Problems for HW 7

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1 HW7 Problem 1

A sphere of radius a has charge density $\rho(r) = -K/r^2$, where r is distance from the center of the sphere and K is a constant.

a) Find the electric field inside the sphere. Give magnitude and direction. Explain your work.

b) Find the potential difference between the center of the sphere (at $r = 0$) and the surface of the sphere (at $r = a$).

c) Find the total charge on the sphere. If the potential is zero at $r \to \infty$, what is the potential at the surface of the sphere?

2 HW7 Problem 2

This problem asks you to compare the expressions for the multipole expansion in Griffiths and derived in class, given in the lecture notes on "Multipoles" on eres.

a) Consider a very thin shell, with outer radius R, very small thickness $d << R$, and charge density within the shell of $\rho = \rho_0 \cos^2(\theta)$. Here, ρ_0 is a constant. Use the equation on p. 7 of the "Multipole" lecture notes to find b_m, for $m = 0$ and $m = 2$ Argue that the other m's vanish. Use these to find $V(r, \theta)$ outside the shell, at $r > R$.

Evaluate your expression along the x-axis, at $r = x$ and $\theta = \pi/2$. How does the potential vary with x along the X-axis ($y = z = 0$)??

b) Now consider the same problem from the standpoint of Eq. 3.95. Using that equation, find the potential for the charged ring at the field point $\vec{r}_Q = (x, 0, 0)$. Note that θ' now measures the angle of the source point with respect to the X-axis. To keep things straight, I suggest the notations:

$$\theta_Z \equiv \arccos(z/r) \qquad \phi_Z \equiv \arctan(x/y) \tag{1}$$

$$\theta_X \equiv \arccos(x/r) \qquad \phi_X \equiv \arctan(z/y) \tag{2}$$

In this notation, the charge density in the shell is $\rho = \rho_0 \cos^2(\theta_Z)$. Show that this approach gives the same result.

c) In a short paragraph, compare Eq. 3.95 in Griffiths with the corresponding expressions in the lecture notes: the expression for $V(r, \theta)$ outside, at $r > R$, on p. 4 and the expression for b_m at the top of p. 7. These expressions look pretty similar. But, they are not identical.

Note that the expressions in the notes include θ twice: once as a variable of integration in calculating b_m (where it is the coordinate of the source point and probably should be called θ'), and once as an argument in the general expression for $V(r, \theta)$ (where it is the coordinate of the field point and might be called θ_Q). The expression in Griffiths includes θ' only, as a variable of integration: it is the coordinate of the source point.

Also, in class we assumed that the charge distribution $\rho(r', \theta')$ is axisymmetric: it may depend on r' and θ', but is independent of ϕ'. Griffiths claims that Eq. 3.95 holds for an arbitrary localized charge distribution (see the remarks just above Eq. 3.91). And, in Eqs. 3.92 through 3.94 he treats the charge distribution as a superposition of point sources, and uses the idea behind HW5 Problem 1 to expand this as a series of Legendre polynomials in θ'. What determines the axis for measurement of θ' in this calculation? Moreover, the argument of V on the left-hand side of Eq. 3.95 is $\vec{r}$: this suggests that Eq. 3.95 holds true for any field point in space.

How can Griffiths' expression involve fewer variables, yet be more general? Explain in a brief paragraph. (Hint: Try the preceding parts before you commit to paper.)

3 Problems from Griffiths

3.28, 3.33, 4.1, 4.5