Problems for HW 2

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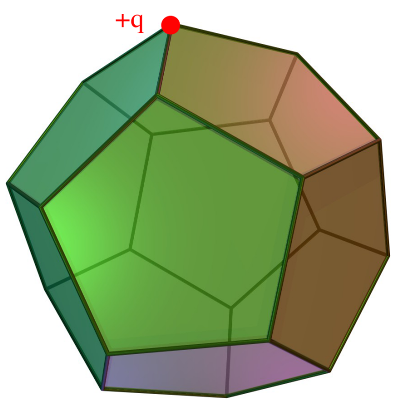
Due 5 Oct 2007, 4 pm



1 HW1 Problem 1

A plane can be tiled with equilateral triangles, squares, or hexagons. Analogously, 3-space can be completely filled with cubes or regular dodecahedra. Suppose that a point charge $+q$ is placed at one corner of a dodecahedron, with length of each edge a.

1. What is the total electric flux to emerge from the 12 faces? What fraction of that flux emerges from the 3 pentagons adjacent to the charge?

2. Argue that, if a charge $+q$ is placed at *each* corner of a dodecahedron, then the electric field at the center of the dodecahedron is zero. What is the E-field if one of the 20 charges is removed? (Useful fact: 8 of the 20 corners of a dodecahedron lie on a cube. The edges of the cube are diagonals of the regular pentagons, that are the faces of the dodecahedron. The ratio of the length of this diagonal to the length of one of the edges of the dodecahedron is the Golden Ratio, $\phi = \frac{\sqrt{5}+1}{2} \approx 1.618...$.)

   For fun: Can you remove *two* charges and have *zero* E-field? Three? Four? Five? And so on!

2 HW1 Problem 2

Consider the electric fields $\vec{E}_A = E_0\vec{r}$, and $\vec{E}_B = \frac{3}{2}E_0\vec{s}$. Here, $\vec{r} = (x\hat{x} + y\hat{y} + z\hat{z})$, and $\vec{s} = (x\hat{x} + y\hat{y})$. Suppose that these functions describe the fields within a distance R of the origin: $|\vec{r}| < R$. Outside of that region, the fields are not stated.

1. Find the charge densities ρ_A and ρ_B that correspond to these E-fields. (For fun, after completing this problem and the rest of the homework, find illustrations of these in Griffiths.)

2. Calculate the curl of these electric fields, and demonstrate that it is zero, as required for a static electric field.

3. Curl and divergence are the same for $\vec{E}_A$ and $\vec{E}_B$. Is this consistent with the Helmholtz Theorem? (Appendix B, and p. 52?) Why or why not?

3 HW1 Problem 3

3a) Electric field lines are tangent to $\vec{E}$ everywhere in space. Suppose that the number of field lines that emerge from positive charges (or disappear in negative ones) is proportional to the charge, and that they emerge from the charge evenly spaced: That is, assume that the emerging field lines will pierce a sufficiently small sphere around the charge at uniform separation. • Show that the density of field lines is proportional to the strength of the electric field throughout space.

3b) Unless you make the assumptions given in part 3a, field lines give the only direction of a vector: not its length. Suppose, just for this part, that you haven't made those assumptions. In this case, how much can field lines tell you about curl? • Specifically, if two vector fields have the same field lines, must they have the same curl? • Indeed, if a vector field has zero curl, must every other field with the same field lines also have zero curl? If so, prove it! If not, give counterexamples. • Please state whether you are considering the vector fields only over a finite region of space (as in Problem 2) or over all of space, and whether they go to zero at $|\vec{r}| \to \infty$.

4 Problems from Griffiths

2.5, 2.6, 2.7, 2.12, 2.16