Synthesising arbitrary quantum states in a superconducting resonator

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The superposition principle is a fundamental tenet of quantum mechanics: The quantum state of a physical system can simultaneously include measurably different physical states, thus allowing a quantum system to be "in two places at the same time." The preparation of such superposed states, and their subsequent use, is the basis for quantum computation and simulation[1]. Creating these complex superpositions in harmonic systems, such as the motional state of trapped ions[2], microwave resonators[3–5] or optical cavities[6], has presented a significant challenge, because these superpositions cannot be obtained with classical control signals. Here we demonstrate the preparation and measurement of arbitrary quantum states in an electromagnetic resonator, superposing states with different numbers of photons in a completely controlled and deterministic manner. We synthesise the states using a superconducting phase qubit to phase-coherently pump photons into the resonator, employing an algorithm[7] that generalises our previously-demonstrated method of generating photon number (Fock) states in a resonator[8]. We completely characterise the resonator quantum state using Wigner tomography, equivalent to measuring the resonator's full density matrix.

The quantum state of a resonator is extraordinarily rich, with infinitely many energy levels,

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of which each can have a non-zero amplitude. However, this richness is difficult to access when driving a resonator with a classical signal, as the two adjustable parameters of an on-resonant drive, the amplitude and the phase, give very limited control. Creating an arbitrary quantum state instead requires a non-linear element and a control scheme with many parameters. Here we demonstrate quantum state generation in a resonator by interposing a highly non-linear Josephson phase qubit[9] between a superconducting resonator and a classical signal. A qubit[4,5,10–14] has two quantum degrees of freedom, the relative amplitude and phase of its ground $|g\rangle$ and excited $|e\rangle$ energy eigenstates. This simplicity allows full quantum control of a qubit with a classical signal[15]. By following a sequence of steps developed for trapped ions[2,7] (and later adapted to charge qubits[16]), where each step involves creating a particular qubit state and then having the qubit interact with the resonator for a controlled time, we synthesise arbitrary states in the resonator. The preparation is deterministic, unlike methods involving probabilistic projective measurements[17]. After the preparation, we analyse the resonator state using Wigner tomography[18–22], mapping out the Wigner quasi-probability distribution[23,24], from which we extract the resonator's full density matrix.

The quantum circuit we used is shown in Fig. 1a. The phase qubit is capacitively coupled to a superconducting coplanar waveguide resonator, and the circuit includes control lines for the qubit and resonator, and a qubit measurement circuit described elsewhere[25]. This circuit is similar to that used previously to generate Fock states in a resonator[8]; here, however, most of the superconducting wiring is made of rhenium in place of aluminium, and we removed unnecessary dielectric, reducing dissipative elements in the circuit.

The qubit frequency $\omega_q/2\pi$ can be externally adjusted, while the resonator frequency $\omega_r/2\pi = 6.570\,\text{GHz}$ is fixed. This allows us to describe the system with a Hamiltonian in the resonator rotating frame, so that the resonator states have zero frequency:

$$\frac{H}{\hbar} = \Delta(t)\sigma_+\sigma_- + \left(\frac{\Omega}{2}\sigma_+a + \frac{\Omega_q(t)}{2}\sigma_+ + \frac{\Omega_r(t)}{2}a^\dagger\right) + \text{h.c.} \tag{1}$$

Here σ_+ and σ_- ($a^\dagger$ and a) are the qubit (resonator) raising and lowering operators, and h.c. is the Hermitian conjugate of the terms in parentheses. The first term is the qubit energy, which appears as the qubit–resonator de-tuning $\Delta(t) = \omega_q(t) - \omega_r$. The first term in the parentheses gives the qubit-resonator interaction, proportional to the fixed interaction strength $\Omega/2\pi = 19\,\text{MHz}$, while the second and third terms give the effect of the external microwave drive signals applied to the qubit and resonator; these parameters $\Omega_q(t)$ and $\Omega_r(t)$ are complex to account for amplitude and phase. All control signals in Eq. (1) vary on a $\sim 2\,\text{ns}$ time scale, long compared to $2\pi/\omega_r$, so counter-rotating terms in Eq. (1) are neglected.

