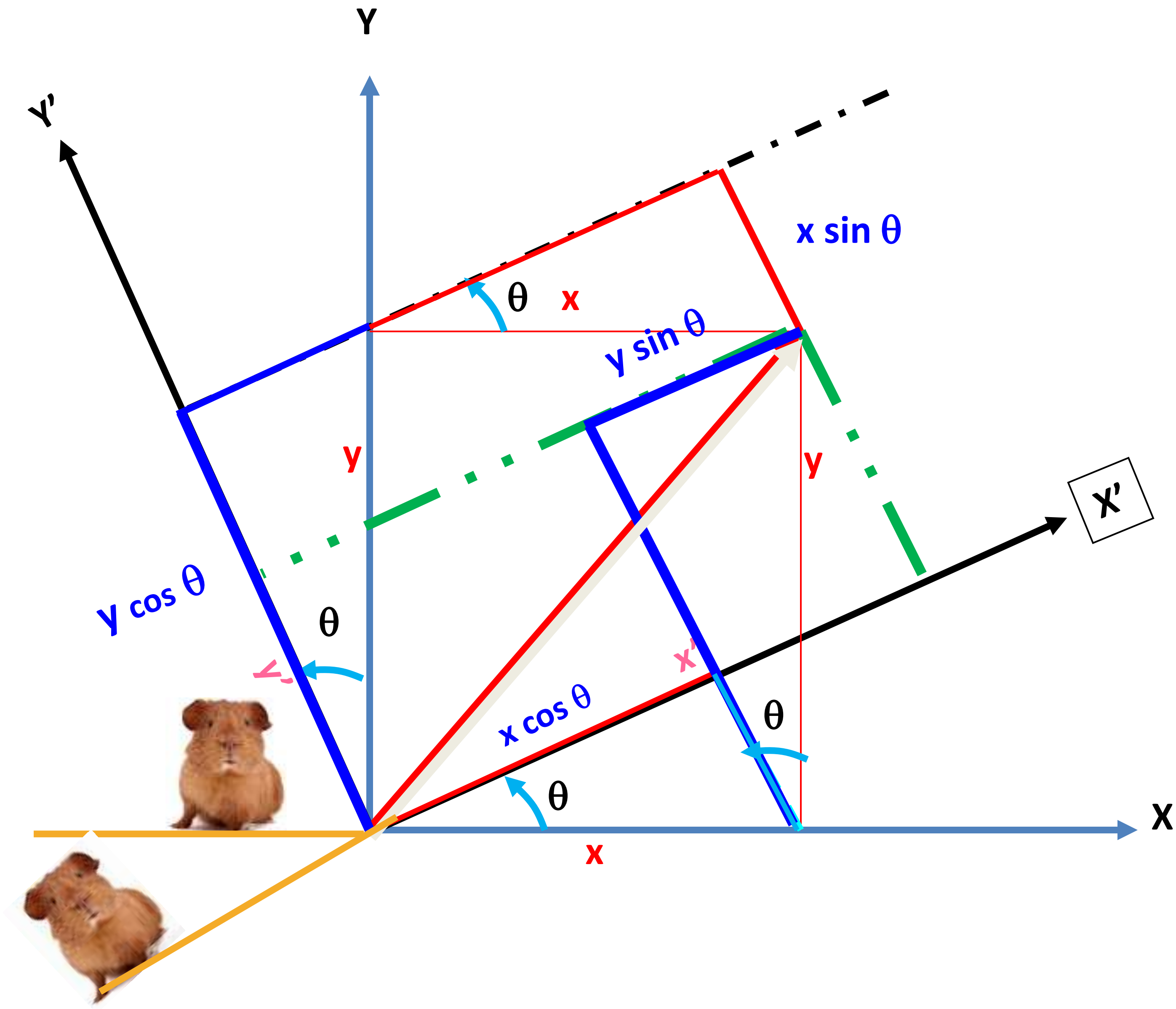
$$L^2 = x^2 + y^2$$

Is

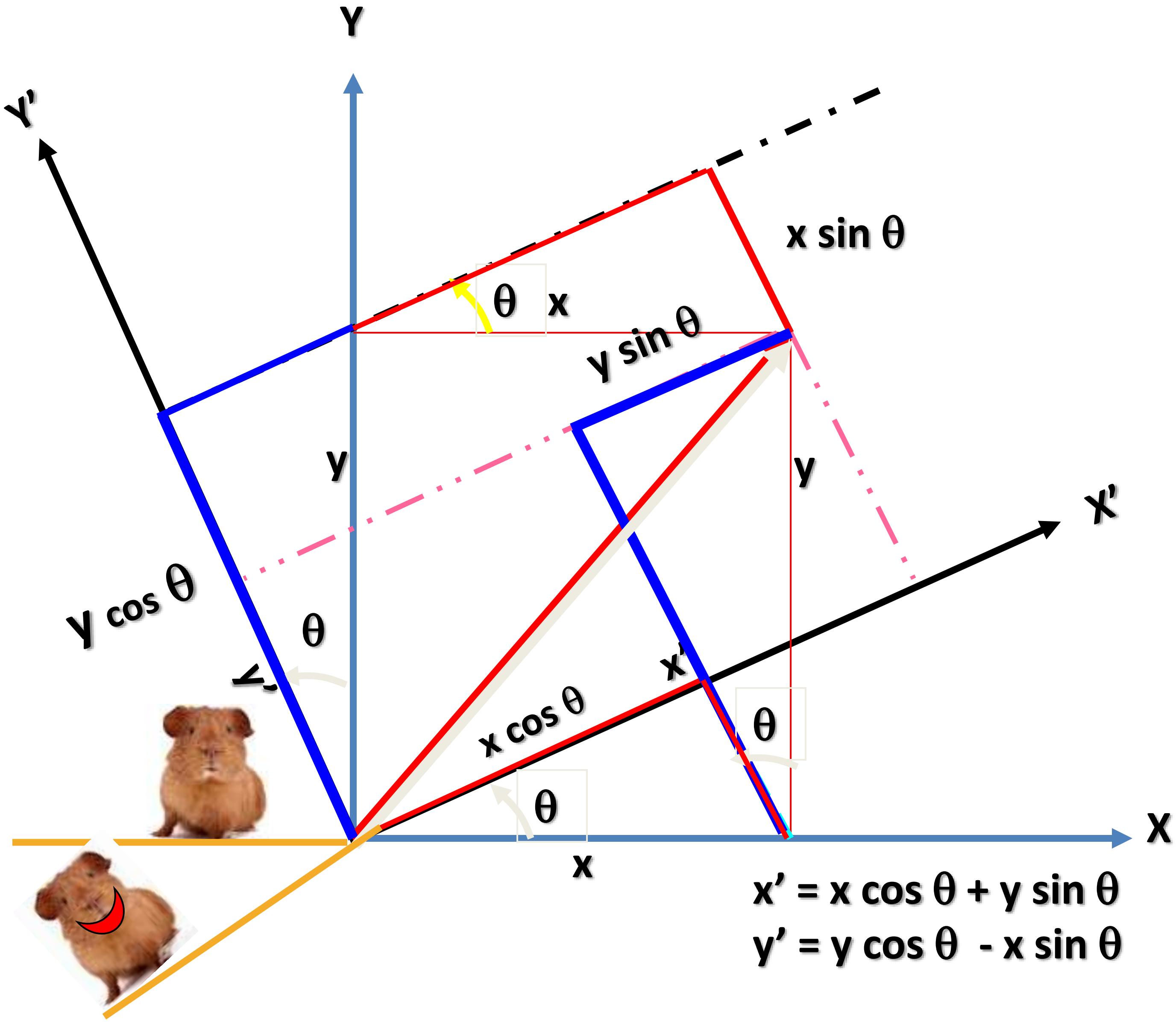
$$L'^2 = x'^2 + y'^2 \,?$$



Can you prove that $L^2 = L'^2$?



Y
Y'
x sin θ
θ
x
y sin θ
y
y
X'
y cos θ
θ
x'
x cos θ
θ
θ
X
x



$$L'^2 = x'^2 + y'^2 = x^2 (\cos^2\theta + \sin^2\theta) + y^2 (\cos^2\theta + \sin^2\theta) = x^2 + y^2 = L^2$$

So, Look! We have derived a rule for describing rotations in the x - y plane:

$x' = x \cos\theta + y \sin\theta$

$y' = -x \sin\theta + y \cos\theta$

for any angle θ

Another way to visualize this rule is to write it in “matrix” notation:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

