

Homework #4

Solutions

Carter 5.3, 5.4, 5.9, 6.7, 6.8, 6.13

Schroeder 4.7, 4.8, 4.11, 4.21abc

Additional Problems 1-3

Carter

5.3 a. $u = u_0 + c_v T - \frac{a}{v}$

$$u - u_0 + \frac{a}{v} = c_v T$$

$$T = \frac{u - u_0}{c_v} + \frac{a}{v c_v}$$

$$\eta = \left(\frac{\partial T}{\partial v} \right)_u = -\frac{a}{v^2 c_v}$$

Clearly, $\eta = 0$ if $a = 0$.

b. $h = u + P v$

We know

$$u = u_0 + c_v T - \frac{a}{v}$$

Eqn of state for Vander Waals:

$$P + \frac{a}{v^2} = \frac{RT}{v-b}$$

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

Combining

$$h = u_0 + c_v T - \frac{a}{v} + \left(\frac{RT}{v-b} - \frac{a}{v^2} \right) v$$

$$h = u_0 + c_v T - \frac{a}{v} + \frac{RTv}{v-b} - \frac{a}{v}$$

$$h = u_0 + c_v T - \frac{2a}{v} + \frac{RTv}{v-b}$$

c. $\mu = \left(\frac{\partial T}{\partial p} \right)_h$ (Carter (5.9))

$$-\frac{1}{c_p} \left(\frac{\partial h}{\partial p} \right)_T = \left(\frac{\partial T}{\partial p} \right)_h = \mu \quad (\text{Carter pg 74})$$

$$\mu = -\frac{1}{c_p} \left(\frac{\partial h}{\partial p} \right)_T = -\frac{1}{c_p} \left(\frac{\partial h}{\partial v} \right)_T \left(\frac{\partial v}{\partial p} \right)_T$$

But $h = u + pv$ so $\left(\frac{\partial h}{\partial v} \right)_T = \left(\frac{\partial u}{\partial v} \right)_T + p + v \left(\frac{\partial p}{\partial v} \right)_T$

$$\left(\frac{\partial u}{\partial v} \right)_T = -c_v \left(\frac{\partial T}{\partial v} \right)_{u,u} \quad (\text{Carter (5.2)})$$

$$= -c_v \eta = -c_v \cdot -\frac{a}{v^2 c_v} = \frac{a}{v^2} \quad (\text{Carter pg. 71})$$

$$\mu = -\frac{1}{c_p} \left(\frac{a}{v^2} + p + v \left(\frac{\partial p}{\partial v} \right)_T \right) \left(\frac{\partial v}{\partial p} \right)_T. \text{ Using } p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\mu = -\frac{1}{c_p} \left(\cancel{\frac{a}{v^2}} + \frac{RT}{v-b} - \cancel{\frac{a}{v^2}} + v \left(\frac{\partial p}{\partial v} \right)_T \right) \left(\frac{\partial v}{\partial p} \right)_T$$

$$= -\frac{1}{c_p} \left(\frac{RT}{v-b} \right) \cdot \left(\frac{\partial v}{\partial p} \right)_T - \frac{v}{c_p}$$

~~But~~ But $\left(\frac{\partial V}{\partial P}\right)_T = -\kappa V$, so

$$\mu = -\frac{1}{c_p} \left(\frac{RT}{v-b}\right) \cdot -\kappa V - \frac{V}{c_p}$$

$$\mu = \frac{\kappa V}{c_p} \frac{RT}{(v-b)} - \frac{V}{c_p}$$

$$d. \kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial P}\right)_T = -\frac{1}{V} \frac{1}{\left(\frac{\partial P}{\partial V}\right)_T}$$

$$\left(\frac{\partial P}{\partial V}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

$$\kappa = \frac{-1}{\left(-\frac{RT}{(v-b)^2} + \frac{2a}{v^3}\right)V} = \frac{1}{\left(\frac{RTv}{(v-b)^2} - \frac{2a}{v^2}\right)}$$

e. If $a=b=0$, $\kappa = \frac{1}{\frac{RTv}{v^2}} = \left(\frac{v}{RT}\right) \checkmark$

$$\mu = \frac{\kappa V}{c_p} \frac{RT}{v} - \frac{V}{c_p} = \frac{v}{RT} \cdot \frac{1}{c_p} \cdot RT - \frac{1}{c_p} = 0 \checkmark$$

$$5.4 \text{ a. } \left(\frac{\partial h}{\partial T} \right)_V = \left(\frac{\partial h}{\partial T} \right)_P + \left(\frac{\partial h}{\partial P} \right)_T \left(\frac{\partial P}{\partial T} \right)_V$$

