Homework #3
Solutions

Carter 3.9, 4.3, 4.4, 4.7, 4.9, 4.13

Schroeder 1.41, 1.43

Additional Problem 1

Carter

3.9 a. $-W =$ work done by gas

$$= \int P\,dV$$

8 kg = how many moles?

For O_2: 32 g/mol = 0.032 kg/mol

$$n = \frac{8\text{ kg}}{0.032\text{ kg/mol}} = 250\text{ mol}$$

$pV = nRT \quad p = \frac{nRT}{V}$; $n = 250$ mol, $R = 8.315$ J/mol·K, $V = 10\text{ m}^3$, $T = 300$ K

$P_0 = 6.24 \cdot 10^4$ Pa

$$-W = \int_{V_i}^{V_f} P_0\,dV = P_0(V_f - V_i) = P_0(10-5)$$

$$-W = 312{,}000\text{ J}$$

b. $-W = \int_{V_i}^{V_f} \frac{nRT}{V}\, dV$ (using $pV = nRT$; $p(V) = \frac{nRT}{V}$

$= nRT(\ln V_f - \ln V_i)$

$= nRT \ln\left(\frac{V_f}{V_i}\right) = 432{,}000\ \text{J}$

c. $pV = nRT$ $\quad T = \frac{p_0 V_f}{nR} = \frac{6.24 \cdot 10^4 \cdot 5\,\text{m}^3}{250 \cdot 8.315}$

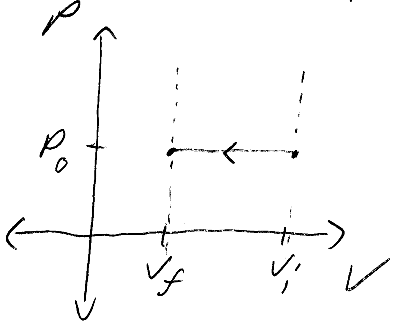
$T = 150\ \text{K}$

d. $pV = nRT$

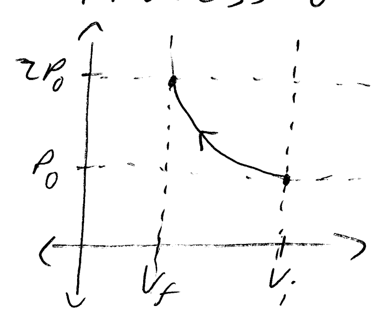
$p = \frac{nRT_0}{V_f} = \frac{8.315 \cdot 250 \cdot 300}{5}$

$p = 1.25 \cdot 10^5\ \text{Pa}$

e. Process A:



Process B



4.3

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \Rightarrow V\beta = \left(\frac{\partial V}{\partial T}\right)_P$$

$$dU = dQ - PdV$$

Differentiating both sides by T at constant P yields:

$$\left(\frac{\partial U}{\partial T}\right)_P = \left(\frac{\partial Q}{\partial T}\right)_P - P\left(\frac{\partial V}{\partial T}\right)_P$$

$\left(\frac{\partial Q}{\partial T}\right)_P$ is C_P. So:

$$\left(\frac{\partial U}{\partial T}\right)_P = C_P - PV\beta$$

4.4 Prove

$$\left(\frac{\partial U}{\partial P}\right)_T = P\kappa V - (C_p - C_v)\frac{\kappa}{\beta}$$

$$dU = \left(\frac{\partial U}{\partial P}\right)_T dP + \left(\frac{\partial U}{\partial T}\right)_P dT$$

Divide both sides by dT holding V constant:

$$\left(\frac{\partial U}{\partial T}\right)_V = \left(\frac{\partial U}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V + \left(\frac{\partial U}{\partial T}\right)_P$$

Solve for $\left(\frac{\partial U}{\partial P}\right)_T$:

$$\left(\frac{\partial U}{\partial P}\right)_T = \left[\left(\frac{\partial U}{\partial T}\right)_V - \left(\frac{\partial U}{\partial T}\right)_P\right]\left(\frac{\partial T}{\partial P}\right)_V$$

For $\left(\frac{\partial U}{\partial T}\right)_P$ use the result of last problem (4.3)

$$\left(\frac{\partial U}{\partial T}\right)_P = C_p - P\left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial U}{\partial P}\right)_T = \left[\left(\frac{\partial U}{\partial T}\right)_V - C_p + P\left(\frac{\partial V}{\partial T}\right)_P\right]\left(\frac{\partial T}{\partial P}\right)_V$$

Note that $\frac{\kappa}{\beta} = \left(\frac{\partial T}{\partial P}\right)_V$.

$$\left(\frac{\partial U}{\partial P}\right)_T = P\kappa V - (C_p - C_v)\frac{\kappa}{\beta}$$