coordinates in the rotated reference frame

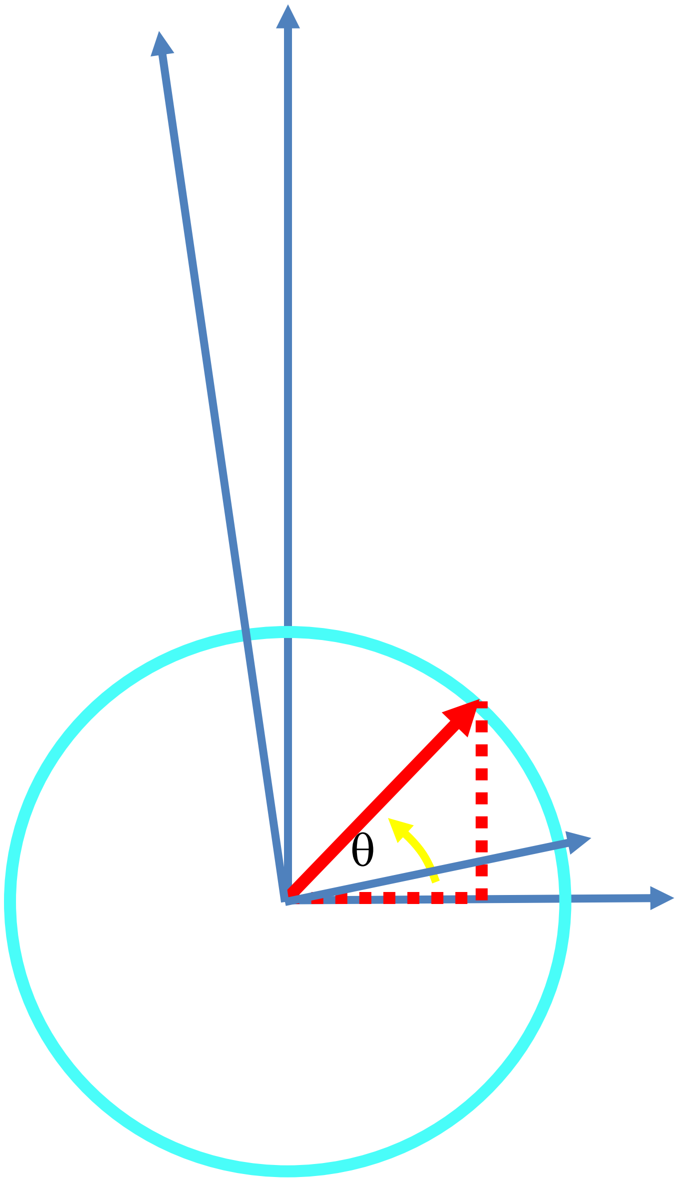
rule we apply

coordinates in the original reference frame

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x' = x\cos(\theta) + y\sin(\theta)$$

$$y' = -x\sin(\theta) + y\cos(\theta)$$



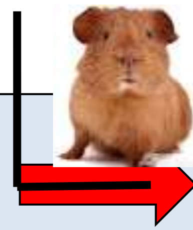
This is the rule that defines all objects with <u>continuous</u> symmetry in a plane – in other words the symmetry of a CIRCLE. This group is called SO(2) or Special Orthogonal Group of order 2.

rotations in the x-y plane:

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$x' = x\cos(\theta) + y\sin(\theta)$$
$$y' = -x\sin(\theta) + y\cos(\theta)$$

