Problems for HW 6

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Due 9 Nov 2007, 5 pm

1 HW6 Problem 1

a) Find a series for the function $\frac{1}{1-\epsilon}$, as a sum over powers of ϵ, near the point $\epsilon = 0$. You may use Taylor series, or look up "binomial expansion" on en.wikipedia.org or elsewhere.

b) Write down the electric potential $V(z)$, along the z-axis for $-1 < z < 1$, for a point charge $+q$ on the z-axis at $z = +1$. Use your result from part **a** to express this as a power series $\sum_{\ell=0}^{\infty} a_\ell z^\ell$. (Note: If you like keeping track of dimensions, it may be easier to place the point charge at $z = +d$, and finding a power series in $(z/d)^\ell$.)

c) Express the potential for a point charge $+q$ on the z-axis at $z = +1$ as a sum of Legendre polynomials in $\cos\theta$ and powers of r, times coefficients a_ℓ. Clearly, this potential depends only on r and θ, never on ϕ, so such an expression must exist. Why is your expression different from that for a point charge at the origin, $V \propto 1/r$? Could one say that a single charge has a dipole moment? Explain.

d) Express $\frac{1}{1+z}$ as a power series in $u = 1/z$, and use your results to find the potential in Legendre polynomials and powers of $1/r$. What is the region of convergence of this power series? What is that of your result from part c? (Note: If you like dimensions, let $u = d/z$).

e) For this simple problem, you could shorten the two-stage process outlined in parts a-c above. Show that you can express the potential of the point charge at $z = +1$, at some field point (r, θ), as a function of r and $\cos(\theta)$:

$$V(r,\theta) = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{(1 - r\cos(\theta))^2 + (r\sin\theta)^2}} = \frac{q}{4\pi\epsilon_0} \left(1 + r^2 - 2r\cos(\theta)\right)^{-1/2} \qquad (1)$$

You *could* then expand the rightmost term in powers of r, for $r < 1$. What would the coefficient of r^ℓ be, in that case?

2 HW6 Problem 2

A thin spherical shell of radius R has uniform charge density σ. Inside the shell is a conducting sphere of radius $\frac{2}{3}R$, at potential V_0. Outside is another conducting sphere, of radius $\frac{3}{2}R$. The outer conducting sphere is *grounded*.

a) Find the potential $V(r)$ between the charged shell and the inner conducting shell, $\frac{2}{3}R < r < R$.

b) Find the potential $V(r)$ between the charged shell and the outer conducting shell, $R < r < \frac{3}{2}R$.

c) Can you specify a potential on the inner shell, V_0, so that the electric field is zero inside the charged shell, $r < R$?

3 Griffiths Problems

3.18, 3.20, 3.21, 3.24