Example: Dimensional Analysis :

[See also HW# 1.9]

Find a formula for the radius (R) for a black hole. The radius could depend on G, M, and c, with the following units:

$[G] = \frac{L^3}{MT^2}$ $[c] = \frac{L}{T}$

$[M] = M$ $[R] = ?$

Answer $R \propto \frac{GM}{c^2}$ Real answer $\frac{2GM}{c^2}$

No mass ; No time

To get rid of mass

$$[GM] = \frac{L^3}{T^2}$$

to get rid of time:

$$\frac{[GM]}{[c]^2} = \frac{\frac{L^3}{T^2}}{\left(\frac{L}{T}\right)^2} = \frac{L^3}{T^2} \cdot \frac{T^2}{L^2} = L \checkmark$$

OR $G^a M^b c^c = R$; $\left(\frac{L^3}{MT^2}\right)^a M^b \left(\frac{L}{T}\right)^c = L$

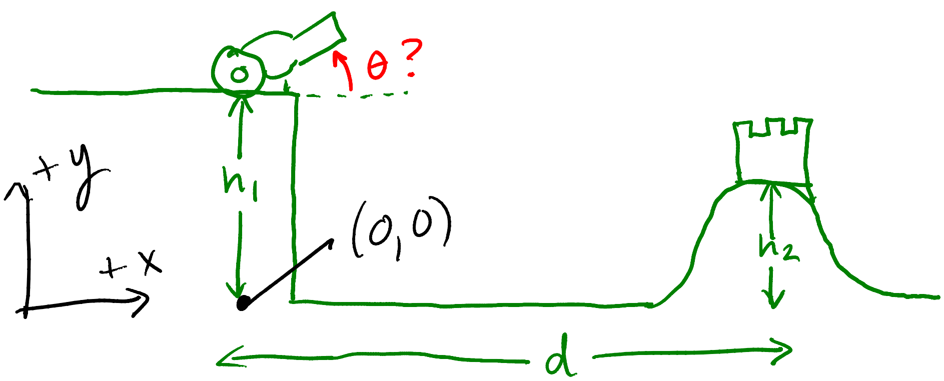
$L^{3a} L^c = L^1$ $T^{-2a} T^{-c} = T^0$ $\Rightarrow$ $3a + c = 1$, $b - a = 0$, $-2a - c = 0$ $\Rightarrow$ $a = b = 1$, $c = -2$

$M^{-a} M^b = M^0$

Example: Kinematics in 2D

HW3 #9, HW3#15
HW 7#10,

Problem 1 (Medieval warfare): A cannon is sitting on a cliff of height h1. There is a castle sitting on a hill of height h2 which is laterally a distance d away from the cannon. What angle should the cannon be set at in order to make sure that a cannonball hits the castle. Neglect air resistance.



$y_0 = h_1$ $\quad$ $v_{oy} = V_0 \sin\theta$ $\quad$ $v_{ox} = V_0 \cos\theta$

$d = v_0 \cos\theta \, t_{cas}$ $\quad$ $\dfrac{d}{v_0 \cos\theta} = t_{cas}$

$$y = y_0 + v_0 \sin\theta \, t - \tfrac{1}{2} g t^2 \Rightarrow h_2 = h_1 + v_0 \sin\theta \left(\frac{d}{v_0 \cos\theta}\right) - \tfrac{1}{2} g \left(\frac{d}{v_0 \cos\theta}\right)^2$$

$$\boxed{(h_2 - h_1) = d \tan\theta - \frac{g d^2}{v_0^2 \, 2 \cos^2\theta}}$$

Example: Newton's Laws

See also HW5 #7, HW 8 #3

5. HRK5 4.E.010. [109445] A 100 kg crate is pushed at constant speed up the frictionless 30.0° ramp shown in Fig. 4-32. What horizontal force F is required? (Hint: Resolve forces into components parallel to the ramp.)

[566] N

The easiest way to solve this problem is to choose x- and y-axes in a good way.

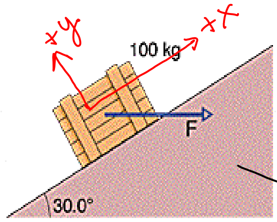
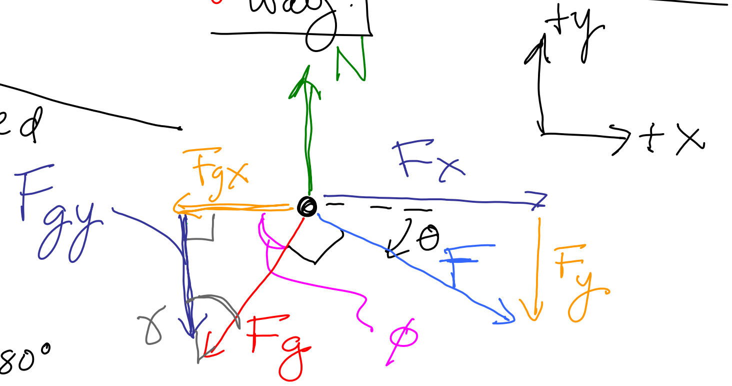


Figure 4-32

rotated



Free body diagram:

Note: This angle γ equals θ

One way to see this:

$\phi + \theta + 90° = 180°$ (plane); $\phi + \gamma + 90° = 180°$

$F_g = mg \qquad F_{gy} = F_g \cos\gamma = mg\cos\theta \qquad F_{gx} = F_g \sin\gamma = mg\sin\theta$

$F_x = F\cos\theta \qquad F_y = F\sin\theta$

Now, the question is asking about constant speed in x-dir.