START HERE ON +x axis:



rotate by 90^0: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

rotate by 180^0: $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

rotate by 270^0: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

rotate by 360^0: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

let's work with 90^0 rotations, but the rule applies to any angle

θ	cos (θ)	sin (θ)
0	1	0
90^0	0	1
180^0	-1	0
270^0	0	-1
360^0	1	0

RESULTS

= IDENTITY ELEMENT

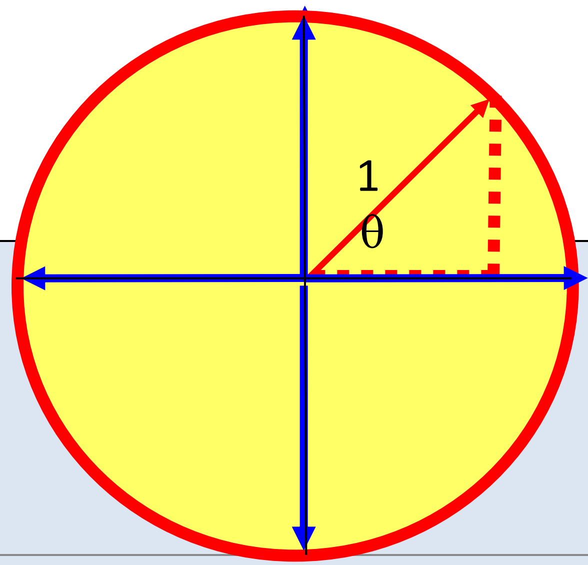
Another view :

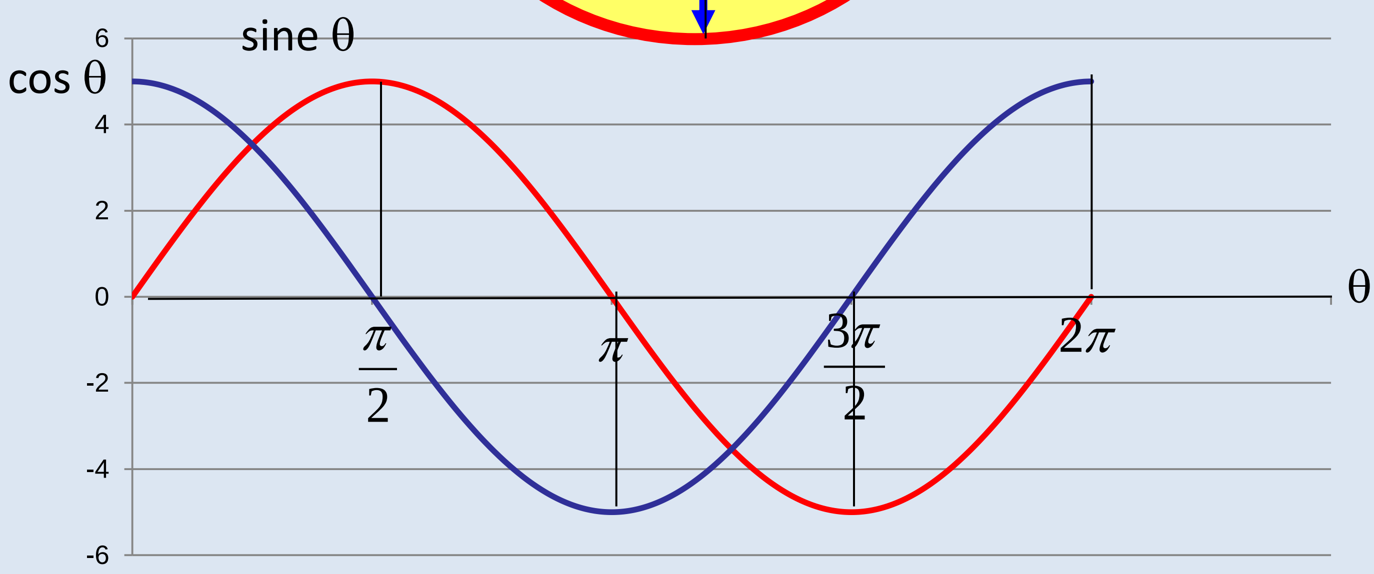
$$radius = 1$$

$$\sin\theta = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{1} = x$$

$$\sin^2\theta + \cos^2\theta = 1$$





We have just defined the group SO(2): Special Orthogonal group of order 2 which describes rotations in the Real plane.

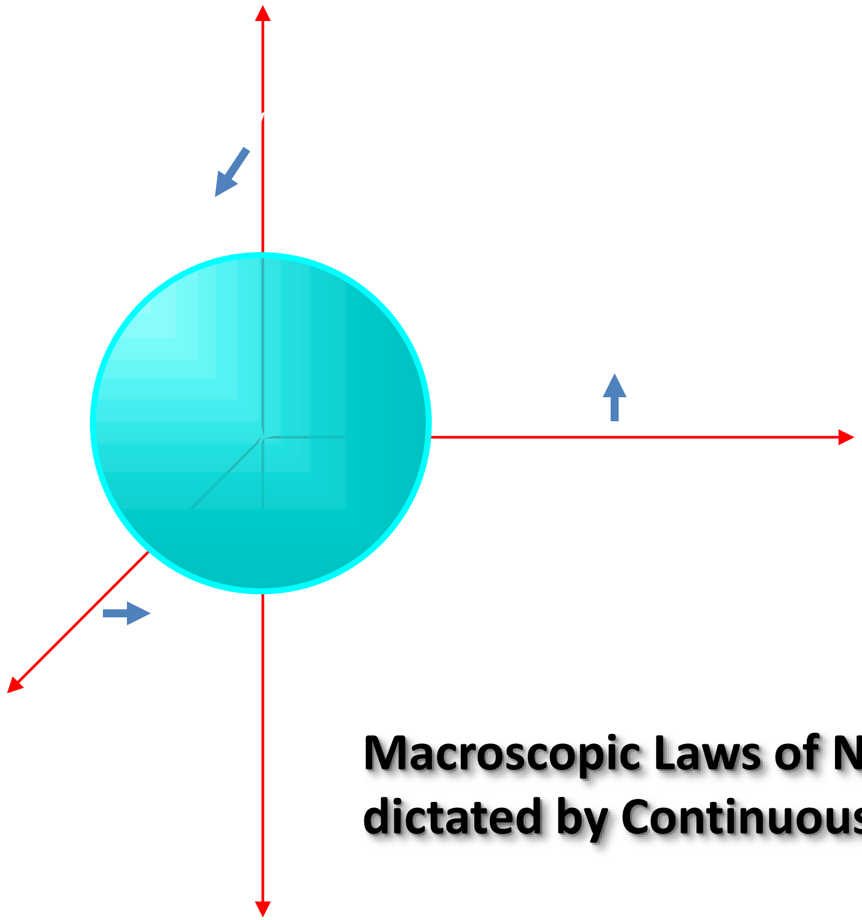
*** The group consists of rotations described by sines and cosines.**

*** The group SO(2) is closed under matrix multiplication.**

*** We found the identity element which "does nothing" to an object (we used a vector, or line segment).**

- **Each element (rotation by θ) has an inverse (rotation by 360- θ), such that $r \otimes r^{-1} = I$.**

A sphere has continuous symmetry in 3 dimensions. Since a rotation of ANY amount leaves the sphere unchanged, we specify the symmetry operations simply by the rotation angles in each of the three spatial directions that we rotate the sphere.



