Violation of Bell's inequality in Josephson phase qubits

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The measurement process plays an awkward role in quantum mechanics, as measurement forces a system to "choose" between possible outcomes in a fundamentally unpredictable manner. Hidden classical processes have therefore been considered as possibly pre-determining measurement outcomes while preserving their statistical distributions [1]. However, the Bell inequalities provide a quantitative measure that can distinguish classically-determined correlations from the stronger quantum correlations, and experiments provide strong evidence that quantum mechanics is a complete description [2–4]. Here we present the first solid-state demonstration of the violation of a Bell inequality. We use a pair of Josephson phase qubits [5–7], acting as spin-$\frac{1}{2}$ particles, and show that the qubits can be entangled [8, 9] and measured so as to violate the Clauser-Horne-Shimony-Holt (CHSH) version of the Bell inequality [10]. We measure a Bell signal S of 2.0732 ± 0.0003, exceeding the maximum value $|S| = 2$ for a classical system by 244 standard deviations. In the experiment, we deterministically generate the entangled state, and measure both qubits in a single-shot manner, closing the "detection loophole"[11]. Since the Bell inequality was designed to test for non-classical behavior without assuming the applicability of quantum mechanics to the system in question, this experiment provides further strong evidence that a macroscopic electrical circuit is really a quantum system [7].

In classical physics, deterministic laws provide a complete description for the evolution of a physical system. Quantum physics purports to provide an equally complete description, but the measurement process involves additional premises, and measurement outcomes are intrinsically uncertain. When performing measurements on entangled particles, however, the unpredictability of measurement is combined with very strong correlations between measurements on the individual particles, leading to the apparently paradoxical thought experiments developed by Einstein, Podolsky, and Rosen [12].

The CHSH protocol [10] describes one such experiment, with a statistical test to distinguish classical pre-determination from quantum theory. This protocol uses a pair of spin-$\frac{1}{2}$ particles A and B, with spin states $|0\rangle$ and $|1\rangle$, which are entangled in the Bell singlet $(|01\rangle - |10\rangle)/\sqrt{2}$. The entangled spins are separated and measured independently along one of two directions (a and a' for spin A, b and b' for spin B). The measurements yield a 0 or 1 for each spin, regardless of the measurement axis. For $x \in a, a'$ and $y \in b, b'$, we define the correlation $E(x, y)$ for two spins as the difference in the probabilities of measuring the same result versus measuring a different result

$$\begin{aligned} E \ (x, y) &= P_{\text{same}}(x, y) - P_{\text{diff}}(x, y) \\ &= P_{00}(x, y) + P_{11}(x, y) - P_{01}(x, y) - P_{10}(x, y) \ . \end{aligned} \tag{1}$$

The Bell signal S is then defined as

$$S = E(a, b) + E(a', b) - E(a, b') + E(a', b'). \tag{2}$$

Classical (predetermined) outcomes result in a Bell signal $|S| \leq 2$, while quantum mechanics permits a larger signal $|S| \leq 2\sqrt{2} = 2.828$, for the appropriate measurement axes. Completely random outcomes result in $S = 0$. An experiment returns a Bell violation if $|S| > 2$, and thus indicates quantum entanglement.

The derivation of the limit $|S| \leq 2$ is based on two assumptions, which, if not met, provide loopholes that, in principle, allow an experiment to return a Bell violation even for a classically predetermined process.

The first loophole is called the "detection loophole" [11] and affects experiments in which the spin measurement is ineffective, for example not detecting one of the spins, or confusing a spin from one pair with that of another. This breaks the assumption that a measurement always returns a 0 or 1 for both particles in the pair, as it allows for the additional measurement outcome of "undetected" or incorrect pair identification. This loophole commonly affects experiments based on the measurement of entangled photons [4], as a fraction of these are missed by even the best available photon detectors. The reported data set then consists of only a subset of the entire ensemble of entangled photons, a subset whose detection could introduce an unknown classical correlation. This loophole is commonly countered by the "fair-sampling hypothesis," which claims that no such correlation should exist.

