Problems for HW X

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These problems will not be graded.

1 HWX Problem 1

Suppose than an object is composed of a linear dielectric material, with constant relative permittivity ϵ_r. The object contains no free charge. Prove that the object contains zero bound volume charge density, ρ_b. (Of course, it can have nonzero bound *surface* charge, σ_b).

2 HWX Problem 2

a) A sphere of polarized material, with uniform polarization $\vec{P} = P_0 \hat{z}$ and radius a, will have a distribution of bound charge on its surface. This bound charge will create an electric field, outside and inside the sphere. Recall that the resulting electric potential outside the sphere ($r > a$) is that of a dipole, with a dipole moment $\vec{p} = \frac{4\pi}{3} a^3 P_0 \hat{z}$. You need not prove this fact.

Show that the electric field inside the sphere is uniform, by matching the potential outside (given above) with the general form for the potential in spherical coordinates (assume no azimuthal dependence: $V = V(r, \theta)$). Find the magnitude and direction of the electric field inside the sphere.

b) Consider a sphere of radius a, composed of dielectric material with uniform relative permittivity ϵ_r. The sphere is placed into a uniform, external electric field $\vec{E}_{ext} = E_0 \hat{z}$. Then, the field inside the sphere is the superposition of the external electric field, and the electric field $\vec{E}_b$ produced by bound surface charge on the sphere. The polarization in the dielectric is proportional to the superposition of these fields:

$$\vec{P} = \chi_e \epsilon_0 \left(\vec{E}_{ext} + \vec{E}_b \right) \tag{1}$$

The proportionality constant χ_e depends on the particular material of the dielectric.

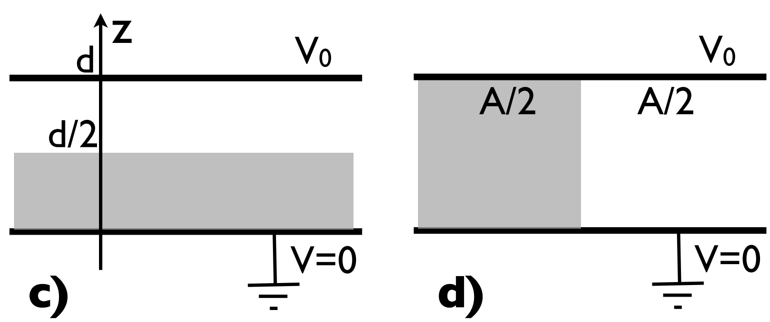
Find the polarization $\vec{P}$ that yields electric field $\vec{E}_b$ that satisfies the above equation, for a given χ_e. (Use your results from part a.) Find the net induced dipole moment of the sphere, and state the electric field outside.

3 HWX Problem 3

A capacitor consists of two conducting plates with large areas A and a small separation d. The lower plate lies at $z = 0$ and is grounded (potential $V = 0$). The upper plate lies at $z = +d$ and is at potential V_0. Because the plates are large, you can ignore fringing fields near the edges of the plates.

a) Assume that the region between the plates contains vacuum. Find the electric field between plates (give magnitude and direction), the charge density on the upper plate, the total charge on the upper plate, and the capacitance of the capacitor.

b) Now suppose that the region between the plates contains dielectric material with relative permittivity ϵ_r. Given the potential of the upper plate V_0, find the electric field between the plates. Find the polarization $\vec{P}$ in the dielectric. Find the bound charge at the surfaces of the dielectric. Find the capacitance.

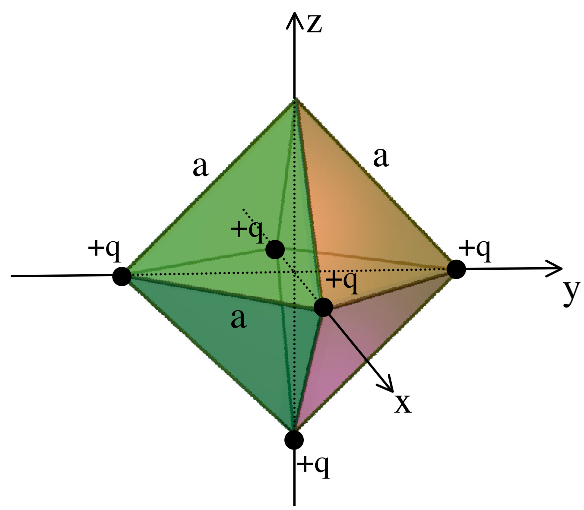


c) Suppose that the dielectric material extends only halfway across the gap between conducting plates, from $z = 0$ to $z = d/2$. Vacuum fills the remainder of the gap. What boundary conditions relate $\vec{E}$ and electric displacement $\vec{D}$ in the two regions, above and below the surface of the dielectric? Find $\vec{E}$ and $\vec{D}$ in these two regions, given that the potential difference between top and bottom plates is V_0. Find the free surface charge, and total free charge, on the top plate. Find the capacitance.