Macroscopic Laws of Nature appear to be dictated by Continuous Symmetries.

SO(n) :
"n" generators of the group, [n(n+1)/2] degrees of freedom

Group	Representations	Degrees of freedom
SO(2)	Circle, motion in a plane	[2(2+1)/2] = 3 d.f. 1 rotation angle, 2 directions of translation
SO(3)	Rotations on a sphere	[3(3+1)/2] = 6 d.f. 3 rotation angles, 3 directions of translation
SO(4) "Poincare Group"	Spacetime	[4(4+1)/2] = 10 d.f. 3 rotation angles, 3 directions of translation, 3 'boosts' 1 direction of time
SO(5)	?	[5(5+1)/2] = 15 d.f.

Time for a break…

Noether's Theorem:

For every continuous symmetry in Nature*, there is a corresponding Conservation Law in physics.

Professor Emmy Noether

Conserved quantities that we can observe:

1. **Conservation of energy**
2. **Conservation of momentum**
3. **Conservation of angular momentum**

1. The laws of physics are invariant to translations in time
2. The laws of physics are invariant to translation in space
3. The laws of physics are invariant to rotations in spacetime

Fundamental Constants of Nature: properties of our universe?

c	G	h
speed of light in a vacuum	universal gravitational constant	Planck's constant

As far as we know, these are constant over all space and all time in our universe.

$$\text{The Planck length:} \quad \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.6 \times 10^{-35} \text{ metres,}$$

$$\text{The Planck mass:} \quad \left(\frac{\hbar c}{G}\right)^{1/2} = 2.1 \times 10^{-8} \text{ kilograms,}$$

$$\text{The Planck time:} \quad \left(\frac{\hbar G}{c^5}\right)^{1/2} = 5.4 \times 10^{-44} \text{ seconds,}$$

$$\text{The Planck energy:} \quad \left(\frac{\hbar c^5}{G}\right)^{1/2} = 1.2 \times 10^{19} \text{ GeV.}$$

Fundamental scales of length, mass, and time are defined in terms of fundamental constants of Nature.

At least, as far as we know, they are fundamental properties that define our universe.

Initial conditions?

Planck length:

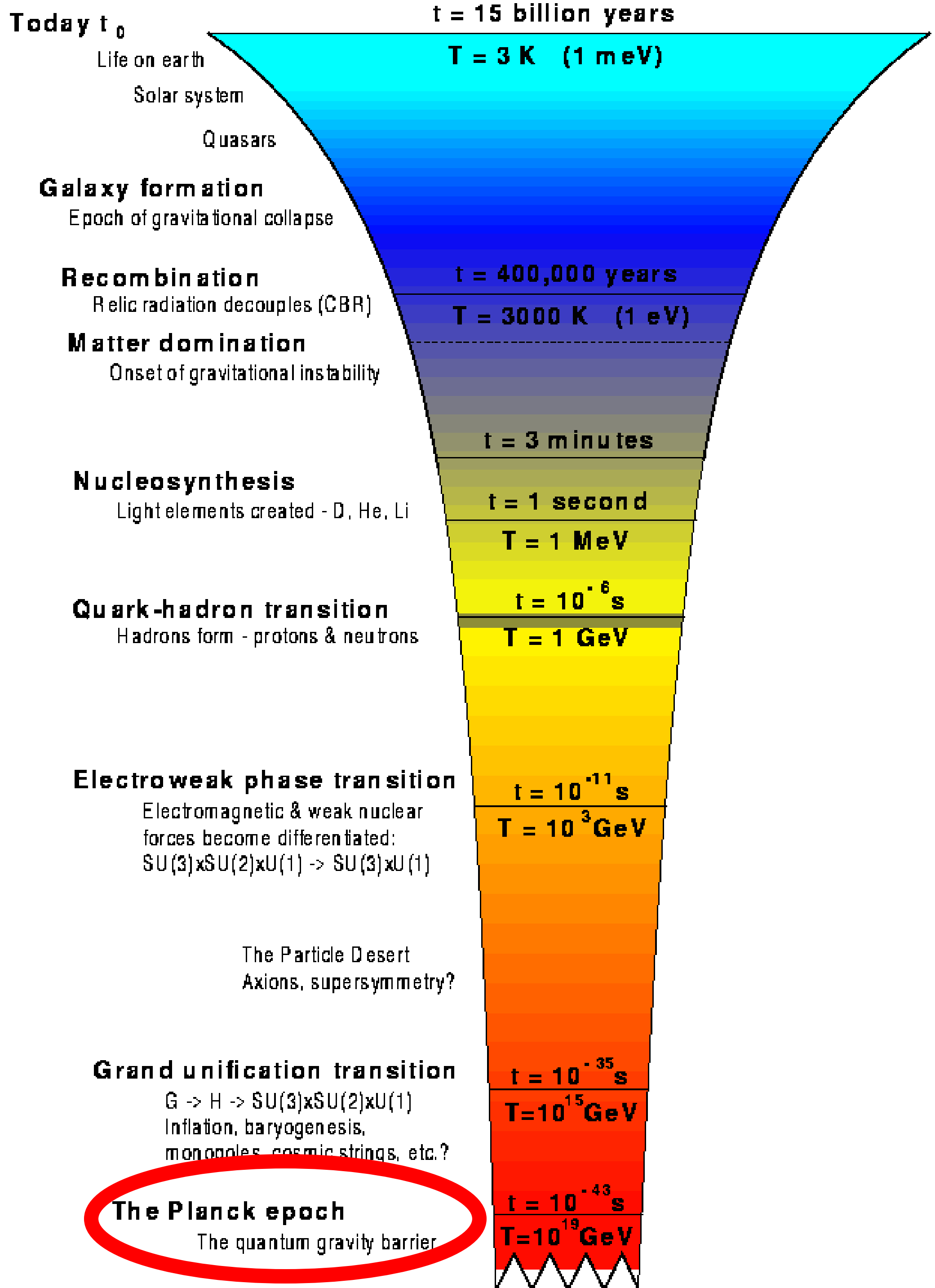
$$l_P = \sqrt{\frac{hG}{2\pi c^3}} \cong 1.6\times10^{-33}cm$$