Recall, $\vec{a} = \frac{d\vec{v}}{dt}$, so if $v_x = \text{constant}$, then $a_x = 0$.

So let's find $F_{NET\,x} = ma_x = 0$.

$$0 = F_{NET\,x} = F_x - F_{gx} = F\cos\theta - mg\sin\theta$$

$$F\cos\theta = mg\sin\theta \Rightarrow \boxed{F = mg\tan\theta}$$

Putting in #'s $F = 100\,kg \cdot 9.8\,m/s^2 \cdot \tan(30°) = 565.8\ N$

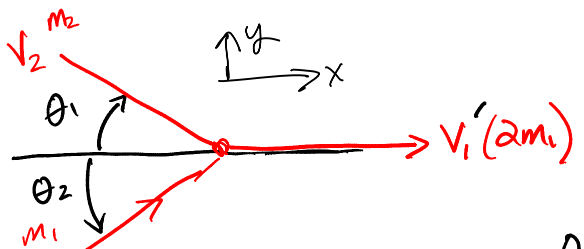
Note: we know the block isn't moving in the y-direction because the y-component of F_g and F are both negative. Therefore N can counteract them and keep $a_y = 0$

Example: collisions & Energy

Also see HW#11.12

7. HRK5 6.E.033. [110583] After a totally inelastic collision, two objects of the same mass and initial speed are found to move away together at 1/4 their initial speed. Find the angle between the initial velocities of the objects.

[151]°



Conservation of momentum in x- & y- directions

$m_1 v_1 \cos\theta_2 + m_1 v_1 \cos\theta_1 = 2m_1 v_1'$ (x-direction mom)

$m_1 v_1 \sin\theta_2 + m_1 v_1 \sin\theta_1 = 0$ (y-direction mom)

(1) $m_1 v_1(\sin\theta_2 + \sin\theta_1) = 0$

(2) $m_1 v_1(\cos\theta_2 + \cos\theta_1) = \frac{2mv_1}{4}$

$\sin\theta_1 = -\sin\theta_2$

$\theta_1 = (-\theta_2)$

$m_1 v_1(\cos\theta_2 + \cos(-\theta_2)) = \frac{2mv_1}{4}$

$\Rightarrow 2\cancel{m_1 v_1}\cos\theta_2 = \frac{2\cancel{mv_1}}{4}$

$\cos\theta_2 = \frac{1}{4}$

$\Downarrow$

$\theta_2 = \cos^{-1}\left(\frac{1}{4}\right) = 75°$

$\theta = 2\theta_2 = 151°$

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in terms of v_1, what fraction of initial Energy (K_i) of system ends up as internal energy?

$\Delta K + \cancel{\Delta U}^{0} + \Delta E_{int} = \cancel{W_{ext}}^{0}$

$\underbrace{\frac{1}{2}mv_1^2 + \frac{1}{2}mv_1^2}_{K_i} + \Delta E_{int} = \frac{1}{2}2m\left(\frac{v_1}{4}\right)^2$

A LOT!

$\Delta E = \frac{mv_1^2}{16} - mv_1^2 = \left(\frac{1}{16} - 1\right)K_i = \frac{15}{16}K_i \Rightarrow \boxed{\frac{15}{16}}$

Conservation of energy

HW problems # 19#1 18#2

#13 a) a ball loses n% of its KE when it bounces back from a concrete walk. w/ what speed must you throw it down from a height of h so it bounces back to same height? Neglect air resistance.

b) Show using dimensional analysis why your answer has the correct units.

a) How would you answer change if there was air resistance?

System: ball + Earth ⇒ No Wext

$\Delta KE + \Delta U + \Delta E_{int} = 0$

1) Right before bouncing: (everything is conservative)

$$\left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + (0 - mgh) = 0$$

$$KE_f = \tfrac{1}{2}mv_f^2 = +mgh + \tfrac{1}{2}mv_i^2$$

2) Right after bounce: ΔE_{int} $(\Delta U = 0)$

$$KE_{after} = KE_f - \underbrace{nKE_f}_{\Delta E_{int}} = KE_f(1-n)$$

3) At the top after bounce

$\Delta K + \Delta U$

$$KE_{top} - KE_{after} + (U_{top} - U_{after}) = 0$$

$$0 - KE_f(1-n) + (mgh - 0) = 0$$

$$mgh = KE_f(1-n)$$

$$mgh = \left[(mgh) + \tfrac{1}{2} m v_i^2\right](1-n)$$

$$mgh - mgh(1-n) = \tfrac{1}{2} m v_i^2 (1-n)$$

$$\frac{2\cancel{m}gh(1-(1-n))}{\cancel{m}(1-n)} = \cancel{\tfrac{1}{2}}\cancel{m} v_i^2$$

$$\sqrt{\frac{2gh(n)}{1-n}} = v_i$$

RHS $\sqrt{\dfrac{[g]}{[h]}} = \sqrt{\dfrac{L^2}{T^2}} = \dfrac{L}{T} = [v_i]$

c) Really think about this. v_i would have to be larger.

Also, look at center of mass & (e.g. HW 12 #2, 4)
impulse problems (e.g. from HW #10, 2)