(Clint Eastwood Relation)

$$= C_p + \left(\frac{\partial h}{\partial P} \right)_T \left(\frac{\partial P}{\partial T} \right)_V$$

Note:

$$\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$$

$$\kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

$$\left(\frac{\partial h}{\partial T} \right)_V = C_p + \left(\frac{\partial h}{\partial P} \right)_T \cdot \frac{\beta}{\kappa}$$

$$= C_p - C_p \left(\frac{\partial T}{\partial P} \right)_h \cdot \frac{\beta}{\kappa} \quad (\text{Carter pg. 74})$$

$$\left(\frac{\partial h}{\partial T} \right)_V = C_p \left(1 - \frac{\mu \beta}{\kappa} \right)$$

$$b. \left(\frac{\partial h}{\partial V} \right)_T = \left(\frac{\partial h}{\partial P} \right)_T \left(\frac{\partial P}{\partial V} \right)_T = \left(\frac{\partial h}{\partial P} \right)_T \cdot -\frac{1}{V\kappa}$$

$$= -C_p \mu \cdot -\frac{1}{V\kappa} = \left(\frac{C_p \mu}{V\kappa} \right)$$

$$c. \left(\frac{\partial T}{\partial V} \right)_h = \left(\frac{\partial T}{\partial P} \right)_h \left(\frac{\partial P}{\partial V} \right)_h = \mu \left(\frac{\partial P}{\partial V} \right)_h = \frac{\mu}{\left(\frac{\partial V}{\partial P} \right)_h}$$

Clint Eastwood relation on denominator:

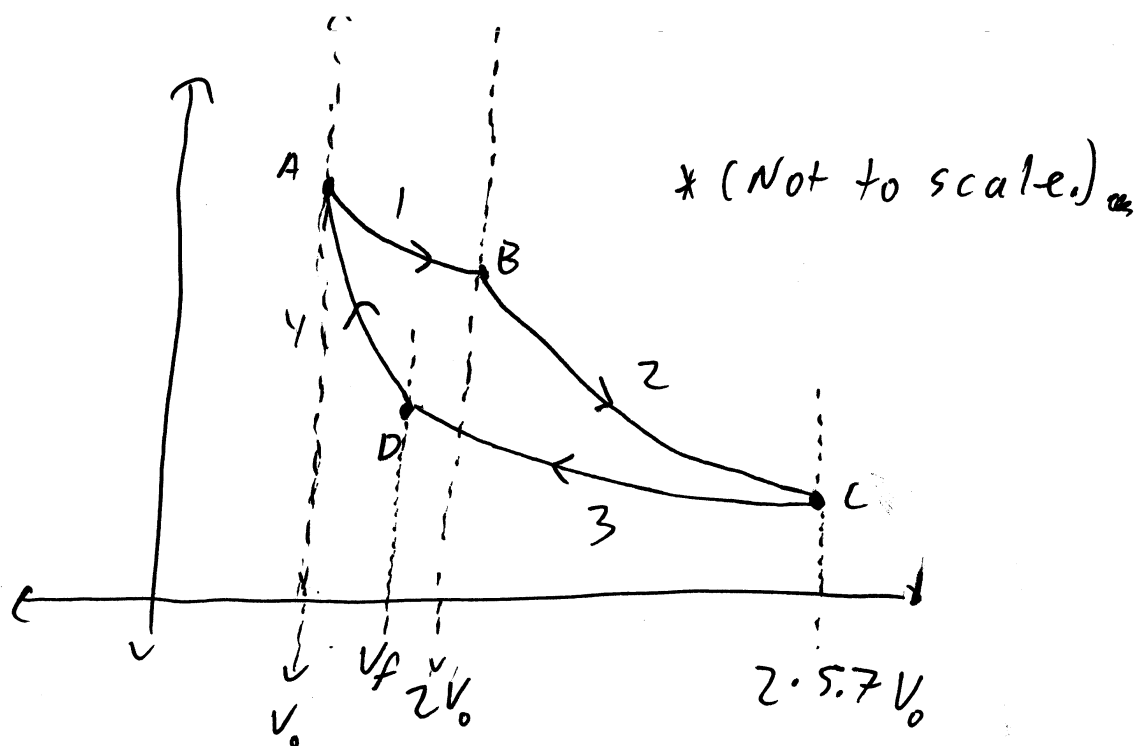
$$\left(\frac{\partial V}{\partial P} \right)_h = \left(\frac{\partial V}{\partial P} \right)_T + \left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial T}{\partial P} \right)_h$$