The second loophole, the "locality/causality loophole", applies when the spin measurements are not performed with true space-like separation, i.e. such that the spins can no longer communicate during the measurement. This loophole affects experiments where the distance d between the spins during measurement does not fulfill $d \geq c\, t_{meas}$, where t_{meas} specifies the time it takes to completely measure the spins and c is the speed of light.

A variety of experiments have shown violations of the Bell inequality [13–15] with one or the other of these loopholes closed. With the caveat that no one experiment has closed both loopholes, it appears that quantum mechanics provides a more accurate description than do local hidden variable theories.

Here we describe measurements on a pair of Josephson phase qubits, serving as spin-$\frac{1}{2}$ particles, which are entangled via an electromagnetic resonator [16, 17]. Since we can generate the entangled pair with certainty and obtain a measurement of each qubit every time, our experiment is not subject to the detection loophole and does not need to invoke the fair-sampling hypothesis. However, the qubits are only separated by 3.1 mm, and with the measurement process lasting around 30 ns our experiment is not able to close the locality loophole.

The Josephson phase qubit, as described previously [18], has qubit states $|0\rangle$ and $|1\rangle$ whose energy difference E_{10} can be adjusted by an external current bias. By also applying microwaves at the transition frequency $E_{10}/h \sim 7\,\mathrm{GHz}$, the qubit state can be completely controlled. The state is measured by applying a pulse of current, which selectively tunnels the $|1\rangle$ state to an auxiliary state that can be readily distinguished from $|0\rangle$ by a classical measurement of magnetic flux.

In previous experiments two qubits were entangled in a Bell singlet through capacitive coupling [19]. This fixed coupling unavoidably keeps the qubits coupled during measurement, so that a measurement of the $|1\rangle$ state of one qubit sometimes produces a $|0\rangle \rightarrow |1\rangle$ transition in the other qubit [20]. This measurement crosstalk introduces a correlation that complicates the Bell measurement [21], and in addition inhibits the adjustment of the measurement pulse for optimum fidelity. To circumvent this problem, here we have coupled the two qubits via a resonator, as depicted in Fig. 1. The resonator, with a resonance frequency of 7.185 GHz, acts as a bandpass filter between the two qubits, so that negligible energy is transferred when a qubit $|1\rangle$ is measured. As a result, measurement crosstalk is now found to be unimportant (0.5%, as shown in the Supplemental Information), and the measurement fidelities of 94.6% and 93.4% are within a few percent of the predicted maximum of 96.6% [22]. The crosstalk reduction was a key to the success of this experiment.

Although the transfer of qubit entanglement through the resonator slightly complicates the control sequence, as shown in Fig. 1, the fidelity is not significantly degraded, as demonstrated in previous qubit-resonator experiments [23, 24]. In Fig. 2 we show high-fidelity swapping of an excitation between the qubits, with the entanglement passing through the resonator. Here, the coupling strengths (splittings) of qubits A and B to the resonator are

$2g = 26.1\,\mathrm{MHz}$ and $36.2\,\mathrm{MHz}$, respectively; swapping is turned off by detuning the qubits from the resonance frequency of the resonator. We chose different off-detuning frequencies, 530 MHz and 430 MHz for qubits A and B, to minimize crosstalk between each qubit's microwave drive and the other qubit. The qubits are maximally entangled when the probabilities P_{01} and P_{10} first cross at 9.9 ns. Using state tomography [19], we find a fidelity of this state with respect to the Bell singlet $|\psi_s\rangle$ of $\mathrm{F}(\rho) = \sqrt{\langle\psi_s|\rho|\psi_s\rangle} = 88.3\%$ and the state's entanglement of formation [25] is 0.378.

