Exchange

- How do spins interact?
  - Magnetostatic dipole-dipole coupling

$$H_{d-d} = -\frac{\mu_0}{4\pi r^3}\left[3(\mathbf{m}\cdot\mathbf{r})(\mathbf{m}'\cdot\mathbf{r}) - \mathbf{m}\cdot\mathbf{m}'\right]$$

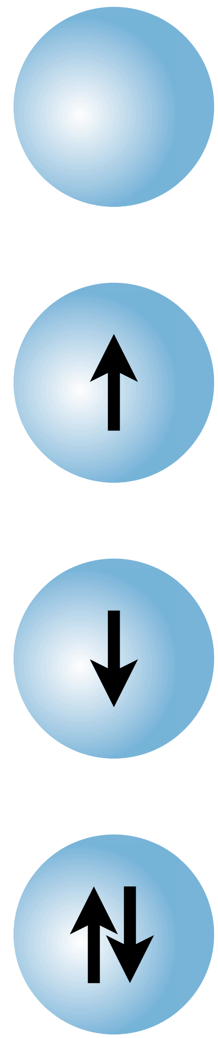
    - This is rather weak, $\lesssim$ 1K for even large spins
  - Electrostatic interaction usually dominates, just as it does inside atoms
    - Indirectly leads to spin coupling through Pauli principle

Exchange

- Generally think of exchange interactions between spins as due to hopping of electrons from one orbital to another
- Many types of exchange, complicated by varieties of orbitals involved, including those on non-magnetic atoms
- We will just discuss the simplest case