$$\left(\frac{\partial V}{\partial P}\right)_h = -V\kappa + V\beta\mu$$

S_0

$$\left(\frac{\partial T}{\partial V}\right)_h = \frac{\mu}{V(\mu\beta - \kappa)}$$

S.9



1 = isothermal at $T = T_0$

2 = adiabatic

3 = isothermal at $T = T_f$

4 = adiabatic

($\gamma = \frac{5}{3}$ for a monatomic ideal gas)

$$\Delta U = 0 = W_{\text{net}} + Q_1 + \cancel{Q_2} + Q_3 + \cancel{Q_4}$$

If we define W_{net} as the work done BY the gas (the output work, in other words) this becomes

$$0 = -W_{\text{out}} + Q_1 + Q_3$$

$$W_{\text{out}} = Q_1 + Q_3$$

First, let's compute T_f ~~in terms of~~ V_f ~~in terms of~~ in terms of known quantities.

We know $U(D) = U(C)$, since

$T(D) = T(C)$. Therefore $\Delta U(2) = \Delta U(4)$

But $Q_2 = Q_4 = 0 \Rightarrow$

$$W_2 = W_4$$

From $pV^\gamma = \text{constant}$, we have:

$$\int_{V_0}^{V_f} \frac{k_4}{V^{5/3}} dV = \int_{2V_0}^{2.5.7V_0} \frac{k_2}{V^{5/3}} dV$$

$$k_4 \left[+ \frac{3}{2} V^{-2/3} \right]_{V_0}^{V_f} = k_2 \left[+ \frac{3}{2} V^{-2/3} \right]_{2V_0}^{2.5.7V_0}$$

Assume V_f is some multiple of V_0 ;

e.g. $V_f = aV_0$. Then

$$k_4 \left((aV_0)^{-2/3} - (V_0)^{-2/3} \right) = k_2 \left((2.5.7V_0)^{-2/3} - (2V_0)^{-2/3} \right)$$

The V_0 s cancel, yielding:

$$a^{-2/3} = 1 + \frac{k_2}{k_4} \left((2.5.7)^{-2/3} - 2^{-2/3} \right)$$

$$\text{But } k_4 = p_A V_A^\gamma = nRT_0 V_0^{\gamma-1}$$

$$\text{and } k_2 = p_B V_B^\gamma = \frac{nRT_0 \cdot (2V_0)^\gamma}{2V_0} = nRT_0 2^{\gamma-1} V_0^{\gamma-1}$$

$$\text{So } \frac{k_2}{k_4} = \frac{nRT_0 2^{\gamma-1} V_0^{\gamma-1}}{nRT_0 V_0^{\gamma-1}} = 2^{\gamma-1} = 2^{2/3}$$

Therefore

$$a^{-2/3} = 1 + \cancel{2^{2/3}} \left(\cancel{2^{-2/3} \cdot 5.7^{-2/3}} - \cancel{2^{-2/3}} \right)$$

$$a = 5.7 \Rightarrow \underline{\underline{V_f = 5.7 V_0}}$$

$$T_f V_f^{\gamma-1} = T_0 V_0^{\gamma-1}$$

$$\underline{\underline{T_f = T_0 \cdot \left(\frac{V_0}{V_f} \right)^{\gamma-1} = T_0 \cdot \left(\frac{1}{5.7} \right)^{2/3}}}$$