This entangled state is used for the Bell violation experiment. In order to measure the Bell state along different axes (a, a') and (b, b'), we first rotate the qubit states using microwave pulses before performing measurements along the z axis of the Bloch sphere [19], as depicted in Fig. 1(c) and (d). Because the coherence of the entangled state degrades in time due to energy decay and dephasing, we additionally shorten the pulse sequence as much as possible, as depicted in Fig. 1(e).

Since the CHSH version of the Bell inequality is not based on any assumptions about the entangled state or the choice of measurement axes, we use search optimization of all relevant parameters in the sequence to maximize S. We find that this search always converges to a violation with $|S| > 2$ using sequence parameters that make physical sense (see Supplementary Information). For example, the measurement axes are close to those expected for maximum violation, with the angle between a and a' (b and b') close to 90°, and the relative in-plane angle between a' and b' close to 45°. However the plane of (a, a') is rotated by an arbitrary azimuthal angle from (b, b') as a phase shift between the states $|01\rangle$ and $|10\rangle$ is produced by the differing qubit frequencies and the tuning pulses that bring the qubits on resonance with the resonator (see Fig. 3).

With optimal parameters, we measure a Bell signal with $S = 2.0732 \pm 0.0003$, which corresponds to a violation by 244 standard deviations. This value is obtained from an average over 34.1 million runs of the sequence. We estimate that with perfect measurement fidelities the Bell signal would be $S = 2.355$ (see Supplementary Information).

Given that this result is only a single number, it is important to perform verification experiments to check for errors. After all, turning off the measurement electronics gives $P_{00} = 1$ and $S = 2$. In Fig. 3(a), we plot S as a function of the azimuthal angle between the (a, a') and (b, b') measurement planes, or, equivalently, the phase between the microwave rotation pulses applied to qubits A and B. The sinusoidal dependence is as expected from theory. In Fig. 3(b) we plot S versus the time delay between the two measurement pulses: We find that S decays with time as expected from the loss of qubit coherence, with no discernable effect when the measurement pulses overlap [20], as expected for negligible crosstalk. Finally, we compare the measured value of S with that expected from theory. Quantum simulations (see Supplementary Information) predict that S is reduced from its theoretical maximum of $2\sqrt{2} = 2.828$ to $S = 2.500$, due to the finite energy relaxation times T_1 that were measured to be 392 ns, 296 ns, and 2550 ns for qubit A, qubit B, and the resonator, respectively. The value of S is reduced further by dephasing to $S = 2.337$ using the measured dephasing times $T_2 = 146\,\mathrm{ns}, 135\,\mathrm{ns}$, and $\sim 5000\,\mathrm{ns}$. Finally, the measurement fidelities of 94.6% and 93.4% give $S = 2.064$, within 0.5% of our measured value. The slightly larger measured S is probably due to our dephasing calculation, which assumes uncorrelated (white) flux noise [26] that slightly overestimates the effects of dephasing, and our maximization of S accounts for small asymmetries in qubit parameters, which are not included in the simulations.

Another comparison of fidelity is shown in Fig. 4, in which we display data and simulations for the qubit-resonator-qubit swap outcome. We observe excellent agreement using only the measured *single* qubit error mechanisms of decoherence and measurement fidelity. With the difference between experiment and theory after four full swap operations (arrow (2)) less than 1%, we believe the errors due to the entanglement operation, corresponding to 2-qubit errors, are less than 1%. This is not unexpected as prior entangling experiments between a qubit and a resonator gave high fidelities [23, 24], and the capacitor coupling elements and resonator used here have low dissipation.

In conclusion, we have measured a violation of the CHSH Bell inequality in a macroscopic solid state quantum system that closes the detection loophole. We note that successfully performing this measurement requires the simultaneous optimization of a number of qubit performance benchmarks, which together form most of what are known as the DiVincenzo criteria for a quantum computational architecture [27]. The very high demands on qubit state preparation, entanglement, and measurement required to achieve a violation of a Bell inequality make this type of measurement useful as a single-number benchmark for any quantum computation implementation, forming the basis for comparing very different physical architectures. We also note that the fast and independent measurement of the entangled qubits should enable, at least in principle, closure of the locality loophole in future extensions of this experiment.