4.7 $c_p = a + bT - \frac{c}{T^2}$

$$Q = \int dq = n \cdot \int_T^{2T} \left(a + BT - \frac{c}{T^2}\right) dT \quad ; \; n = 1000 \text{ mol}$$

$$= \left[aT + b \cdot \frac{T^2}{2} + \frac{c}{T}\right]_T^{2T} \cdot 1000$$

$$Q = \left[aT + \frac{3}{2} bT^2 - \frac{c}{2T}\right] \cdot 1000$$

4.9 a. Adiabatic: $Q = 0$

$W_{net} = \Delta U$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$
(Adiabatic process)

$$P_2 V_2 = nRT_2$$
$$P_1 V_1 = nRT_1$$
(Equation of state)

$$P_1 = \frac{nRT_1}{V_1}$$
$$P_2 = \frac{nRT_2}{V_2}$$

$$\frac{\cancel{nR}T_1}{V_1} V_1^\gamma = \frac{\cancel{nR}T_2}{V_2} V_2^\gamma$$

$$T_1 \cdot V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\ln\left(\frac{T_2}{T_1}\right) = (\gamma-1)\ln\left(\frac{V_1}{V_2}\right)$$

b. $$\ln\left(\frac{2}{5}\right) = (\gamma-1)\ln\left(\frac{1}{2}\right)$$

$$-0.916 = (\gamma-1) \cdot -.693$$

$\gamma = 2.32$ But γ must $= \frac{C_P}{C_V} = \frac{n+2}{n}$, where is an integer ≥ 3. The largest γ can be for known ideal gases is 1.67. Not enough.

4.13 $\Delta T_1 = 20°C$ $\Delta T_2 = 100°C$ $\Delta T_3 = 300°C$

C_p solid $= C_{p(s)} = 0.55 \frac{kcal}{kg}$ $L_v = 538$ kcal/kg

C_p liquid $= C_{p(l)} = 1$ kcal/kg $L_f = 80$ kcal/kg

C_p gas $= C_{p(g)} = 0.48$ kcal/kg

$$\Delta h = C_{p(s)} \cdot \Delta T_1 + L_f + C_{p(l)} \Delta T_2 + L_v + C_{p(g)} \Delta T_3$$

$= 873$ kcal

$= 3.66 \cdot 10^6 J$

Schroeder

1.41 a. $m_w c_w \Delta T_w = 250 \cdot 4.186 \cdot 4$

$= 4186 J$

b. Heat lost by the metal must be the same by energy conservation.

c. $C_m = \frac{Q}{\Delta T_m} = 55 J/°C$

d. $\frac{C_m}{100g} = 0.55 J/g°C$

1.43 Let's say we have 1 mol of water; mass $= 18g$. $N = 6.02 \cdot 10^{23}$ molecules.

$C =$ Total heat capacity $= 18g \cdot 4.186\, J/gK$

$C_N =$ Total heat capacity per molecule

$$= \frac{18g \cdot 4.186\, J/g \cdot K}{6.02 \cdot 10^{23}\ \text{molecules}}$$

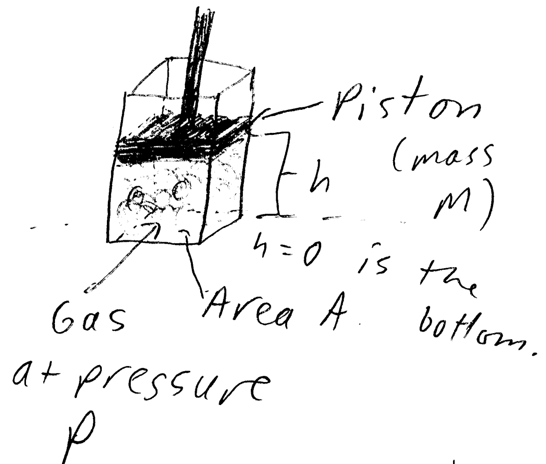
$$= 1.25 \cdot 10^{-22}\, J/K \cdot \text{molecule}$$

$$= 9.07k$$

Each quadratic degree of freedom should have heat capacity $k/2$, so this means water needs about 18 quadratic degrees of freedom.

Additional Problem

A.



$V = Ah$

Net force on piston =

$PA - Mg = F_{net} = Ma_p$, where a_p = acceleration of piston.

Equilibrium occurs when $F_{net} = 0$.

$$PA - Mg = 0$$

$$PA = Mg$$

We need $P(h)$.

$$PV = nRT \qquad P = \frac{nRT}{V}$$

$n = 1$; T = constant; $V = Ah$.

$$P(h) = \frac{1 \cdot RT}{Ah}$$

$$PA = \frac{RTA}{Ah_f} = Mg \qquad h_f = \frac{RT}{Mg}$$

B.

$$Ma_p = M\ddot{h} = \frac{RT}{Ah} \cdot A - Mg$$

$$M\ddot{h} = \frac{RT}{h} - Mg$$

C. By conservation of energy:

Let $\Delta h = h_0 - h_f$.

$K = mg\Delta h$ - Work done by the piston on the gas

$$= mg\Delta h - \left(-\int P\,dV\right) \; ; \quad dV = A\,dh$$

$$= mg\Delta h + \int_{h_0}^{h_f} \frac{RT}{Ah} \cdot A\,dh$$

$$= mg\Delta h + RT\left[\ln h\right]_{h_0}^{h_f}$$

$$= mg\Delta h + RT\left(\ln h_f - \ln h_0\right)$$

(We need to take the negative of $\int P\,dV$ since $\int P\,dV$ is always work done BY the gas.)

$$= mg\Delta h + \underbrace{RT\ln\left(\frac{h_f}{h_0}\right)}_{}, \text{ where } h_f = \frac{RT}{mg}, \; \Delta h = h_0 - h_f$$

Since $h_f < h_0$, this will be negative.

D. The "configuration" contribution is $-\int P\,dV$, so $K + \text{Config.} = mgh$.

This makes sense because, when everything's said and done, all that's changed is that a mass of mass M has fallen a distance $h_0 - h_f$.