Phys 119A HW#1

Due: 8:50 AM, Friday, Oct. 1 in the 119A HW Box

READING: CARTER, CHAPTERS 1, 2.1- 2.4 AND Appendix A
SCHROEDER, CHAPTER 1, PAGES 1-16

BOOK PROBLEMS: CARTER 1.8, 2.3, A-1, A-4
SCHROEDER 1.16 AND 1.17 (parts a, b and c).

ADDITIONAL PROBLEMS:

1) To evaluate a line integral we must choose a path of integration.
The following exercise is intended to remind us how this works for a simple mathematical example.

a) Calculate the integral: $I = \int_{x_i = y_i = 0}^{x_f = y_f = 1} 2xydx + x^2 dy$ along the following four paths.

i) y(x)=x (a straight line)
ii)y(x) = x^2 (a parabola)
iii) y(x) = x^3
iv) an "L shaped" path, (0,0) -> (1,0) -> (1,1) in two straight line segments.

b) Show that the integral is expected to be path independent, so that we could have guessed that all the paths attempted above would yield the same result.

c) Find a function F(x,y) that satisfies $\partial F/\partial x|_y = 2xy$ and $\partial F/\partial y|_x = x^2$.

d) Show that the integral I has the value F(1,1) – F(0,0). Why does this make sense?

2) The general formula for the volume of an ellipsoid with radii a,b and c is $V = \frac{4}{3}\pi abc$.

There is no simple general expression for the area on the surface of the ellipsoid, but the expression

$$S = 4\pi\left(\frac{a^p b^p + a^p c^p + b^p c^p}{3}\right)^{\frac{1}{p}}$$

with $p \approx 1.6075$ provides a very good approximation to the ellipsoid's surface area.

Derive an expression for the change in volume associated with changing radius a of the ellipsoid while holding both S and radius b constant. (The algebra here becomes much simpler than might first appear through the judicious use of partial derivative relationships).