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Author Contributions: M.A. performed the experiment and analyzed the data, while H.W. fabricated the sample. J.M.M. and E.L. designed the custom electronics and M.H. developed the calibrations for it. M.A. and M.N. provided software infrastructure. All authors contributed to various infrastructure, such as the fabrication process, qubit design, or experimental set-up.

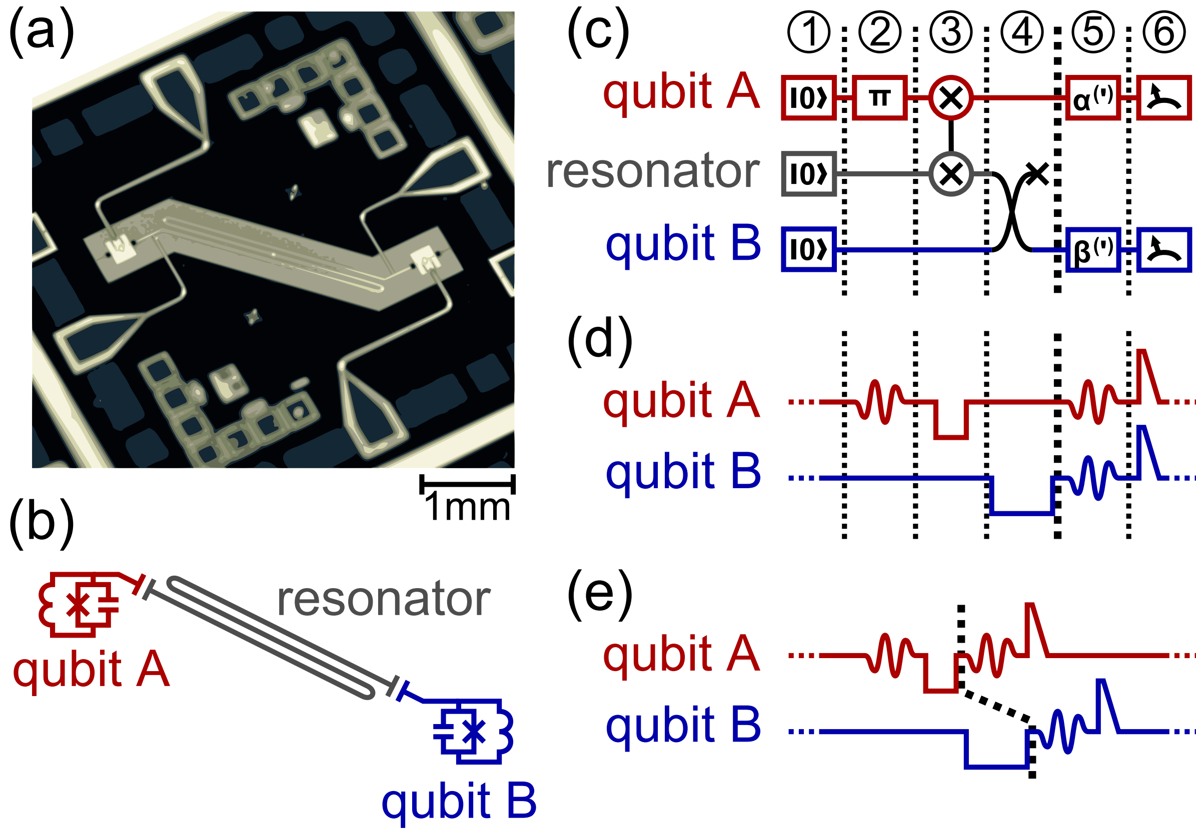

FIG. 1: Resonator-coupled qubits. (a) Photograph of qubit die showing two qubits coupled via a coplanar waveguide resonator. (b) Circuit diagram of (a). (c) Experimental sequence in quantum circuit notation: (1) Initialize to $|00\rangle$ state. (2) Create state $|10\rangle$ via π-pulse to qubit A. (3) Entangle qubit A with resonator via square-root of i-SWAP coupling [19]. (4) Resonator entanglement swapped to qubit B, creating a generalized Bell state $(|10\rangle - e^{i\theta}|01\rangle)/\sqrt{2}$. (5) Rotate state for change of measurement axes. (6) Tunneling measurement (along z axis) of qubit states. (d) Control sequence used to implement (c): Downward square pulses represent tuning the qubits into resonance with the resonator ((3) and (4)), whereas oscillations represent microwave excitations ((2) and (5)); triangular pulses at end are for measurement ((6)). (e) Shortened control sequence: Removal of dead-time places measurement of qubit A about 11 ns before B.