Now we can compute Q_1 & Q_3 easily: $\Delta U_1 = 0$ (isotherm) $\Rightarrow$

$$Q_1 = -W_1 = \int_{V_0}^{2V_0} \frac{n R T_0}{V} dV = n R T_0 \ln 2$$

$$Q_3 = -W_3 = \int_{2 \cdot 5.7 V_0}^{5.7 V_0} \frac{n R T_0 \cdot \left(\frac{1}{5.7} \right)^{2/3}}{V} dV$$

$$= \frac{n R T_0}{(5.7)^{2/3}} \ln \left(\frac{1}{2} \right) = -\frac{n R T_0}{(5.7)^{2/3}} \ln 2$$

$$W = Q_1 + Q_3 = 9 \cdot 10^5 \text{ J}$$

$$n R T_0 \ln 2 \left(1 - \frac{1}{(5.7)^{2/3}} \right) = 9 \cdot 10^5$$

$$n = \underset{\text{mol}}{1000}, \quad R = 8.315 \text{ J/mol} \cdot \text{K}$$

$$T_0 = 227 \text{ K}$$

$$T_f = \frac{T_0}{(5.7)^{2/3}} = 71.3 \text{ K}$$

1 & 2 are simply shorthand for
6.7 the conditions at states 1 & 2.

$$\begin{aligned} \text{a. } \Delta S &= \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{dU + P dV}{T} \\ &= \int_1^2 \frac{n C_v dT}{T} + \int_1^2 \frac{n R T}{V T} dV \end{aligned}$$

$$\Delta S = n C_v \ln\left(\frac{T_2}{T_1}\right) + n R \ln\left(\frac{V_2}{V_1}\right)$$

$$\text{b. } V_1 = \frac{n R T_1}{P_1} \quad V_2 = \frac{n R T_2}{P_2}$$

$$\frac{V_2}{V_1} = \frac{\cancel{n R} T_2}{P_2} \cdot \frac{P_1}{\cancel{n R} T_1} = \frac{T_2 P_1}{T_1 P_2}$$

$$\begin{aligned} \Delta S &= n C_v \ln\left(\frac{T_2}{T_1}\right) + n R \ln\left(\frac{T_2}{T_1} \cdot \frac{P_1}{P_2}\right) \\ &= n C_v \ln\left(\frac{T_2}{T_1}\right) + n R \ln\left(\frac{T_2}{T_1}\right) + n R \ln\left(\frac{P_1}{P_2}\right) \\ &= n (C_v + R) \ln\left(\frac{T_2}{T_1}\right) + n R \ln\left(\frac{P_1}{P_2}\right) \end{aligned}$$

$$\Delta S = n C_p \ln\left(\frac{T_2}{T_1}\right) + n R \ln\left(\frac{P_1}{P_2}\right)$$

6.8 Using our result from 6.7:

a. ~~isobaric process~~

$$\Delta S = n C_p \ln \left(\frac{T_2}{T_1} \right) + n R \ln \left(\frac{P_1}{P_2} \right)$$

$$\Delta s = C_p \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{P_1}{P_2} \right)$$

$P_1 = P_2$ (isobaric) so:

$$\Delta s = C_p \ln \left(\frac{T_2}{T_1} \right)$$

$$P_1 V_1 = n R T_1$$

$$P_2 V_2 = n R T_2$$

$$\Delta s = C_p \ln \left(\frac{V_2}{V_1} \right)$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \Rightarrow \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

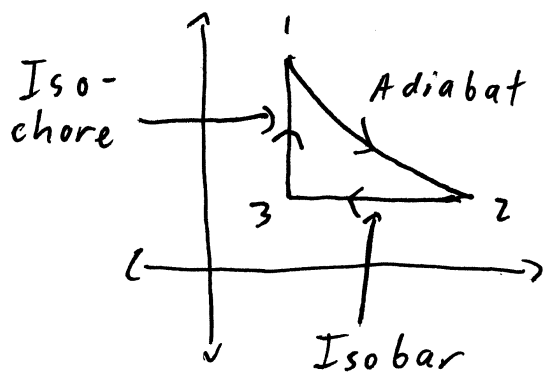
b. $\Delta s = C_v \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right)$

Isothermal $\Rightarrow T_1 = T_2$

$$\Delta s = R \ln \left(\frac{V_2}{V_1} \right)$$

c. a. is larger, by $(C_p - R) \ln \left(\frac{V_2}{V_1} \right)$
 $= C_v \ln \left(\frac{V_2}{V_1} \right)$.