Although the coupling Ω is fixed, we control the qubit-resonator interaction by adjusting the qubit frequency between two operating points, one with qubit and resonator exactly on resonance ($\Delta_\text{on} = 0$), the other with the qubit well off-resonance ($|\Delta_\text{off}| \gg \Omega$). On resonance, the coupling will produce an oscillation where a single photon transfers between qubit and resonator with unit probability, alternating between states with e.g. the qubit in its ground state with n photons in the resonator, $|g\rangle \otimes |n\rangle = |g, n\rangle$, and the qubit in its excited state with $n - 1$ photons in the resonator, $|e, n - 1\rangle$; this occurs at the n-photon "Rabi-swap" frequency $\sqrt{n}\Omega$. Off resonance,

the system oscillates at a higher frequency $\sqrt{n\Omega^2 + \Delta^2}$ but with reduced $|e, n-1\rangle$ probability

$n\Omega^2/(n\Omega^2 + \Delta^2) < 1$. This de-tuning dependence is shown in Fig. 1 for $n = 1$ photon and

small de-tunings $|\Delta| \lesssim \Omega$. At our typical off-resonance operating point $\Delta_{\text{off}} \approx -25\Omega$, the photon

transfer probability is only $0.0016\, n$, so the coupling is essentially turned off.

We determine from Fig. 1 the flux bias for on-resonance tuning ($\Delta = 0$) and the one-photon

swap time. Using these parameters, we can pump photons one at a time into the resonator by

repeatedly exciting the de-tuned qubit from $|g\rangle$ to $|e\rangle$ using a qubit microwave π-pulse, followed

by a controlled-time, on-resonance photon swap[8], where we scale the swap time for the n-th photon

by $1/\sqrt{n}$. Precise scaling of this swap time is crucial for proper control, and was verified for up to

15 photons (see Supplementary Information).

Our goal is to synthesise arbitrary N-photon states in the resonator with the qubit in its

ground state, disentangled from the resonator. Our target state for the coupled system is

$$|\psi\rangle = |g\rangle \otimes \sum_{n=0}^{N} c_n |n\rangle \tag{2}$$

with complex amplitude c_n for the n-th Fock state. Law and Eberly[7] showed that these states can

be generated by sequentially exciting the qubit into the proper superposition of $|g\rangle$ and $|e\rangle$, and then

performing a partial transfer to the resonator. As illustrated in Fig. 2, and detailed in Table 1, a

sequence generating the desired state can be found by solving the time-reversed problem: Starting

with the desired final state, we first transfer the amplitude of the highest occupied resonator Fock

state to the qubit, then remove the excitation from the subsequently de-tuned qubit using a classical

microwave signal, and repeat until the ground state $|g, 0\rangle$ is reached. The actual control signals are

sequenced in the normal (un-reversed) order to generate the desired final state from the initial ground state. We note that the Law and Eberly protocol[7] assumes an adjustable phase for the qubit-resonator coupling Ω, which Eq. (1) does not allow; instead, we correct the relative phases of $|g, n\rangle$ and $|e, n-1\rangle$ by adjusting the time t_n over which the qubit and resonator are de-tuned.

To calibrate the actual microwave signals needed to implement this sequence, it is impractical to individually tune each sequence step, because the intermediate states are quite complex and measuring them time-consuming. Instead we perform careful calibrations of the experimental system independent of the particular state preparation (see Supplementary Information).

An initial check of the outcome of the preparation is to determine if the qubit ends up in the ground state $|g\rangle$, as desired. We find that this holds with a probability typically greater than $80\,\%$, the remaining $20\,\%$ being compatible with decoherence during the preparation sequence (see Supplementary Information).

With the qubit near its ground state and not entangled with the resonator, we can use the qubit to measure the resonator state. By bringing the qubit and resonator into resonance for a variable time τ and subsequently measuring the probability P_e for the qubit excited state, we can determine[8] the n-photon probabilities $P_n = |c_n|^2$, correcting for measurement fidelity and initial qubit state probability (see Supplementary Information). In Fig. 2c we compare $P_e(\tau)$ for the experimentally prepared states $|\psi_a\rangle = |1\rangle + |3\rangle$ and $|\psi_b\rangle = |1\rangle + i|3\rangle$, showing the expected superposed oscillations corresponding to the $|1\rangle$ and $|3\rangle$ Fock states. This measurement however only yields the probabilities P_n: The relative phases of the Fock states are lost, so the states $|\psi_a\rangle$

and $|\psi_b\rangle$ cannot be distinguished.