Direct exchange

Assume individual
S=1/2 ions



$E=0$

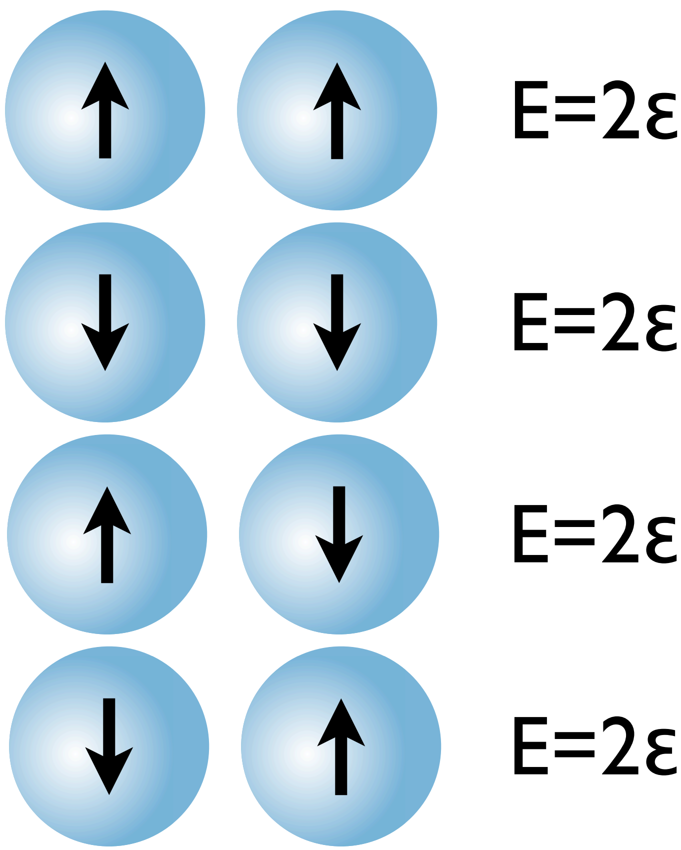
$E=\varepsilon$

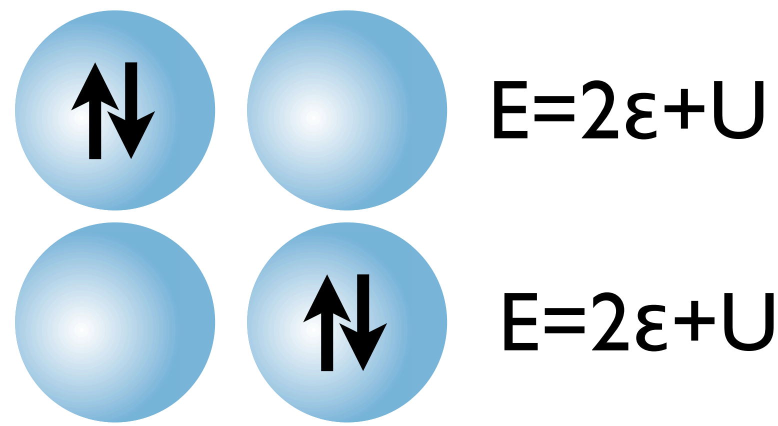
$E=\varepsilon$

$E=2\varepsilon+U$

Direct exchange

- Two ions, 2 electrons. With no contact:

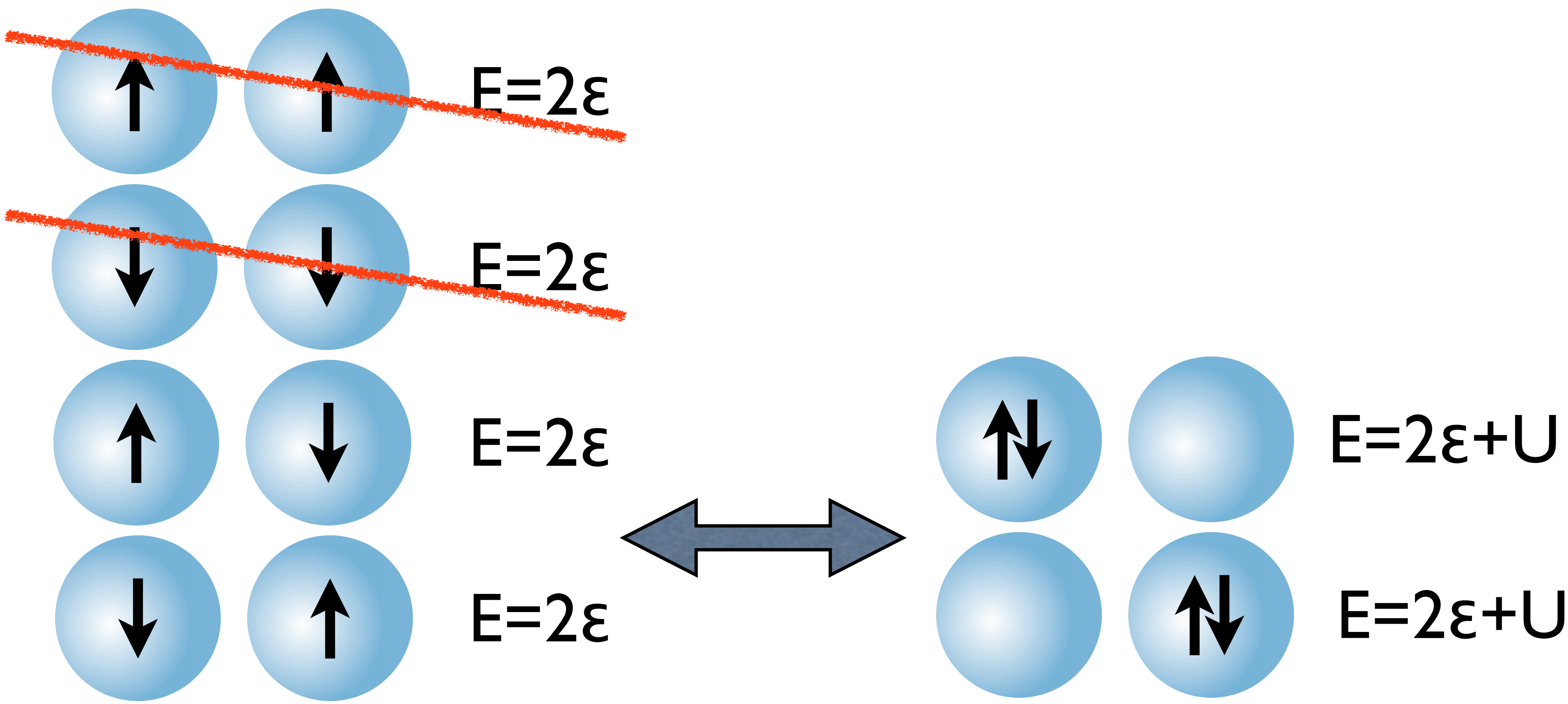




Now include
electron hopping

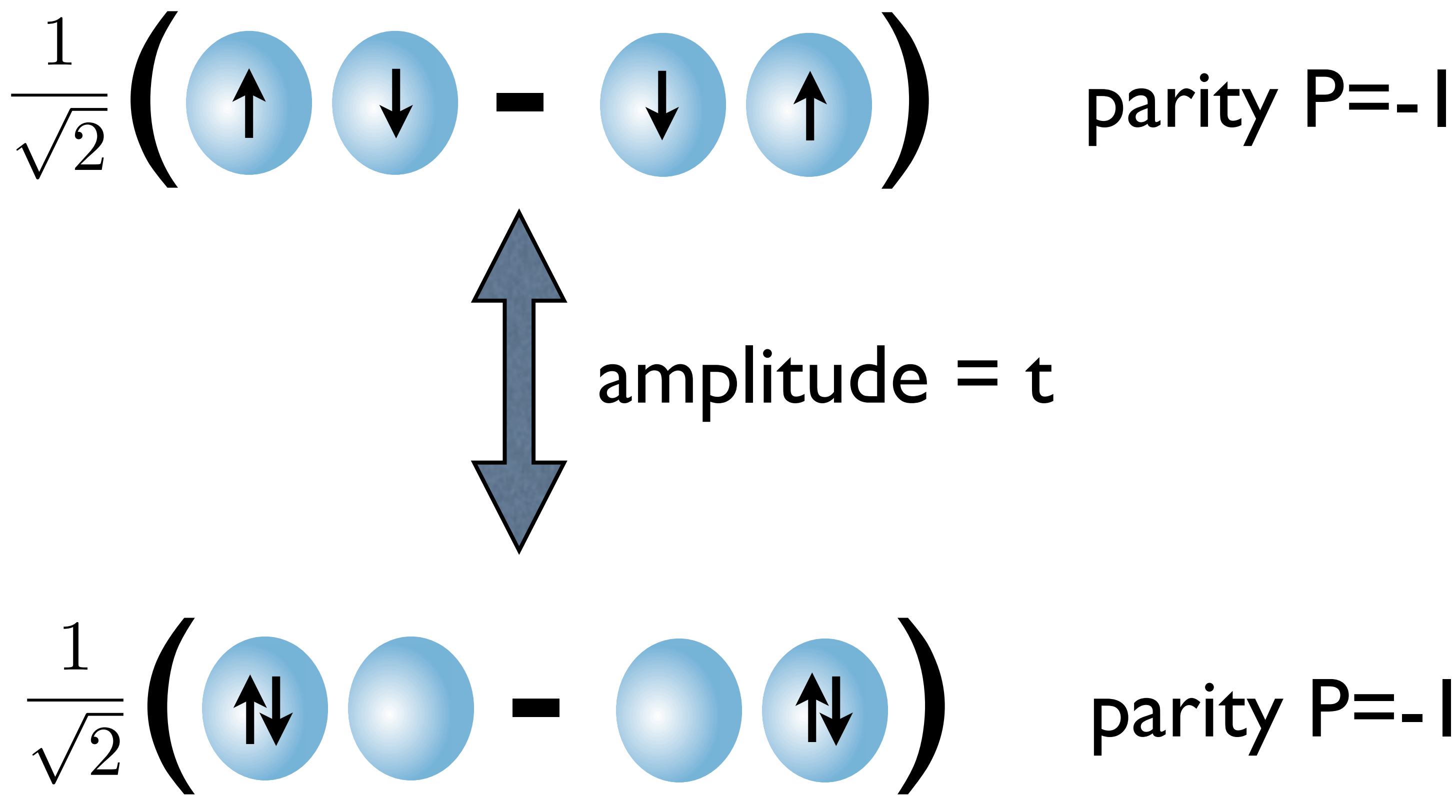
Direct exchange

- Hopping? Prohibited by Pauli for parallel spins (S=1)