6.13



Path $\overline{12}$: $\Delta Q = 0$
by definition.

$$\Rightarrow \Delta S = 0$$

Path $\overline{23}$:

Use our result from 6.7 b.

$$\Delta S = n C_V \ln\left(\frac{T_2}{T_1}\right) + n R \ln\left(\frac{V_2}{V_1}\right)$$

Let's rewrite this in ~~the~~ terms of p & V instead of T & V :

$$\frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1}$$

$$\Delta S = n C_V \ln\left(\frac{P_2 V_2}{P_1 V_1}\right) + n R \ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S = n C_V \ln\left(\frac{P_2}{P_1}\right) + n C_P \ln\left(\frac{V_2}{V_1}\right)$$

~~The~~ Now we just need P_2 in terms of P_1, V_1 .

$$P_2 V_2^\gamma = P_1 V_1^\gamma \quad P_2 = P_1 \cdot \left(\frac{V_1}{V_2}\right)^\gamma$$

Monatomic ideal gas $\Rightarrow \gamma = \frac{5}{3}$

$$P_2 = P_1 \cdot \left(\frac{2}{4}\right)^{5/3} = 0.315 P_1$$

$$\Delta S \text{ for } \overline{23} = n C_v \ln\left(\frac{P_3}{P_2}\right) + n C_p \ln\left(\frac{V_3}{V_2}\right)$$

$$P_3 = P_2 ; \quad V_3 = \frac{1}{2} V_2 .$$

$$\Delta S_{\overline{23}} = n C_v \cdot 0 + n C_p \cdot \ln\left(\frac{1}{2}\right)$$

$$C_p = \frac{5}{2} R \text{ for a monatomic ideal gas.}$$

$$\Delta S_{\overline{23}} = 1000 \cdot \frac{5}{2} \cdot 8.315 \text{ J/mol} \cdot \text{K} \cdot \ln\left(\frac{1}{2}\right) = -19908 \text{ J/K}$$

$$\Delta S_{\overline{23}} = -19908 \text{ J/K}$$

$$\Delta S \text{ for } \overline{31} = n C_v \ln\left(\frac{P_1}{P_3}\right) + n C_p \ln\left(\frac{V_1}{V_3}\right)$$

$$\text{But } V_1 = V_3 \text{ \& } P_3 = P_2 = 0.315 P_1$$

$$\Delta S_{\overline{31}} = n \cdot C_v \cdot \ln\left(\frac{P_1}{0.315 P_1}\right)$$

$$= 1000 \cdot \frac{3}{2} R \cdot 1.155 \text{ J/K}$$

$$\Delta S_{\overline{31}} = 19908 \text{ J/K}$$

Clearly,

$$\Delta S_{\overline{23}} + \Delta S_{\overline{31}} + \Delta S_{\overline{12}} = 0$$

Schroeder

4.7 The waste heat is always greater than the heat removed from the cold reservoir. If the air conditioner is in the middle of a building, it has no place to dump the waste heat except right back into the room! This means that the ~~reverse~~ air conditioner's net effect will be to INCREASE the room's temperature.



4.8 Temporarily, yes, as the cold air already in the fridge mixes with the surroundings.

But after the kitchen and the refrigerator's interior equalize in temperature, it's basically the same thing as having an air conditioner with nowhere to dump its waste heat (since, all along, the fridge has been dumping its waste heat into the kitchen).

4.14

a. $COP \text{ usually} = \frac{\text{Benefit}}{\text{Cost}}$

Benefit = Q_h (you want to heat the house)

Cost = W (Energy you must expend)

$$COP = \frac{Q_h}{W}$$

b. By conservation of energy

$$Q_h = Q_c + W \text{ for a full cycle.}$$

$$COP = \frac{Q_h}{Q_h - Q_c} = \frac{1}{1 - \frac{Q_c}{Q_h}}$$

Always > 1 , since

$$Q_h > 0 \ \& \ Q_c > 0 \ \& \ Q_h > Q_c.$$

c. ~~By~~ The entropy of the universe ~~must always~~ can never decrease.