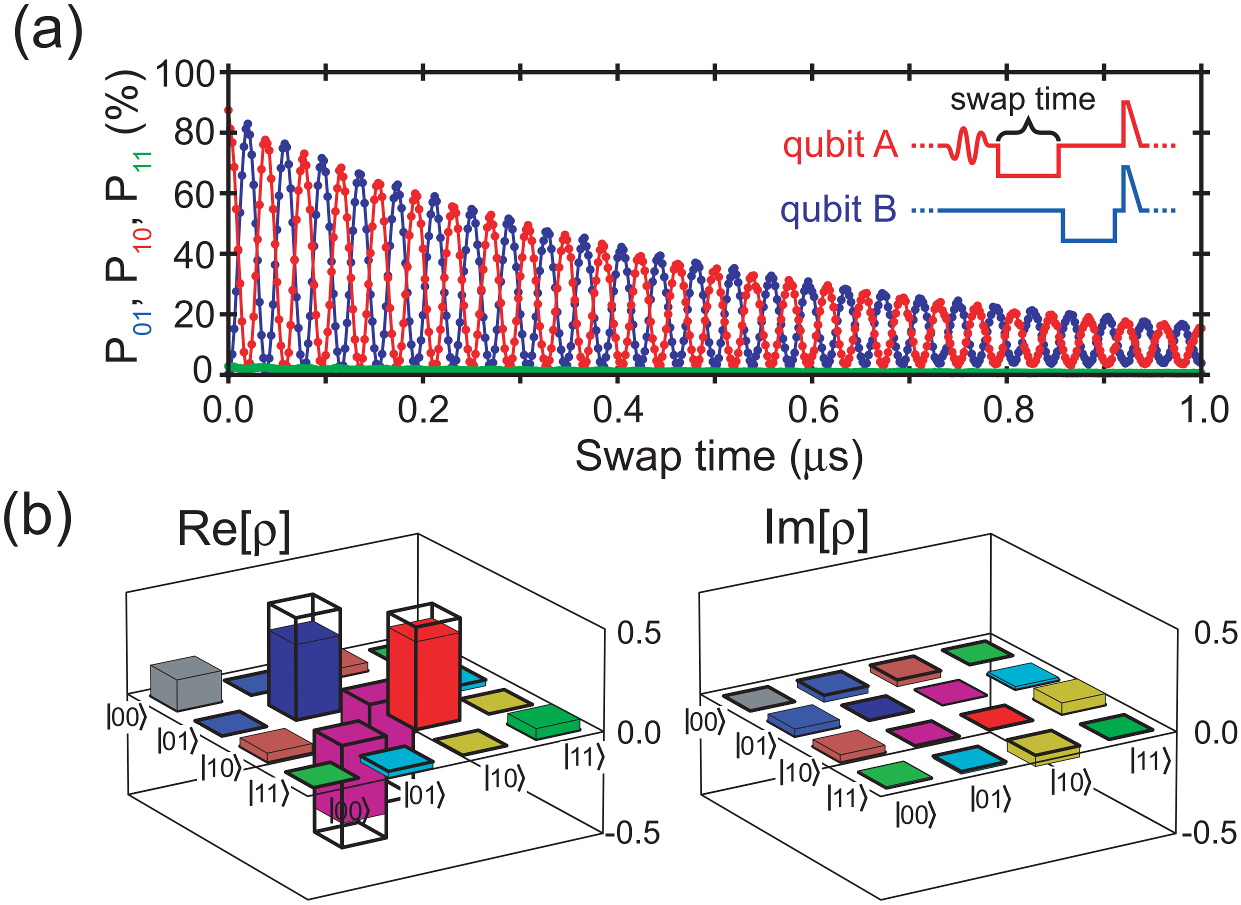

FIG. 2: Entanglement analysis. (a) Measurement probabilities of $|01\rangle$ (blue), $|10\rangle$ (red), and $|11\rangle$ (green) plotted as a function of swap time between qubit A and the resonator. Insert shows experiment sequence, where the length of the coupling pulse for qubit B is set to produce a state-swap operation between the resonator and qubit. The qubits are maximally entangled at the first crossing of P_{01} and P_{10}, at an interaction time of 9.9 ns. (b) Measurement of the density matrix for this entangled state using state tomography [19], after including an azimuthal (z-axis) rotation of 18° to compensate for the accumulated relative phase θ due to coupling pulses and differing qubit frequencies. The fidelity of our entangled state with respect to the ideal Bell singlet $|\psi_s\rangle$ (box outlines) is $F(\rho) = \sqrt{\langle\psi_s|\rho|\psi_s\rangle} = 88.3\%$. To better understand the actual state of the entangled pair that will undergo the Bell measurement, the density matrix can be corrected for measurement errors to give a state fidelity of 92.1%. The state's entanglement of formation [25] is 0.378 (0.449 corrected).