Direct exchange

- Since spin is conserved, only S=0 state can hop



Direct exchange

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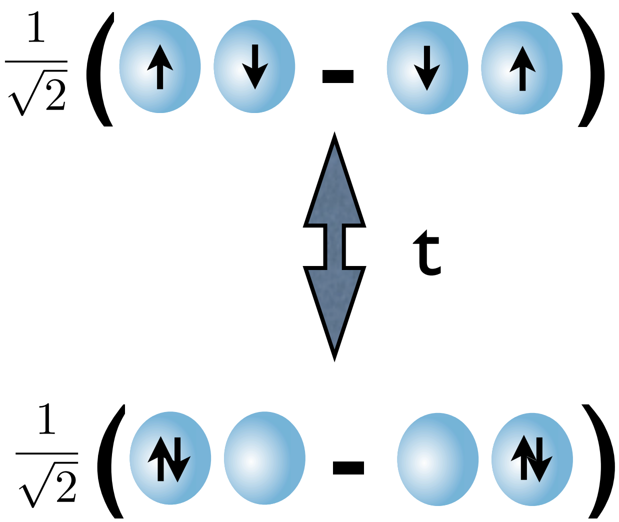


$E_0=2\varepsilon$

$E_0=2\varepsilon+U$

$$H = \begin{pmatrix} 2\epsilon & t \\ t & 2\epsilon + U \end{pmatrix}$$

$$E_- \approx 2\epsilon - \frac{t^2}{U}$$

singlet energy lowered by $O(t^2/U)$

Direct exchange

- Write as an "effective Hamiltonian":

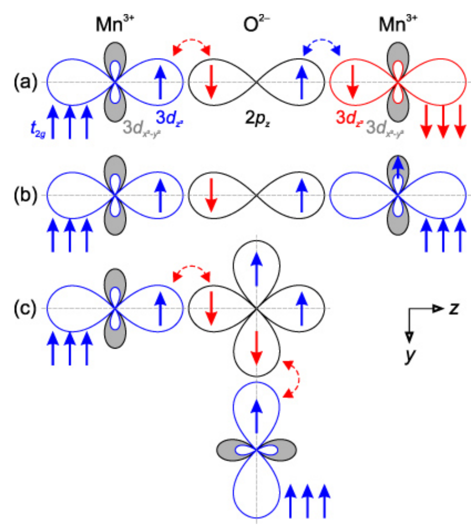
$$
\begin{aligned}
H_{\text{eff}} &= -\frac{t^2}{U}\hat{P}_{S=0} \\
&= -\frac{t^2}{U}\left[1 - \frac{1}{2}\left(\mathbf{S}_i + \mathbf{S}_j\right)^2\right] \\
&= \text{const.} + J\,\mathbf{S}_i \cdot \mathbf{S}_j
\end{aligned}
$$

- With J ~ t^2/U. Note this is typically *antiferromagnetic* interaction: favors singlet/ anti-parallel spins

J can be as large as 1000K, or as small as a few K

Other exchanges

- Super-exchange: exchange due to electrons hopping through an intermediate orbital



usually antiferromagnetic

this can be ferromagnetic

Other exchanges

- RKKY exchange:
  - interaction of spins mediated by delocalized metallic electrons
  - only in a metal, obviously
- Double exchange
  - more exotic exchange in some metals with some localized electrons

Magnetic order

- In a crystal with a periodic lattice of spins, exchange interactions typically induce an *ordered* state at low temperature

- For example, the ferromagnetic Heisenberg model:

$$H = -J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \qquad\qquad J > 0$$

- Wants every spin parallel to its neighbor, so they choose a global axis

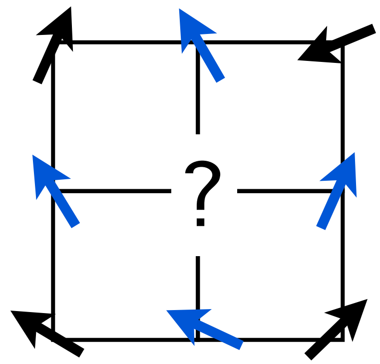
$$\langle \mathbf{S}_i \rangle = \mathbf{m}$$