Thus the entropy expelled during the cycle must be $\geq$ the entropy absorbed:

$$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c} \Rightarrow \frac{T_c}{T_h} \geq \frac{Q_c}{Q_h}$$

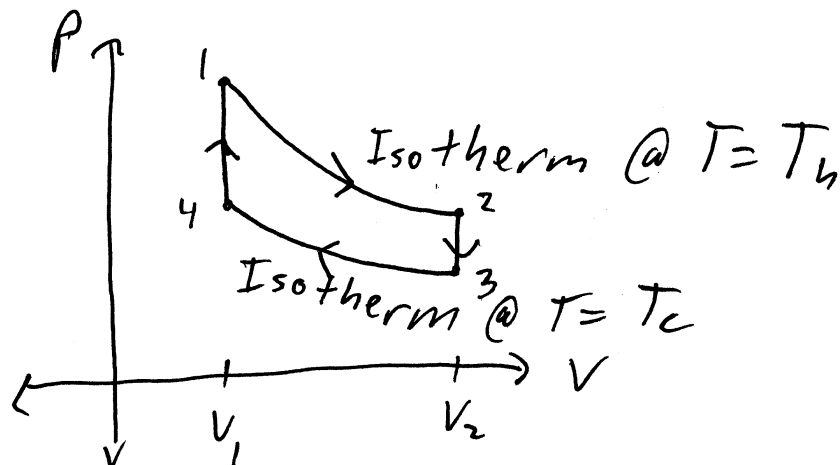
Since $\frac{Q_c}{Q_h} \leq \frac{T_c}{T_h}$,

$1 - \frac{Q_c}{Q_h} \geq 1 - \frac{T_c}{T_h}$, which means

the $COP \leq \frac{1}{1 - T_c/T_h} = \frac{T_h}{T_h - T_c}$.

d. For an electric heater, all the electrical energy (W) is converted to heat (Q_h), so $W = Q_h$
 $COP = 1$. An ideal heat pump, however, always has a $COP > 1$ (E.g. a heat pump operating between $25^\circ C$ & $0^\circ C$ could have a COP as high as $\frac{298}{298 - 273} = 12!$).

4.21 a.



Note: w
here refers
to work
done BY the
gas $= + \int P dV$.

b. Power stroke (12):

$$W_{12} = \int_{V_1}^{V_2} \frac{NkT_h}{V} dV = NkT_h \ln\left(\frac{V_2}{V_1}\right)$$

Compression stroke (34):

$$W_{34} = \int_{V_2}^{V_1} \frac{NkT_c}{V} dV = -NkT_c \ln\left(\frac{V_2}{V_1}\right)$$

$$\text{Net work} = W_{12} + W_{34}$$

$$= Nk(T_h - T_c) \ln \frac{V_2}{V_1}$$

Heat input during power stroke:

$$T = 0 \Rightarrow \Delta U = 0$$

$$\text{so } Q_{12} = W_{12} = NkT_h \ln\left(\frac{V_2}{V_1}\right)$$

During step $\overline{YI}$ $W=0$, so

$$Q_{YI} = \Delta U_{YI} = \frac{f}{2} Nk(T_h - T_c)$$

Total heat ~~Q~~ input,

$$Q_h = NkT_h \ln\left(\frac{V_2}{V_1}\right) + \frac{f}{2} Nk(T_h - T_c)$$

$$e = \frac{W}{Q} \quad \text{Easier to compute } \frac{1}{e} :$$

$$\frac{1}{e} = \frac{Q_h}{W} = \frac{\cancel{Nk} T_h \ln(V_2/V_1) + f/2 Nk(T_h - T_c)}{Nk(T_h - T_c) \ln(V_2/V_1)}$$

$$\frac{1}{e} = \frac{T_h}{T_h - T_c} + \frac{f}{2 \ln(V_2/V_1)}$$

$$e = \frac{1}{\frac{T_h}{T_h - T_c} + \frac{f}{2 \ln(V_2/V_1)}}$$

But $\frac{T_h}{T_h - T_c} = \frac{1}{e_c}$, where $e_c =$
efficiency of
Carnot engine.