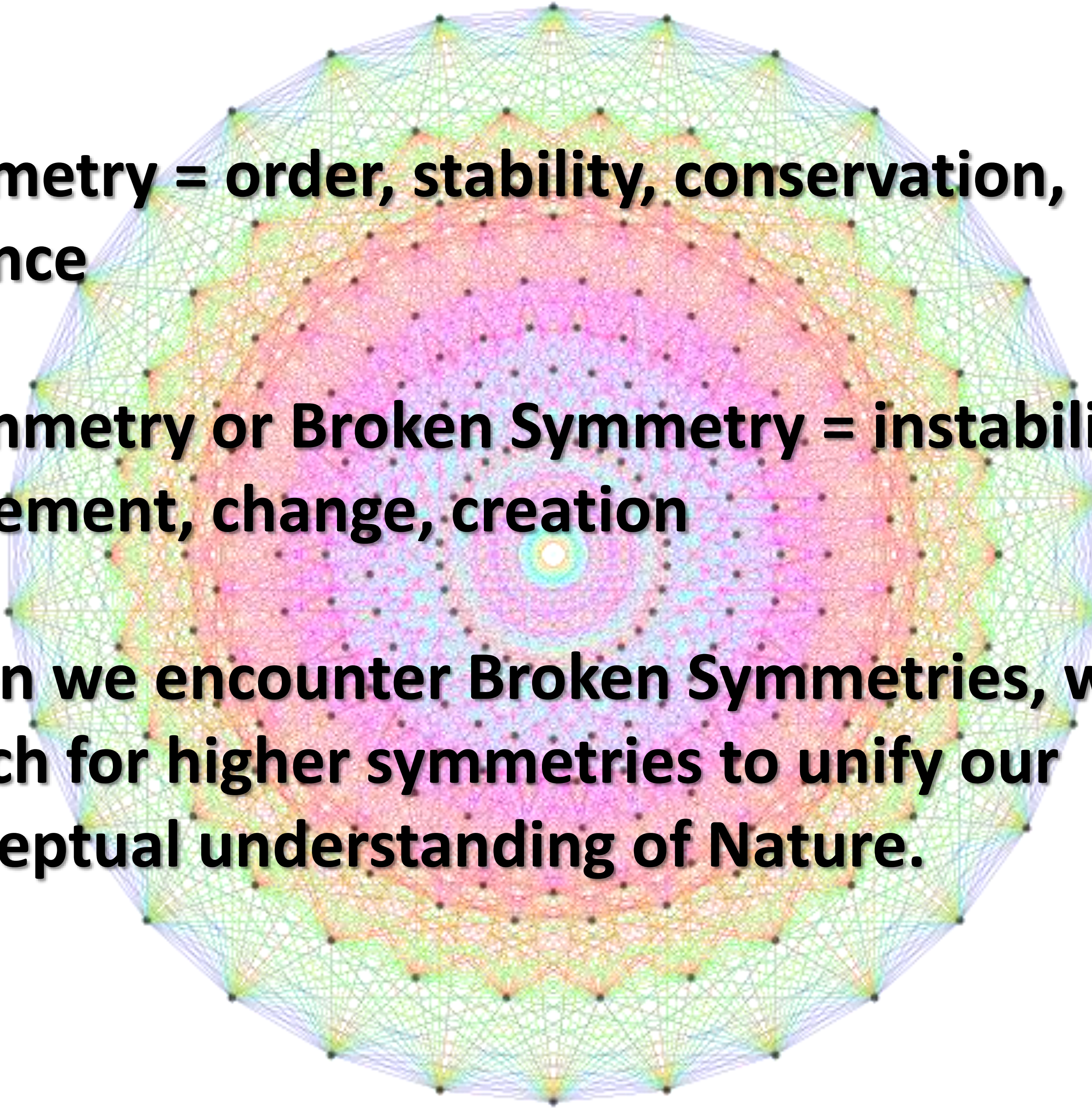
Planck mass:

$$m_P = \sqrt{\frac{hc}{4\pi G}} \cong 1.5\times10^{-5}gr$$

Planck energy:

$$10^{19}GeV \cong 1.8\times10^{-5}gr$$





Symmetry = order, stability, conservation, balance

Asymmetry or Broken Symmetry = instability, movement, change, creation

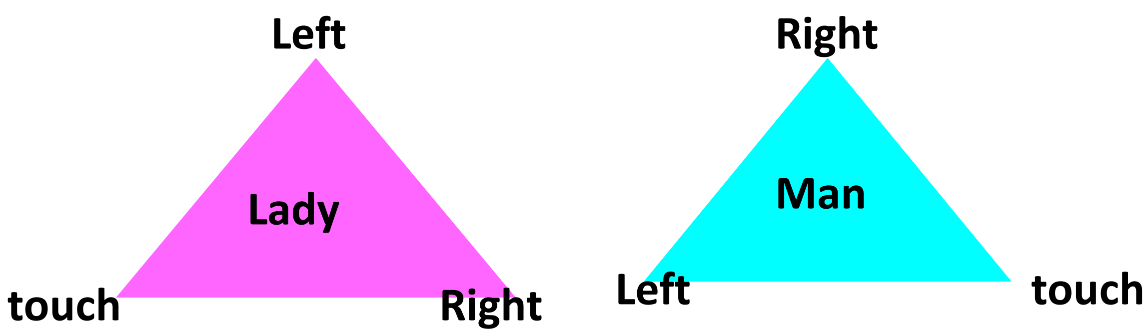
When we encounter Broken Symmetries, we search for higher symmetries to unify our conceptual understanding of Nature.

folk dance as culturally-situated mathematics

Scandinavian folk dances as representations of D_3

In many Scandinavian couple dances, the Lady and Man rotate simultaneously, 120^0 out of phase with each other, and the music is in 3 / 4 time.