To measure the complex amplitudes c_n, we need to probe the interference between the superposed Fock states. This may be done using Wigner tomography[19,21,24], which maps out the Wigner quasi-probability distribution $W(\alpha)$ as a function of the phase space amplitude α of the resonator (see Supplementary Information). Wigner tomography is performed by following the functional definition

$$W(\alpha) = \frac{2}{\pi}\langle\psi|D^\dagger(-\alpha)\,\Pi\,D(-\alpha)|\psi\rangle. \tag{3}$$

The resonator state $|\psi\rangle$ is first displaced by the operator $D(-\alpha)$, implemented with a microwave drive pulse $-\alpha = (1/2)\int\Omega_r(t)\mathrm{d}t$. The photon number probabilities P_n are then measured and finally the parity $\langle\Pi\rangle = \sum_n(-1)^n P_n$ evaluated. The corresponding pulse sequence is depicted in Fig. 2b.

Measured and calculated Wigner functions are shown in Fig. 3 for the resonator states $|0\rangle + |N\rangle$, with $N = 1$ to 5. The structures of the Wigner functions match well, including fine details, indicating that the superposed states are created and measured accurately. The density matrices for each state are also calculated (see Supplementary Information) and are as expected. The Wigner function of non-classical states has been measured previously, either calculated via an inverse Radon transform[18,26,27], or measured at enough points to fit the density matrix[3,28], from which the Wigner function is reconstructed. The high resolution direct mapping of the Wigner function used here is an important verification of our state preparation. The good agreement in shape shows that very pure superpositions of $|0\rangle$ and $|N\rangle$ have been created. Slight deviations in amplitude can be

due to small errors in the read-out process, the relative amplitudes of the $|0\rangle$ and $|N\rangle$ states, or statistical mixtures with other Fock states.

The data in Fig. 3 do not demonstrate phase control between Fock states, as a change in the relative phase of a two-state superposition only rotates the Wigner function. The phase accuracy may be robustly demonstrated by preparing states with a superposition of three Fock states, as changing the phase of one state then changes the shape of the Wigner function. Figure 4 shows Wigner tomography for a superposition of the $|0\rangle$, $|3\rangle$ and $|6\rangle$ Fock states, where the phase of the $|3\rangle$ state has been changed in each of the five panels. The shape of the calculated and measured Wigner functions again agree, including small details, indicating that precise phase control has been achieved. The calculated and measured density matrices also match well.

In conclusion, we have generated and measured arbitrary superpositions of resonator quantum states. State preparation is deterministic and "on-demand", requiring no projective measurements, and limited to about ten photons, mainly by decoherence[29]. The accuracy of the prepared states demonstrates that a qubit, when controlled with high fidelity, is ideally suited for synthesising and measuring arbitrary quantum states of light.

Author contributions: M.H. performed the experiments and analysed the data. H.W. improved the resonator design and fabricated the sample. J.M.M. and E.L. designed the custom electronics and M.H. developed the calibrations for it. M.A. and M.N. provided software infrastructure. All authors contributed to fabrication process, qubit design or experimental setup. M.H., J.M.M., and A.N.C. conceived the experiment and co-wrote the paper.

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$\lvert\psi\rangle$		$\lvert g\rangle(0.707\lvert 1\rangle + 0.707i\lvert 3\rangle)$
S_3	$\tau_3\Omega$	1.81
Q_3	q_3	3.14
$\lvert\psi_2\rangle$		$\lvert g\rangle(-0.557i\lvert 0\rangle + 0.707\lvert 2\rangle) + 0.436\lvert e\rangle\lvert 1\rangle$
Z_2	$t_2\Delta$	4.71
S_2	$\tau_2\Omega$	1.44
Q_2	q_2	$-2.09 - 2.34i$
$\lvert\psi_1\rangle$		$(0.553 - 0.62i)\lvert g\rangle\lvert 1\rangle - (0.371 + 0.416i)\lvert e\rangle\lvert 0\rangle$
Z_1	$t_1\Delta$	3.26
S_1	$\tau_1\Omega$	1.96
Q_1	q_1	$-2.71 - 1.59i$
$\lvert\psi_0\rangle$		$(0.197 - 0.98i)\lvert g\rangle\lvert 0\rangle$