Thus

$$e = \frac{e_c}{1 + e_c \cdot \frac{f}{2 \ln(V_2/V_1)}} < e_c$$

For example: if $f = 5$ (5 degrees of freedom), $\frac{V_2}{V_1} = \text{compression ratio} = 10$, and $\frac{T_h}{T_c} = 2$, we have

$$e_c = \frac{2T_c - T_c}{2T_c} = \frac{1}{2}$$

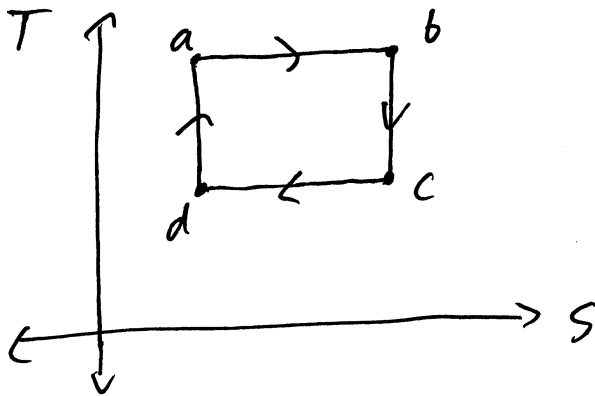
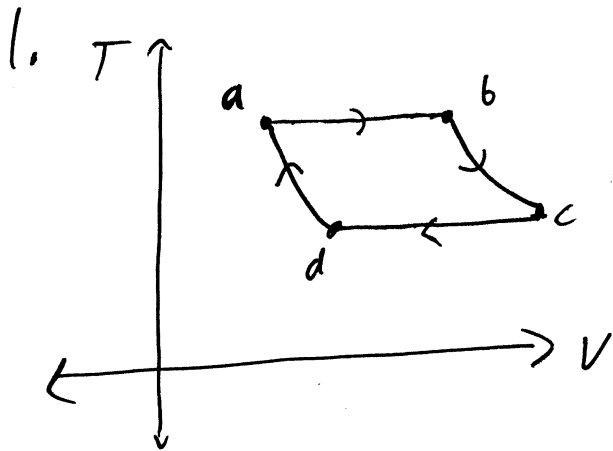
$$e = \frac{e_c}{1 + e_c \cdot \frac{5}{2 \ln 10}} = \sim .32 < e_c$$

C. With an ideal regenerator, the heat input during step $\overline{41}$ comes for free, because it's exactly the same $\left(\frac{f}{2} N k (T_h - T_c)\right)$ as the heat output during step $\overline{23}$.

Therefore only Q_2 counts in computing e . If we run through the calculations again, we find

$$\frac{1}{e} = \frac{\cancel{Nk} T_h \ln(\cancel{V_2/V_1})}{\cancel{Nk} (T_h - T_c) \ln(\cancel{V_2/V_1})} = \frac{T_h}{T_h - T_c}$$
$$= \frac{1}{e_{cc}} \quad \text{so } (e = e_c)$$

Additional Problems



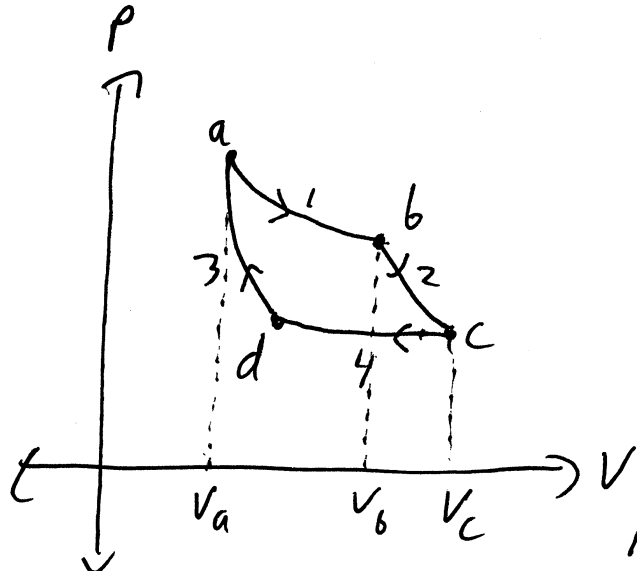
$$pV^\gamma = \text{constant}$$

$$TV^{\gamma-1} = \text{constant}$$

$$T = \frac{k}{V^{\gamma-1}}$$

$$dS = \frac{dQ}{T}$$

2.