d) Now suppose that the dielectric material extends all the way across the gap, but covers only half of the area of the plates. Vacuum fills the remainder of the region between the plates. What boundary conditions relate $\vec{E}$ and electric displacement $\vec{D}$ in the two regions, inside and outside the dielectric? Find $\vec{E}$ and $\vec{D}$ in these two regions, given that the potential difference between top and bottom plates is the same: V_0. Find the free surface charge on the top plate, in the two regions. Find the capacitance.

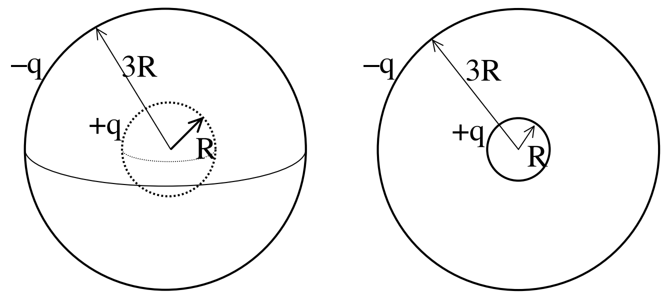
4 HWX Problem 4

A regular octahedron has sides of length a, and is centered at the origin. The 6 vertices, or "corners" of the octahedron lie on the $\pm x$, $\pm y$, and $\pm z$ axes. A charge $+q$ is placed on each of 5 of the 6 vertices: the charge on the $+z$-axis is *omitted.*
a) Find the electric field $\vec{E}$ at the center of the octahedron. Give magnitude and direction. Explain your reasoning.
Why is Gauss's Law not a suitable approach for determining the electric field in this case? Would Gauss's Law be suitable if the charge on the $+z$-axis were present? Explain your answers in a sentence or 2.
b) Find the electric potential V at the center of the octahedron. Explain your reasoning.



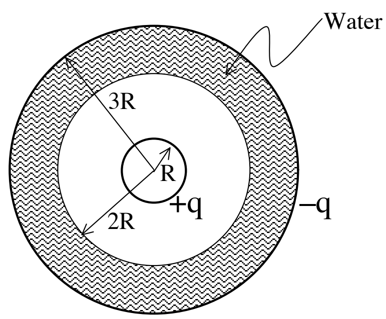
5 HWX Problem 5

a) A charge $+q$ is placed on a metal sphere of radius R. A second spherical metal shell, concentric with the first and with radius $3R$, is given a charge $-q$.
• Give the electric field between the shells.
• Find the potential difference between the shells, ΔV.



Cross-Section

b) • Find the capacitance of the pair of conducting spherical shells.
• Give the energy stored in the capacitor. You need not derive your answer.



Cross-Section

c) Now water is added to the space between the two spherical shells, from radius $2R$ to radius $3R$. (Very thin insulating shells lie at $2R$ and $3R$, confining the water and keeping charges $\pm q$ on the metal shells.) The water is a dielectric, with electric susceptibility χ_e, or relative dielectric constant $\epsilon_r = 1 + \chi_e$.
• Find $\vec{D}$ in the region between the spheres.
• Find the electric field $\vec{E}$ in the region between the spheres.

d) • Find the polarization $\vec{P}$ in the water.
• Find the bound volume charge ρ_b *within* the water, and the bound surface charge σ_b on *both* surfaces of the water.
• What is the *total* bound charge on the water? Explain your answer in a sentence or two.

6 HWX Problem 6

The $x - y$ plane has the potential

$$V(x, y, z = 0) = V_0 \cos\left(\frac{2\pi x}{L}\right). \tag{2}$$

No charge lies in the upper half-space $z > 0$, or the lower half-space $z < 0$. Far from the $x - y$ plane, at $|z| \to \infty$, the potential falls to zero: $v \to 0$.

• Write down the potential in the upper half-space $V(x, y, z > 0)$, and that in the lower half-space $V(x, y, z > 0)$. Explain why this potential is unique, for the boundary conditions. (You need not derive your answer.)

• Find the surface charge density on the $x - y$ plane, $\sigma(x, y)$.