Table 1: Sequence of operations to generate the resonator state $\lvert\psi\rangle = \lvert 1\rangle + i\lvert 3\rangle$, used for the measurements described in Fig. 2. The sequence is computed top to bottom, but applied bottom to top. The area and phase for the n-th qubit drive Q_n is $q_n = \int \Omega_{\mathrm{q}}(t)e^{i\Delta_{\mathrm{off}}t}\mathrm{d}t$ ($t = 0$ being the time when the qubit is tuned into resonance directly after the step Q_n), the time on resonance for the qubit-resonator swap operation S_n is τ_n, and the time off resonance (mod $2\pi/\Delta$) for the phase rotation Z_n is t_n. We note that the initial state $\lvert\psi_0\rangle$ differs by an overall phase factor from the ground state $\lvert g\rangle\lvert 0\rangle$, but this is not detectable.

Figure 1 Circuit diagram and one photon Rabi-swap oscillations between qubit and resonator. **a**, The qubit (black) is made from a Josephson junction (cross) and a capacitor, biased through a shunting inductor. The qubit de-tuning Δ is adjusted through a flux bias coil, and the qubit state is read out by a three-Josephson junction superconducting quantum interference device (SQUID). The coplanar waveguide resonator (blue) has fixed capacitive coupling Ω to the qubit, and small capacitors couple external microwave signals Ω_q and Ω_r to the qubit and resonator. The device was measured in a dilution refrigerator at $25\,\mathrm{mK}$. The qubit relaxation and dephasing times were $T_{1,q} \approx 650\,\mathrm{ns}$ and $T_{2,q} \approx 150\,\mathrm{ns}$ and the resonator relaxation time was $T_{1,r} \approx 3.5\,\mu\mathrm{s}$ with no measurable dephasing. **b**, Schematic of Rabi-swap pulse sequence. The qubit starts in its ground state, de-tuned at its typical off-resonance point by $\Delta_{\mathrm{off}}/2\pi = -463\,\mathrm{MHz} \approx -25\,\Omega/2\pi$ from the resonator. A resonant qubit microwave π-pulse brings the qubit to its excited state $|e\rangle$, injecting one quantum of energy into the system. A flux bias pulse reduces the qubit de-tuning Δ from the resonator for a controlled time τ, and the qubit state is then measured with a current pulse. **c**, Excited state probability P_e versus de-tuning Δ and interaction time τ. P_e is obtained by averaging 600 repetitions. **d**, Fourier transform of c, showing the expected hyperbolic relation between de-tuning Δ and swap frequency $\sqrt{\Omega^2 + \Delta^2}/2\pi$ (dotted line), and the expected fall-off in probability (colour scale). Resonance $\Delta = 0$ corresponds to lowest swap frequency and maximum probability amplitude.

Figure 2 Sequence to synthesise an arbitrary resonator state. **a**, Qubit-resonator energy ladder. Levels are depicted by dotted and solid lines when tuned ($\Delta = 0$) and

de-tuned, respectively. Three types of operations are used in state preparation: Qubit drive operations Q_n, indicated by undulating lines; qubit–resonator swap operations S_n, indicated by straight horizontal lines; and phase rotations of the qubit state Z_n, indicated by circles. Each operation affects all the levels in the diagram. **b**, Microwave pulse sequence. The sequence is computed in reverse order by emptying energy levels from top to bottom. To descend the first step of the ladder in a, a swap operation S_N transfers the highest occupied resonator state to the qubit, $|g, N\rangle \rightarrow |e, N-1\rangle$. This operation also performs incomplete transfers on all the lower-lying states, as do the succeeding steps. A qubit microwave drive Q_N then transfers all the population of $|e, N-1\rangle$ to $|g, N-1\rangle$ (in general this step is not a π pulse as $|g, N-1\rangle$ is not completely emptied by pulse S_N). For the second step down the ladder, a rotation Z_{N-1} first adjusts the phase of the qubit excited state $|e\rangle$ relative to the ground state $|g\rangle$. The succeeding swap pulse S_{N-1} can then move the entire population of $|g, N-1\rangle$ to $|e, N-2\rangle$. This sequence is repeated N times until the ground state $|g, 0\rangle$ is reached. Steps Q_n are performed with resonant qubit microwave pulses of amplitude q_n, swaps S_n achieved by bringing the qubit and resonator on resonance for time τ_n, and phase rotations Z_n completed by adjusting the de-tuning time t_n; see Table 1 for a detailed example. After state preparation, tomographic read-out is performed: A displacement $D(-\alpha)$ of the resonator is performed by a microwave pulse R to the resonator, then the resonator state is probed by a qubit-resonator swap S for a variable interaction time τ, and finally the qubit state measured by the measurement pulse M. **c**, Plot of the qubit excited state probability P_e versus interaction time τ for the resonator