2 & 3 are
adiabats
($dS=0$).

1 & 4 are
isotherms ($\Delta U=0$)

$$\text{So } \Delta S_{\text{net}} = \Delta S_1 + \Delta S_4.$$

$$\Delta S_1 = \int_1 \frac{dQ}{T_1} = \int_1 \frac{dU}{T_1} + \frac{P dV}{T_1}$$

$$= \int_{V_a}^{V_b} \frac{nRT_1}{V} \cdot \frac{1}{T_1} dV = nR \ln\left(\frac{V_b}{V_a}\right)$$

$$\Delta S_4 = \int_4 \frac{nRT_4}{V} \cdot \frac{1}{T_4} dV = \int_{V_c}^{V_d} \frac{nR}{V} dV$$

$$= nR \ln\left(\frac{V_d}{V_c}\right)$$

Now we just need V_d, V_c in terms
of V_b, V_a . Adiabatic Equations:

$$T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1}, \quad T_1 V_a^{\gamma-1} = T_2 V_d^{\gamma-1}$$

(path 2) (path 3)

Dividing these two equations,
we see that

$$\frac{T_1 V_b^{r-1}}{T_1 V_a^{r-1}} = \frac{T_2 V_c^{r-1}}{T_2 V_d^{r-1}}$$

$$\left(\frac{V_b}{V_a}\right)^{r-1} = \left(\frac{V_c}{V_d}\right)^{r-1} \Rightarrow \frac{V_c}{V_d} = \frac{V_b}{V_a}$$

$$\begin{aligned} \text{So } \Delta S_y &= n R \ln \left(\frac{V_a}{V_b} \right) = -n R \ln \left(\frac{V_b}{V_a} \right) \\ &= -\Delta S_x \end{aligned}$$

In other words

$$\Delta S_{\text{net}} = \Delta S_y + \Delta S_x = 0. \quad \checkmark$$

Since every state function has a net change of 0 around a closed cycle, this might lead us to conclude that S is a state function. By itself, however, this example is not sufficient to PROVE that S is a state function.

$$3. \Delta S = \int \frac{dQ}{T}$$

$$\begin{aligned} \Delta S_{\text{net}} &= \int_{253 \text{ K}}^{273 \text{ K}} \frac{C_p(s)}{T} dT + \frac{L_f}{273 \text{ K}} + \int_{273}^{373} \frac{C_p(l)}{T} dT + \frac{L_v}{373} \\ &\quad + \int_{373}^{673} \frac{C_p(g)}{T} dT \\ &= C_p(s) \ln\left(\frac{273}{253}\right) + \frac{L_f}{273} + C_p(l) \ln\left(\frac{373}{273}\right) \\ &\quad + \frac{L_v}{373} + C_p(g) \ln\left(\frac{673}{373}\right) \end{aligned}$$

$$\begin{aligned} C_p(s) &= \text{Heat capacity of solid water (ice)} \\ &= 2.30 \text{ J/g}\cdot\text{K} = 2.30 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

$$\begin{aligned} C_p(l) &= \text{Heat capacity of liquid water} \\ &= 4.18 \text{ J/g}\cdot\text{K} = 4.18 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

$$\begin{aligned} C_p(g) &= \text{Heat capacity of gaseous} \\ &\quad \text{water (steam)} = 2.01 \text{ J/g}\cdot\text{K} = 2.01 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \end{aligned}$$

$$L_f = 333 \text{ J/g} = 333 \text{ kJ/kg}$$

$$L_v = 2257 \text{ J/g} = 2257 \text{ kJ/kg}$$

$$\Delta S_{net} = 0.175 + 1.22 + 1.305 \\ + 6.05 + 1.186 \text{ kJ/k}$$

$$= 9.94 \cdot 10^3 \text{ J/k}$$