Problems for HW 5

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Due 3 Nov 2009, 5 pm

1 HW4 Problem 1

a) Use Gauss's law in integral form, and appropriate symmetries and boundary conditions, to find the electric field $\vec{E}$ as a function of position *inside* each of these charge distributions:

- A solid sphere of radius R and charge density ρ_0, at $r < R$.
- A solid cylinder of radius S and charge density ρ_0, at $s < S$.
- A solid slab of thickness D and charge density ρ_0, at $z < D/2$.

In each case, sketch the charge density and the Gaussian surface that you use. Show which faces (if any) of that surface contribute nothing to the integral (in other words, where $\vec{E} \cdot \vec{da} = 0$). Give magnitude, direction, and dependence on position in each of these cases.

b) Use Gauss's law in integral form to show that the electric field *outside* of the shapes in the previous section is the same as the electric field outside of:

- A hollow spherical shell with surface charge σ_S and radius R_S.
- A line charge λ.
- A thin sheet of surface charge σ_P.

In each case, sketch the figure and your Gaussian surface. Find the relationships between σ_S, R_S, λ, σ_P on the one hand; and ρ_0, R, S, and D on the other.

2 HW5 Problem 2

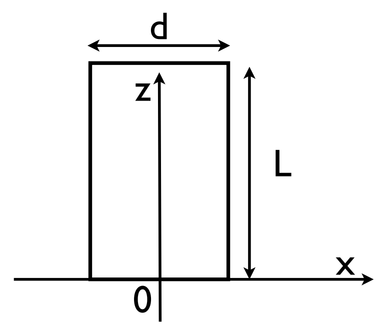
a) A long, rectangular tube has sides of length L (long sides) by d (short sides). Both long sides and one short side are grounded ($V = 0$). The un-grounded short side has a piecewise linear, continuous potential:

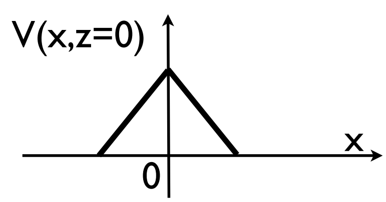
$$V(x) = \begin{cases} V_0\,(d/2 + x), & -d/2 < x < 0 \\ V_0\,(d/2 - x), & d/2 > x > 0 \end{cases} \tag{1}$$

Here, for convenience I have taken the midpoint of the un-grounded side as the origin, with the z-axis normal to that face. The x-axis runs along the un-grounded face, perpendicular to the long sides. You may find it useful to recall from lecture that the potential along the ungrounded face is (part of) a triangle wave, the integral of a square wave. The triangle wave has Fourier series

$$T(x) = \frac{8}{\pi^2} \sum_{k=1,\ k \text{ odd}}^{\infty} \frac{1}{k^2} \cos\left(\frac{2\pi}{2d} kx\right) \tag{2}$$

- Find the potential within the tube, $V(x, z)$.





b) (Optional, no credit, not graded) You might find it interesting to check out triangle waves on the web: particularly Wolfram's MathWorld (an offshoot of Mathematica). Why are there sines there, and cosines here? What about the alternating signs there? Is the form here really the integral of a square wave? And what about the difference in arguments – 2π vs π?