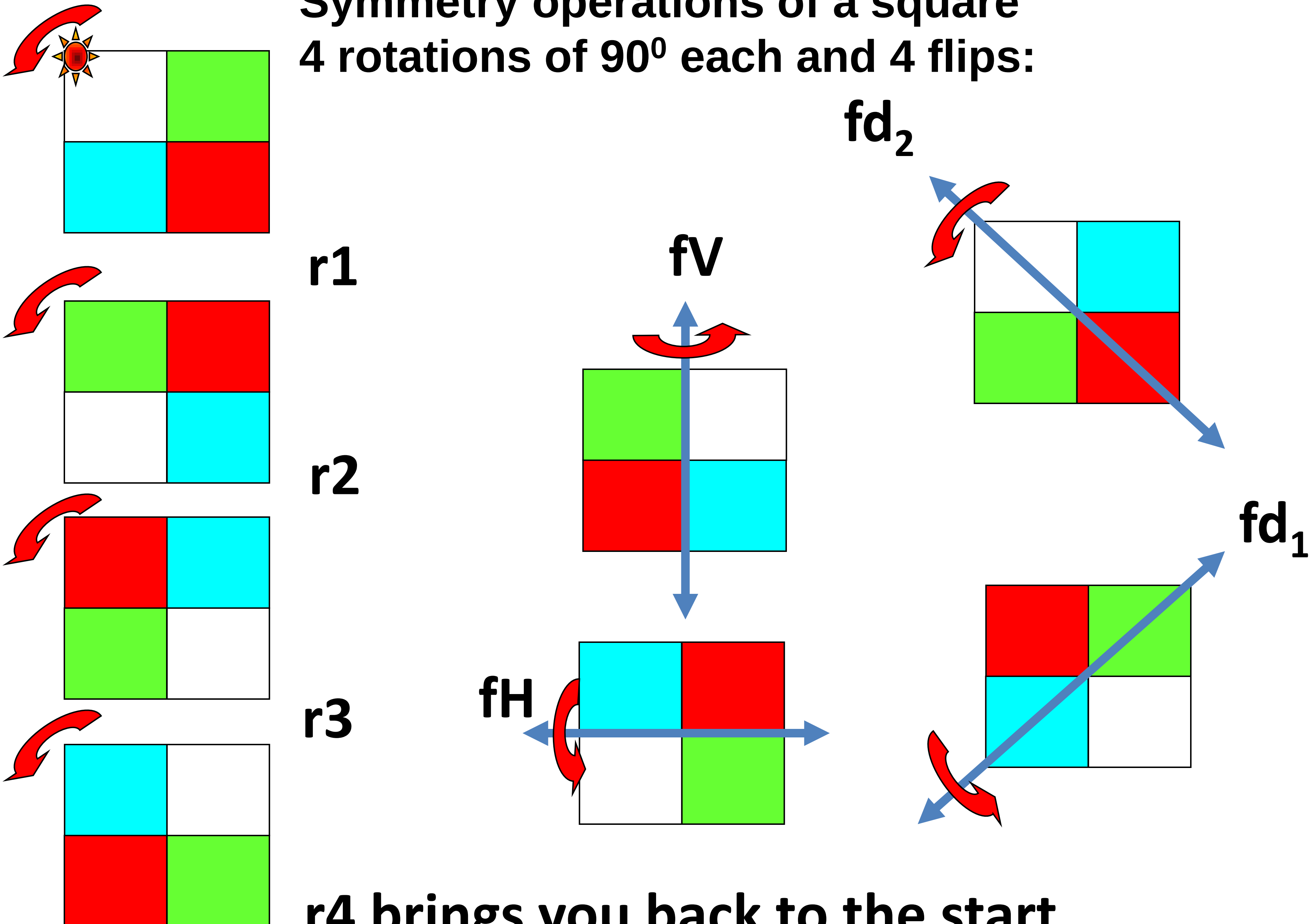
https://www.youtube.com/watch?v=nJYwODr8700



Contra Dance is based on permutations of a group of 4.

Embedded in the dance structure
is the tension between symmetry breaking
and a return to order.

Symmetry operations of a square
4 rotations of 90^0 each and 4 flips:



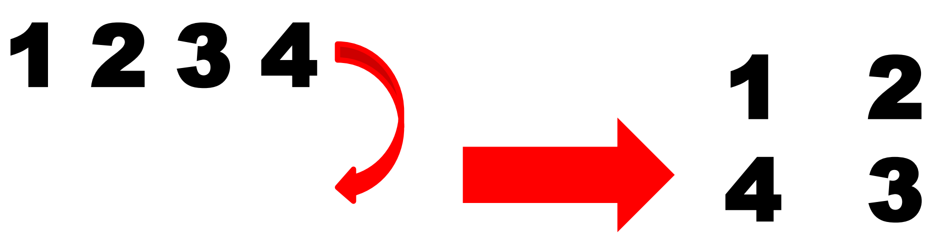
r4 brings you back to the start

Permutations of a set of four numbers:

1 2 3 4

There are 4! (4 x 3 x 2 x 1) = 24 unique ways to permute a set of four numbers (or any four individual objects).

For convenience, let's bend the line of numbers into a square:



8 of the permutations of the set of 4 numbers are the same as the 8 symmetry operations of a square:

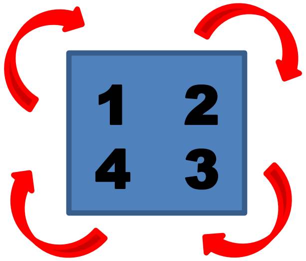
“Rotations”

4 1 2 3
3 4 1 2
2 3 4 1
1 2 3 4

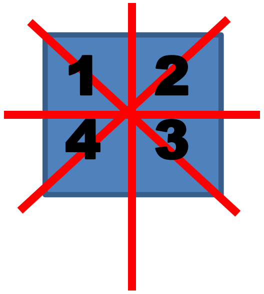
“Reflections”