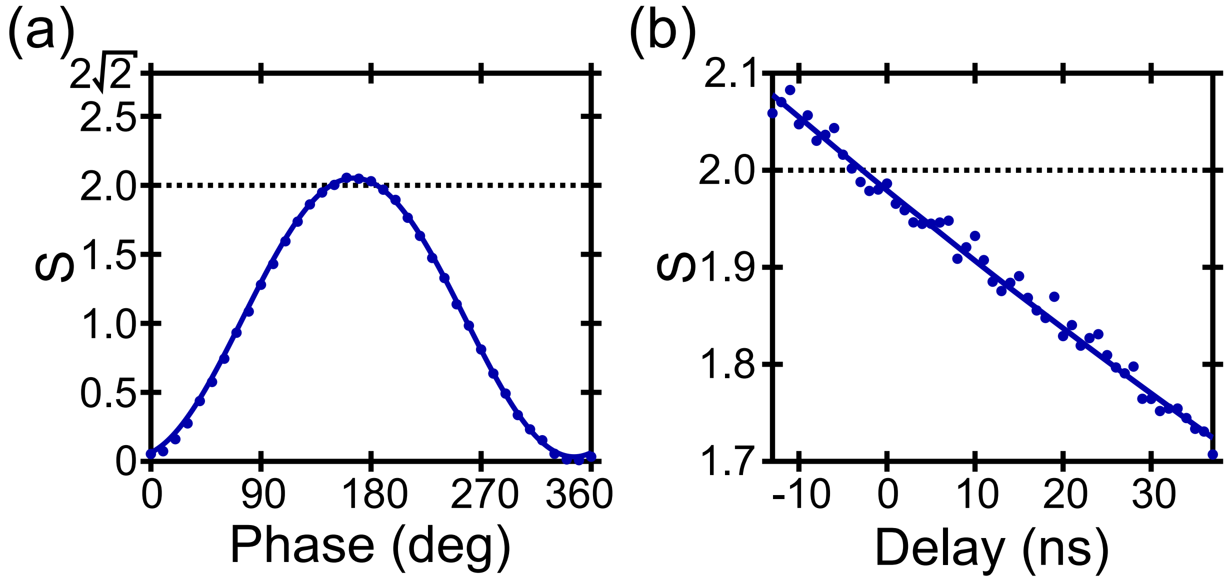

FIG. 3: Verification experiments. Points are experimental data, and lines are theory. (a) Plot of S versus the phase difference between the microwave pulses applied to qubits A and B. This is equivalent to varying the azimuthal (z-axis) angle between the plane containing the measurement axes a and a' with respect to the (b, b') plane. Sinusoidal modulation is observed, as expected. The phase-offset of the oscillation results from accumulated phases between the qubits due to coupling pulses and differing qubit frequencies. (b) Plot of S versus relative time delay between the measurement pulses applied to qubits A and B. A reduction in S versus delay is observed, consistent with the loss of qubit coherence.

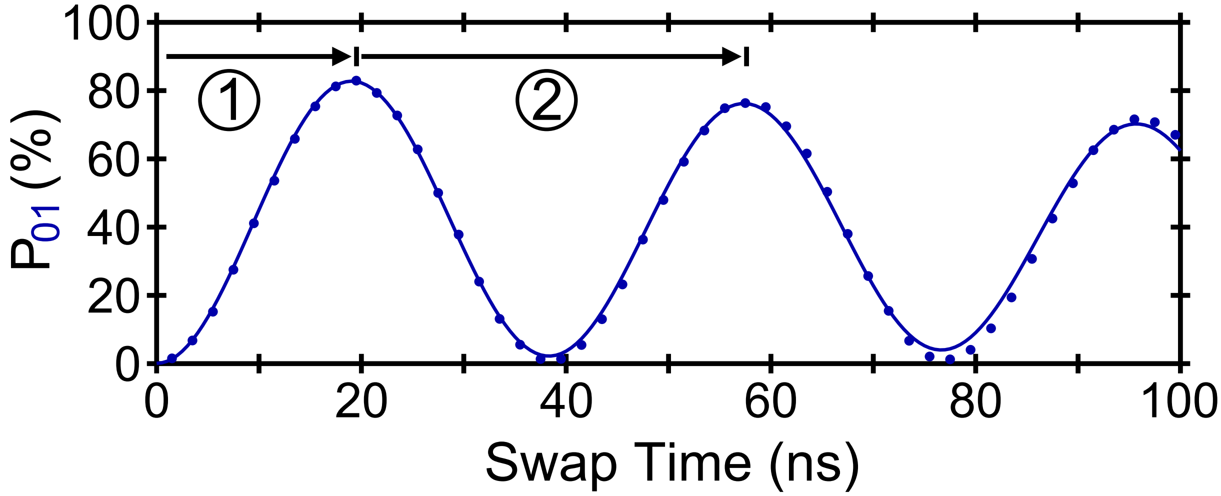

FIG. 4: Entanglement analysis. Probability of measuring the $|01\rangle$ final qubit state versus the qubit A–resonator swap time (dots) along with results of quantum simulations (line), as in Fig. 2a. Arrow (1) indicates the time for a qubit–resonator swap, while arrow (2) is the time for three full swap operations; both (1) and (2) are followed by a resonator–qubit B swap prior to measurement (not shown). Quantum simulations (see supplemental information), which only account for *single* qubit coherence and measurement fidelity, match the data with excellent agreement even after four full swaps (2). The increasing disagreement after $\sim$70 ns is caused by finite settling times in the flux bias circuitry, making qubit A slowly drift off-resonance from the resonator. We conclude that errors due to the entanglement operation, corresponding to 2-qubit errors, are less than 1%.

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