Mean field theory

- When kT$\gtrsim$ J, spins will fluctuate thermally, and **m** will be reduced.
- We can study this with *mean field theory*

$$
\begin{aligned}
H &= -J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \\
&\rightarrow -J \sum_{\langle ij \rangle} \left[\langle \mathbf{S}_i \rangle \cdot \mathbf{S}_j + \mathbf{S}_i \cdot \langle \mathbf{S}_j \rangle - \langle \mathbf{S}_i \rangle \cdot \langle \mathbf{S}_j \rangle \right] \\
&= -zJ \sum_i \mathbf{m} \cdot \mathbf{S}_i + \text{const.}
\end{aligned}
$$

Mean field theory



$$g\mu_B \mathbf{H}_{\mathrm{eff}} = J\langle \nwarrow + \nearrow + \nwarrow + \nwarrow \rangle$$

- This reduces the problem to independent spins in an *effective* "exchange field"
- Note: this exchange field can be a thousand times larger than physical laboratory fields!

Mean field theory

- Define h = z J m (= g μ_B H_{eff})
- Then we know for a single spin

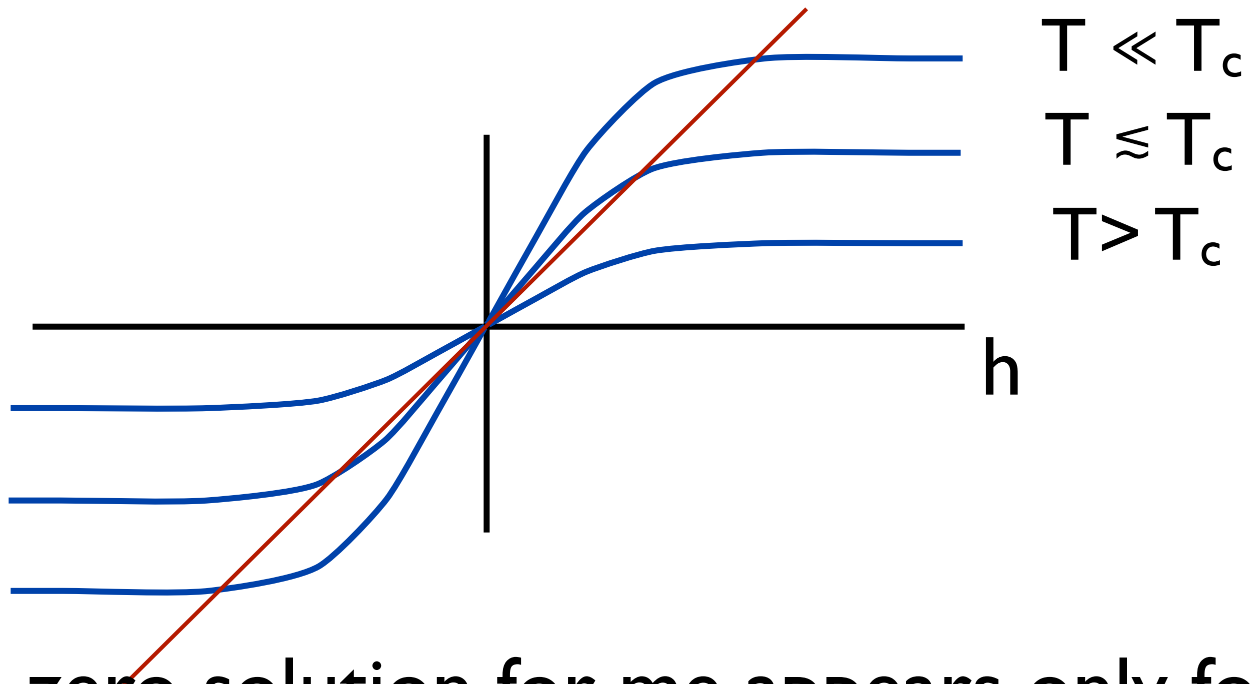
$$|\langle \mathbf{S}_i \rangle| = m = S B_S(\beta h S)$$

$$\Rightarrow \quad m = S B_S(\beta z J S m)$$

- For example for S=1/2

$$m = \frac{1}{2} \tanh \left[\frac{zJm}{2kT} \right]$$

Mean field theory



- Non-zero solution for me appears only for $T<T_c$
- equality of slopes implies kT_c = z J/4 $(= \frac{zJS(S+1)}{3})$