2 1 4 3
4 3 2 1
3 2 1 4
1 4 3 2

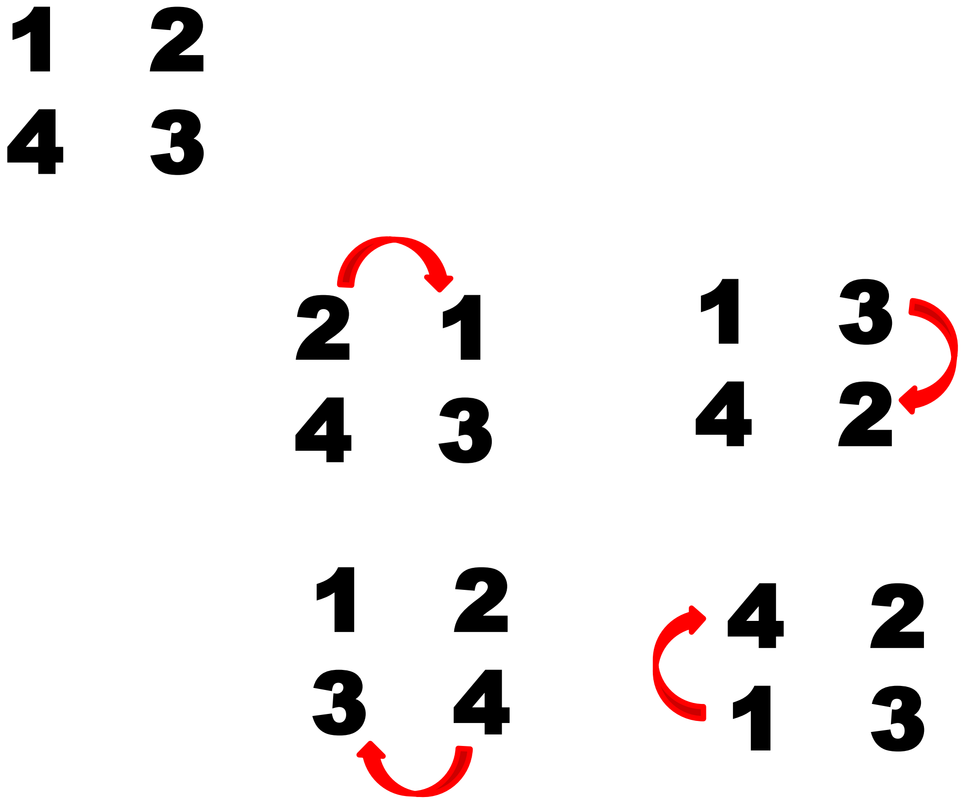
like rotating the vertices of a square



like reflecting about horizontal, vertical, and diagonals



Permutations of the 4 numbers that flip two numbers are like "twists" of the square:



Note that twists are NOT symmetry operations, because they distort the square into a bow tie, thus creating tension…

Symmetry-preserving Moves

Any combination of rotations and reflections that preserves the symmetry of the square (keeps partners together).

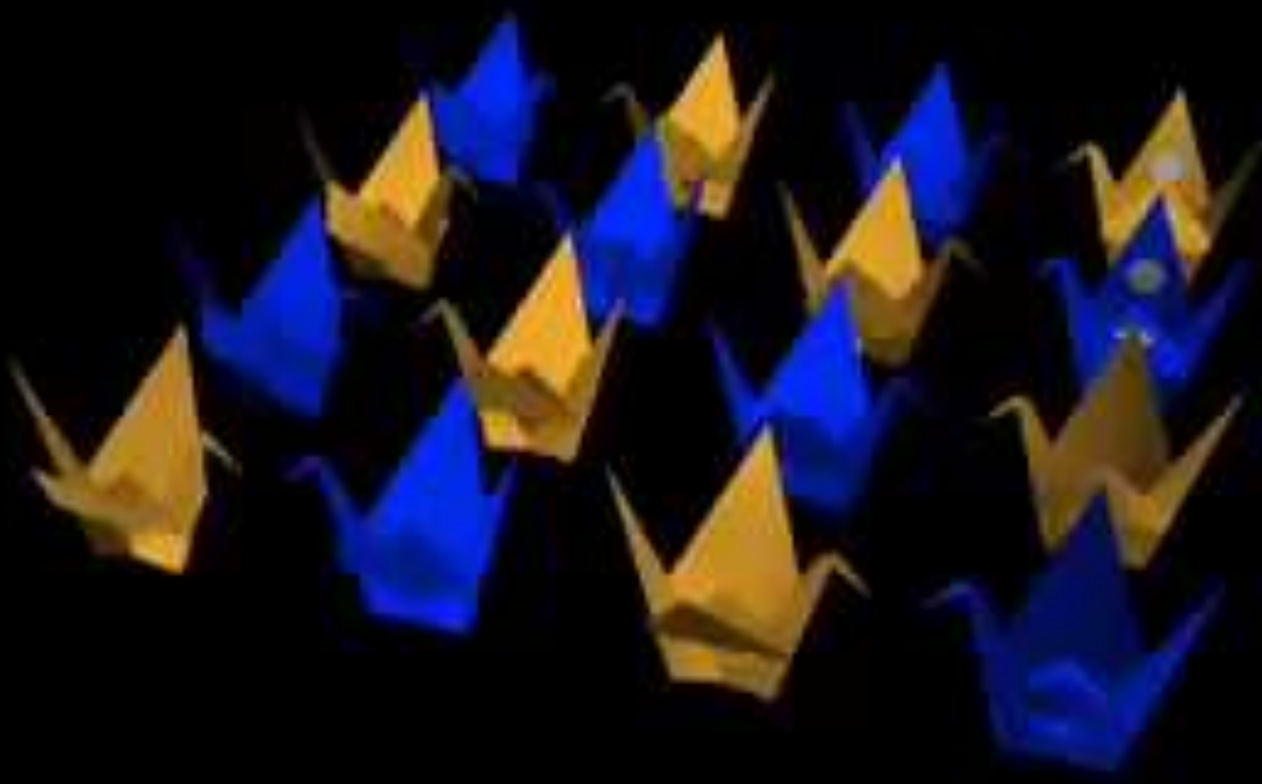
Balance, Order, & Harmony

Symmetry-breaking Moves

Twists that destroy the original symmetry of the square (mixes the couples).