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states $|\psi_a\rangle = |1\rangle + |3\rangle$ (blue) and $|\psi_b\rangle = |1\rangle + i|3\rangle$ (red), taken with $\alpha = 0$. We clearly

observe oscillations at the $|1\rangle$ and $|3\rangle$ Fock state frequencies. Nearly identical traces for

$|\psi_a\rangle$ and $|\psi_b\rangle$ indicate the same photon number probability distribution, as expected. **d**,

Photon number distributions for $|\psi_a\rangle$ and $|\psi_b\rangle$. Both states are equal superpositions of $|1\rangle$

and $|3\rangle$ but the phase information that distinguishes the two states is lost.

Figure 3 Wigner tomography of superpositions of resonator Fock states $|0\rangle + |N\rangle$. The

top row displays the theoretical form of the Wigner function $W(\alpha)$ as a function of the com-

plex resonator amplitude α in photon number units, for states $N = 1$ to 5. The measured

Wigner functions are shown in the second row, with colour scale bar on the far right. Neg-

ative quasi-probabilities are clearly measured. The experimental Wigner functions have

been rotated to match theory, compensating for a phase delay between the qubit and res-

onator microwave lines; the measured area is bounded by a dotted white line. The third

row displays the calculated (grey) and measured (black) values for the resonator density

matrix ρ, projected onto the number states $\rho_{mn} = \langle m|\rho|n\rangle$. The magnitude and phase of

ρ_{mn} is represented by the length and direction of an arrow in the complex plane (for scale,

see legend on right). The fidelities $F = \sqrt{\langle\psi|\rho|\psi\rangle}$ between the desired states $|\psi\rangle$ and the

measured density matrices ρ are, from left to right, $F = 0.92$, 0.89, 0.88, 0.94, and 0.91.

Each of the 51 by 51 pixels (61 by 61 for $N = 5$) in the Wigner function represents a local

measurement. The value of $W(\alpha)$ is calculated at each pixel from 50 (41 for $N = 4$ and

5) interaction times τ, each repeated 900 times to give $P_e(\tau)$. This direct mapping of the

Wigner function takes $\sim 10^8$ measurements or ~ 5 hours.

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Figure 4 Wigner tomography of the states $|0\rangle + e^{ik\pi/8}|3\rangle + |6\rangle$ for five values of phase $k = 0$ to 4. The first row is calculated, whereas the second row shows measurements. The third row displays the calculated (grey) and measured (black) values for the density matrix obtained from the Wigner functions, displayed as for Fig. 3. The fidelities between the expected states and the measured density matrices are, from left to right, $F = 0.89$, 0.91, 0.91, 0.91, and 0.91.

a

qubit drive —$\Omega_q(t)$—||— qubit —Ω— ||O resonator O|| —$\Omega_r(t)$— res. drive

flux bias —$\Delta(t)$

measure

b

resonator

Δ

qubit τ

$|e\rangle$ probability, P_e

0.0　　0.2　　0.4　　0.6　　0.8　　1.0

c

Detuning, $\Delta/2\pi$ (MHz)

Interaction time, τ (ns)

d

Frequency (MHz)

a

$|g,N\rangle$ ⟷ $|e,N-1\rangle$

$|g,N-1\rangle$ ⟷ $|e,N-2\rangle$

$|g,1\rangle$ ⟷ $|e,0\rangle$

$|g,0\rangle$

b

calculation

resonator

q_1 τ_1 t_1 α τ

qubit

Q_N S_N

Q_1 S_1 Q_2 S_2 R   S   M
          Z_1

preparation   tomography

c

$|e\rangle$ probability, P_e

$|1\rangle + |3\rangle$
$|1\rangle + i|3\rangle$

Interaction time, τ (ns)

d

P_n

Photon number, n