Problems for HW 8

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1 HW8 Problem 1

A slab of thickness $2w$ contains polarizing material with polarization $\vec{P}$. The polarization is constant inside, but drops linearly to zero at the top and bottom surfaces over a distance h. Thus, if we take the origin at the center of the slab and the z-axis normal to the surfaces of the slab, the polarization is:

$$\vec{P}(z) = \begin{cases} P_0 \hat{z}, & -w < z < w \\ P_0 \cdot ((w+h) - z)/h \; \hat{z}, & +w < z < +w + h \\ P_0 \cdot (z + (w+h))/h \; \hat{z}, & -w - h < z < -w \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

To make some sense of these rather-complicated equations, note that the polarization is constant, except for the zones of width h starting at $z = \pm w$, where it varies linearly. It increases from 0 to $P_0 \hat{z}$ over the range $-h - w < z < -w$, and decreases from $P_0 \hat{z}$ to 0 over $w < z < w + h$.

a) Find the electric field as a function of z. Explain your work.

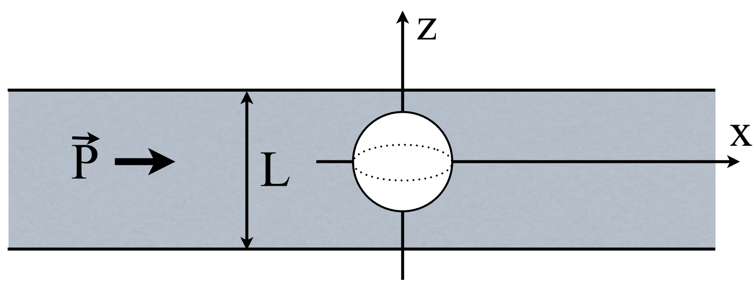
b) Find the electric displacement $\vec{D}$ as a function of z. Explain your work.

c) Now suppose that the polarization is *parallel* to the surfaces of the slab: $\vec{P} = P_0 \hat{x}$. Otherwise, the situation is just as described above, with constant polarization within the slab and a linear decline over narrow regions at the top and bottom surfaces. What is the electric field $\vec{E}$ as a function of z? What is the electric displacement $\vec{D}$ as a function of z?

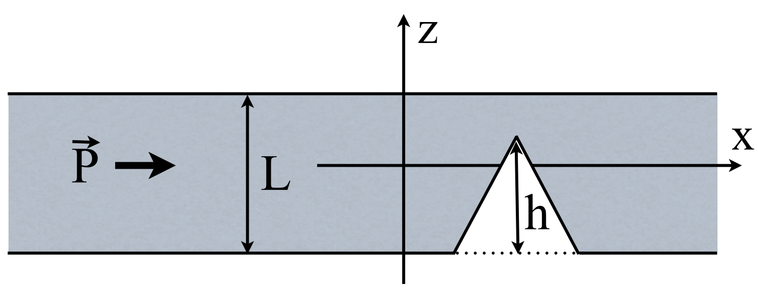
2 HW8 Problem 2

A thick slab containing polarized material has thickness L. The origin of the coordinate system is at the middle of the slab, halfway from either surface, and the z-axis is normal to the surfaces. Assume that the slab is infinite in the x and y directions. The polarization inside the slab is $\vec{P} = P_0\hat{x}$, parallel to the surfaces of the slab.

a) What is the bound volume charge within the slab? What is the bound surface charge on the surfaces of the slab? What is the electric field outside the slab, at $|z| > L/2$? Explain your answers.



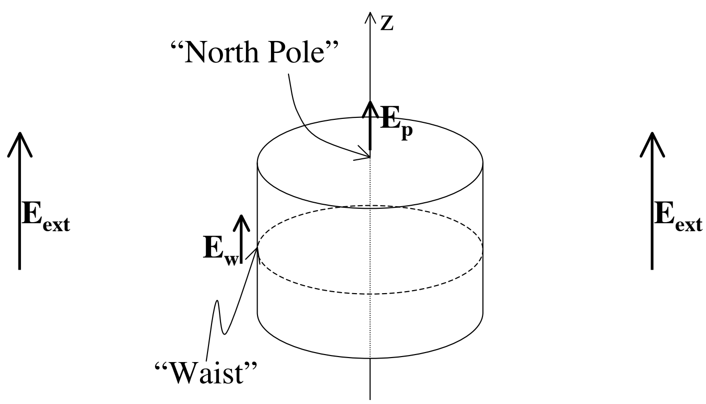
b) A spherical bubble of radius $a = L/3$ lies within the slab, with its center at the origin. The bubble contains zero polarization. What is the electric potential from sphere plus slab, outside the slab? What is the electric field outside the slab?



c) A hollow pyramid is excavated from the bottom face of the slab. The pyramid has a square base, of side ℓ, and it protrudes into the slab for height h. The sides of the pyramid are at some unknown angle to the coordinate axes. What is the electric potential of slab and pyramid, far away from the pyramid? Take, as your field point, $\vec{r}_Q = (x_Q, y_Q, z_Q)$. Assume that $|\vec{r}_Q| >> \ell$, and that $|\vec{r}_Q| >> h$. Give the smallest, non-zero term contributing to the potential.

3 HW8 Problem 3

A small conducting cylinder, carrying zero net charge, is placed in a constant external electric field $\vec{E}_{ext} = E_0\hat{z}$. The cylinder has equal height and diameter, both ℓ. The axis of the cylinder is aligned with the z-axis, along the external electric field. The resulting charge distribution on the surface of the cylinder is difficult to determine. Of course, this charge distribution will produce an electric field that may be quite large close to the cylinder. However, symmetry considerations apply, and we can state the boundary conditions.



a) Because of symmetry, the *net* electric field at the waist of the cylinder, exactly halfway between the ends, $\vec{E}_w$, must be *parallel* to $\hat{z}$, and so to the surface of the cylinder. What is the magnitude of the E-field, $|E_w|$, *just oustide* the cylinder at its waist? What is the surface charge at the waist of the cylinder? Explain your answer.

b) Also because of symmetry, the *net* electric field at the axis of the cylinder at its "north pole", $\vec{E}_p$, is *perpendicular* to the surface of the cylinder, and so parallel to $\hat{z}$. What is the surface charge on the cylinder at the north pole, σ_p, in terms of the magnitude of the E-field there, $|\vec{E}_p|$? Explain your answer.

4 HW8 Problem 4

A spherical shell has inner radius a and outer radius b. The material in the shell has radially-outward polarization $\vec{P} = P_0\hat{r}$. Note that the magnitude of polarization is constant, but its direction changes.

a) Show that the shell has uniform, opposite bound surface charges on its inner and outer surfaces. Because they have different areas, the net charges do not balance.

b) Find the bound charge density between the shells. What is the *net* bound charge on the shell? (You may work this part in any way you wish, so long as you explain your reasoning).

c) What is the electric field inside the shell? What is the electric field outside? (Hint: Use Gauss's law in integral form).

d) What is the electric displacement as a function of radius r?

e) What is the electric field $\vec{E}$ inside the shell?

5 HW8 Problem 5

A sphere of radius a has charge density $\rho(r) = \rho_0 r \cos^3(\theta)$, where r is distance from the center of the sphere, ρ_0 is a constant, and θ is the angle from the z-axis.

a) Find the electric potential $V(r_Q, \theta_Q)$ outside the sphere. Explain your work.

b) Far away from the sphere, the potential is that of a dipole. Explain why, in a couple of sentences. What is the dipole moment? (Hint: You need not do any more math to answer this!)