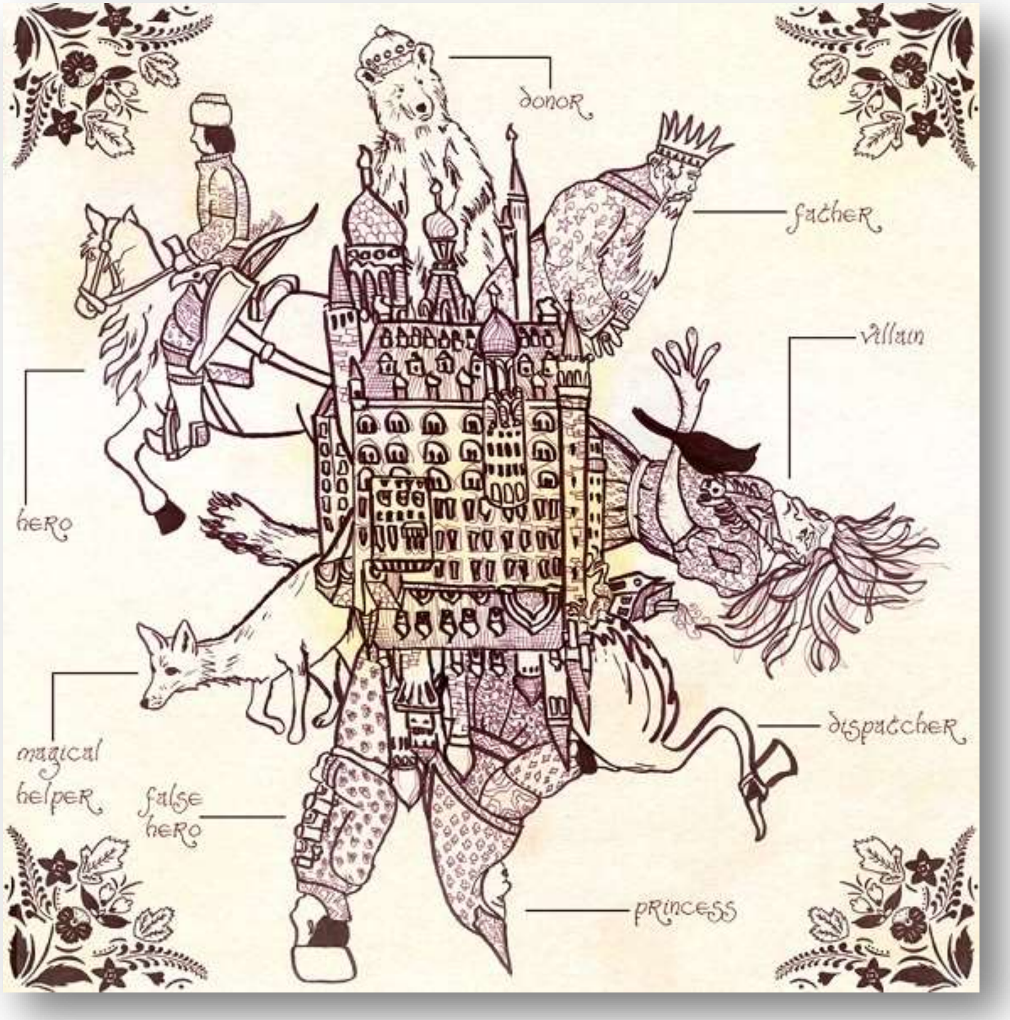
Tension waiting to be resolved

Symmetry in contra dance – "Dutch Crossing" illustrated by Origami cranes



Aside:

Note that fairy tales and myths are all stories of symmetry breaking and the search for a return to harmony.



Asymmetry in Balkan music

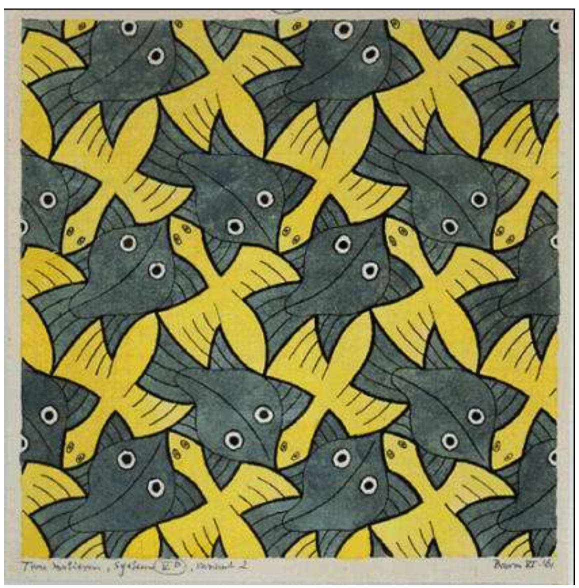
- "There is no excellent beauty that hath not some strangeness in its proportions."

7/8 meter

11/8 meter

another 7/8 meter

combining 9 and 7 by a contemporary band



20th century artist M.C. Escher - represented symmetry and broken symmetry, and challenged our notions of space, time, and gravity

Next: photos from my visit to the Escher Museum in Den Haag, July 2013

10.

In class activity – do now, with a partner or on your own:

Choose a representation of any symmetry group you like, for example:

- **dihedral groups with n-rotation angles of 360/n and n reflection axes**
- **SO(2) – the circle**
- **SO(3) – the sphere**

Or choose another that you like

Start messing around with a paper, ruler, and pencil and come up with some ways to show symmetry operations of rotation, reflection, and translation such that you always have a representation of your group (shape).

If you want to play with words or with the piano, that's fine, too.

Symmetry demonstration – art project due next week. Plus Livio chapters & RR

From the syllabus, which is in the reader and on line:

Choose one manifestation of symmetry that is most interesting to you and create a representation that you will present in class. This does not have to be a drawing, but you are free to work in any medium you choose: drawing, painting, sculpture, music, dance, computer simulation, or something else.

Turn in:

1. Your symmetry demonstration, presented in class;

2. A one-page write up in which you discuss the symmetry group your demonstration represents, the symmetry operations apparent in it, and any other information that would be interesting for your audience to know, such as why you chose it, how you created it, why you chose a particular medium or materials, and any symbolic meanings that you have chosen which you would like to explain.

Examples from previous years: