

Physics 119A Midterm

Oct. 27, 2010

NAME: _____ PERM: _____

Instructions: The last page of the exam contains some useful information. Please verify that your exam contains 8 pages (7 of exam and 1 of information; 3 pages are intentionally left blank for additional work space). Show all work and box your final answers to receive full credit.

Question I, 12 points (2 points each): True or False: Provide a brief justification or no credit.

- a) $\Delta S < 0$ is impossible for any naturally occurring process.

False $\Delta S < 0$ possible if non-adiabatic.

- b) The result $\Delta U = 0$ for the adiabatic expansion of a gas into vacuum (Joule experiment) is true for both ideal and real gasses.

True both q & $w = 0$ for Joule expansion
 $\Delta U = q + w = 0$

- c) The constant pressure heat capacity for a monoatomic ideal gas is $3/2 nR$.

False $C_p = \frac{5}{2} nR$

- d) The following three quantities are all extensive: temperature, entropy, pressure.

False entropy extensive; T, p are intensive

- e) Any process that is carried out quasi-statically is necessarily reversible.

False a process can be carried out infinitely slowly, but still not be reversible (paddle wheel, etc.)

- f) In principle, you could air condition your kitchen by opening the door to the refrigerator. It just isn't very efficient.

False. A refrigerator will take some heat in from the room, but will put out more heat. The room will get warmer, not cooler.

Question II, 12 points: Later in the quarter we will define a useful new state function called the Helmholtz free energy, A . The total differential for A may be written $dA = -PdV - SdT$. Based solely on this expression and the rules of calculus, you can immediately write down simple results for the below three quantities. Do so.

$$\begin{array}{ll}
 \text{a) } \left. \frac{\partial A}{\partial V} \right|_T = -P & \left. \begin{array}{l} \text{Since } dA = \left. \frac{\partial A}{\partial V} \right|_T dV + \left. \frac{\partial A}{\partial T} \right|_V dT \\ \text{and compare to } dA = -PdV - SdT \end{array} \right\} \\
 \text{b) } \left. \frac{\partial A}{\partial T} \right|_V = -S & \\
 \text{c) } \left. \frac{\partial P}{\partial T} \right|_V = \left. \frac{\partial S}{\partial V} \right|_T & \text{from } \frac{\partial^2 A}{\partial T \partial V} = \frac{\partial^2 A}{\partial V \partial T}
 \end{array}$$

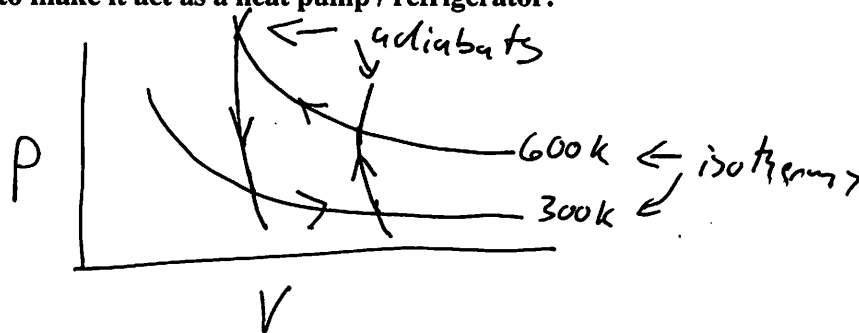
Question III, 16 points: Referring to the function A defined in question II, derive an expression for the thermodynamic derivative $\left. \frac{\partial A}{\partial V} \right|_U$. Your answer should include the quantities $(P, S, C_v, \left. \frac{\partial U}{\partial V} \right|_T)$.

$$\begin{aligned}
 \left. \frac{\partial A}{\partial V} \right|_U &= \left. \frac{\partial A}{\partial V} \right|_T + \left. \frac{\partial A}{\partial T} \right|_V \left. \frac{\partial T}{\partial V} \right|_U \\
 &= -P - S \left. \frac{\partial T}{\partial V} \right|_U ; \quad \left. \frac{\partial T}{\partial V} \right|_U \left. \frac{\partial V}{\partial U} \right|_T \left. \frac{\partial U}{\partial T} \right|_V = -1 \\
 &\quad \left. \frac{\partial T}{\partial V} \right|_U = -\frac{1}{C_v} \left. \frac{\partial U}{\partial V} \right|_T
 \end{aligned}$$

$$\left. \frac{\partial A}{\partial V} \right|_U = -P + \frac{S}{C_v} \left. \frac{\partial U}{\partial V} \right|_T$$

Question IV, 25 points: 1 mol of a real gas undergoes a reversible Carnot cycle operating between the temperatures 600K and 300K.

- a) (10 points) Draw the cycle on a P,V state diagram. Indicate which legs of the cycle are isothermal and the corresponding temperatures, which legs are adiabatic and draw arrows indicating the direction around the cycle if the cycle is carried out in such a way that the surroundings perform work on the system to make it act as a heat pump / refrigerator.



- b) (5 points) If the cycle is run as an engine, what is the ratio of the heat absorbed by the system at 600K to the work done by the system?

$$\underbrace{\frac{Q_H}{-W}}_{\text{-W: work done by system}} = \frac{Q_H}{Q_H + Q_L} = \left(1 + \frac{Q_L}{Q_H}\right)^{-1} = \left(1 - \frac{T_L}{T_H}\right)^{-1} = 2$$

- c) (5 points) If $C_p = 13/2 nR$ and $C_v = 11/2 nR$ for the working gas of the engine and the cycle is carried out such that the gas expands by exactly a factor of two along the 600K leg of the cycle, what do we know about the signs of ΔU and ΔS for the Carnot cycle?

$$\Delta U = \Delta S = 0 \quad \text{since } S, U \text{ are state functions}$$

no change for complete cycle

- d) (5 points) An engineer creates a real heat engine that operates between two heat reservoirs at 600K and 300K. The engine happens to produce exactly the same amount of work in a given cycle as the idealized Carnot engine described above in part b. Does this engine draw in more or less heat at 600K than the idealized Carnot engine?

More any real engine will be less efficient than Carnot $\eta \equiv \frac{-W}{Q_H}$

Question V, 35 points: Calculate $w, q, \Delta H, \Delta U$ and ΔS for a process in which 1 mol of water undergoes the transition $\text{H}_2\text{O}(\text{liquid}, 373 \text{ K}) \rightarrow \text{H}_2\text{O}(\text{gas}, 460 \text{ K})$ at 1 atm pressure. The molar volume of liquid water at 373K is $1.89 \times 10^{-5} \text{ m}^3 \text{ mol}^{-1}$. The molar volume of steam at 373K and 460K is $3.03 \times 10^{-2} \text{ m}^3 \text{ mol}^{-1}$ and $3.74 \times 10^{-2} \text{ m}^3 \text{ mol}^{-1}$, respectively. For steam, the molar heat capacity at constant pressure is $C_p = 33.58 \text{ J mol}^{-1} \text{ K}^{-1}$ and can be assumed constant over the temperature range considered. The latent heat of vaporization for water is $40.656 \text{ kJ mol}^{-1}$. (MAKE CERTAIN THAT YOU USE THE CONVENTION $\Delta U = q + w$ in your reporting of heat and work! Put your final answers in the table below.)

w	q	ΔH	ΔU	ΔS
$-3.79 \times 10^3 \text{ J}$	$4.36 \times 10^4 \text{ J}$	$4.36 \times 10^4 \text{ J}$	$3.98 \times 10^4 \text{ J}$	116 J/K

$$\begin{aligned}
 \text{At const. } P, \quad q = \Delta H &= n L_{\text{vap}} + n C_p \Delta T \\
 &= 40,656 \text{ J} + 33.58 \frac{\text{J}}{\text{K}} (460 - 373) \text{ K} \\
 &= 4.36 \times 10^4 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 w &= -P \Delta V \\
 &= -1 \text{ atm} \times (3.74 \times 10^{-2} - 1.89 \times 10^{-5}) \text{ L} = -37.4 \text{ L} \cdot \text{atm} \\
 &= -3.79 \times 10^3 \text{ J}
 \end{aligned}$$

$$\Delta U = q + w = 3.98 \times 10^4 \text{ J}$$

$$\begin{aligned}
 \Delta S &= \frac{n L_{\text{vap}}}{T_{\text{vap}}} + \int_{T_{\text{vap}}}^{T_{\text{final}}} dT \frac{n C_p}{T} \\
 &= \frac{40,656 \text{ J}}{373 \text{ K}} + 33.58 \frac{\text{J}}{\text{K}} \ln \left(\frac{460}{373} \right)
 \end{aligned}$$

$$109 + 7.04 = 116 \text{ J/K}$$