PHILOSOPHICAL

TRANSACTIONS.

I. The Bakerian Lecture. Experiments and Calculations relative to physical Optics. By Thomas Young, M. D. F.R.S.

Read November 24, 1803.

I. EXPERIMENTAL DEMONSTRATION OF THE GENERAL LAW OF THE INTERFERENCE OF LIGHT.

IN making some experiments on the fringes of colours accompanying shadows, I have found so simple and so demonstrative a proof of the general law of the interference of two portions of light, which I have already endeavoured to establish, that I think it right to lay before the Royal Society, a short statement of the facts which appear to me so decisive. The proposition on which I mean to insist at present, is simply this, that fringes of colours are produced by the interference of two portions of light; and I think it will not be denied by the most prejudiced, that the assertion is proved by the experiments I am about to relate, which may be repeated with great ease, whenever MDCCCIV.

B

2

the sun shines, and without any other apparatus than is at hand to every one.

Exper. 1. I made a small hole in a window-shutter, and covered it with a piece of thick paper, which I perforated with a fine needle. For greater convenience of observation, I placed a small looking glass without the window-shutter, in such a position as to reflect the sun's light, in a direction nearly horizontal, upon the opposite wall, and to cause the cone of diverging light to pass over a table, on which were several little screens of card-paper. I brought into the sunbeam a slip of card, about one-thirtieth of an inch in breadth, and observed its shadow, either on the wall, or on other cards held at different distances. Besides the fringes of colours on each side of the shadow, the shadow itself was divided by similar parallel fringes, of smaller dimensions, differing in number, according to the distance at which the shadow was observed, but leaving the middle of the shadow always white. Now these fringes were the joint effects of the portions of light passing on each side of the slip of card, and inflected, or rather diffracted, into the shadow. For, a little screen being placed a few inches from the card, so as to receive either edge of the shadow on its margin, all the fringes which had before been observed in the shadow on the wall immediately disappeared, although the light inflected on the other side was allowed to retain its course, and although this light must have undergone any modification that the proximity of the other edge of the slip of card might have been capable of occasioning. When the interposed screen was more remote from the narrow card, it was necessary to plunge it more deeply into the shadow, in order to extinguish the parallel lines; for here the light, diffracted from the edge of the object, had entered further into

the shadow, in its way towards the fringes. Nor was it for want of a sufficient intensity of light, that one of the two portions was incapable of producing the fringes alone; for, when they were both uninterrupted, the lines appeared, even if the intensity was reduced to one-tenth or one-twentieth.

Exper. 2. The crested fringes described by the ingenious and accurate GRIMALDI, afford an elegant variation of the preceding experiment, and an interesting example of a calculation grounded on it. When a shadow is formed by an object which has a rectangular termination, besides the usual external fringes, there are two or three alternations of colours, beginning from the line which bisects the angle, disposed on each side of it, in curves, which are convex towards the bisecting line, and which converge in some degree towards it, as they become more remote from the angular point. These fringes are also the joint effect of the light which is inflected directly towards the shadow, from each of the two outlines of the object. For, if a screen be placed within a few inches of the object, so as to receive only one of the edges of the shadow, the whole of the fringes disappear. If, on the contrary, the rectangular point of the screen be opposed to the point of the shadow, so as barely to receive the angle of the shadow on its extremity, the fringes will remain undisturbed.

II. COMPARISON OF MEASURES, DEDUCED FROM VARIOUS EXPERIMENTS.

If we now proceed to examine the dimensions of the fringes, under different circumstances, we may calculate the differences of the lengths of the paths described by the portions of light, which have thus been proved to be concerned in producing those

Dr. Young's Experiments and Calculations

4

fringes; and we shall find, that where the lengths are equal, the light always remains white; but that, where either the brightest light, or the light of any given colour, disappears and reappears, a first, a second, or a third time, the differences of the lengths of the paths of the two portions are in arithmetical progression, as nearly as we can expect experiments of this kind to agree with each other. I shall compare, in this point of view, the measures deduced from several experiments of NEWTON, and from some of my own.

In the eighth and ninth observations of the third book of NEWTON'S Optics, some experiments are related, which, together with the third observation, will furnish us with the data necessary for the calculation. Two knives were placed, with their edges meeting at a very acute angle, in a beam of the sun's light, admitted through a small aperture; and the point of concourse of the two first dark lines bordering the shadows of the respective knives, was observed at various distances. The results of six observations are expressed in the first three lines of the first Table. On the supposition that the dark line is produced by the first interference of the light reflected from the edges of the knives, with the light passing in a straight line between them, we may assign, by calculating the difference of the two paths, the interval for the first disappearance of the brightest light, as it is expressed in the fourth line. The second Table contains the results of a similar calculation, from NEWTON's observations on the shadow of a hair; and the third, from some experiments of my own, of the same nature: the second bright line being supposed to correspond to a double interval, the second dark line to a triple interval, and the succeeding lines to depend on a continuation of the progression. The unit of all the Tables is an inch.

relative to physical Optics.

TABLE I. Obs. 9. N.

Distance of the knives from	the aper	ture	s 901 10	ustance.	i aui ua	101.
Distances of the paper from the knives -	I <u>7</u> ,	313,	8 <u>3</u> ,	32,	96,	131.
Distances between the edges of the knives, op- posite to the point of						om om
concourse, fille and and	.012,	.020,	.034,	.057,	.081,	.087.
Interval of disappearance	0000122,	.0000155	.0000182,	.0000167,	.0000166,	.0000166.

TABLE II. Obs. 3. N.

Breadth of the hair	- 1
Distance of the hair from the aperture	- 144.
Distances of the scale from the aperture 15	0, 252.
(Breadths of the shadow	
Breadth between the second pair of bright lines -	4
Interval of disappearance, or half the difference of the paths .000	0151, .0000173.
Breadth between the third pair of bright lines	mint out logohil
Interval of disappearance, ‡ of the difference000	0130, 0000143.

TABLE III. Exper. 3.

Breadth of the object	01.000	alizza aito	-	.434.
Distance of the object from the aperture	-	inis circi		- 125.
Distance of the wall from the aperture	onjecture	bat I ci	noini	250.
Distance of the second pair of dark lines from	each other	erned, fre	conc	1.167.
Interval of disappearance, $\frac{1}{3}$ of the difference	iman	From its	-ither	.0000149.

Exper. 4.

Breadth of the wire -		- incor	1	- and		.082.
Distance of the wire from the aperture		-	- of the		-	22.
Distance of the wall from the aperture		: 50 001	od of	ar assur	2911 85	250
(Breadth of the shadow, by three measu	remen	ts .815.	.826. 0	or .827:	mean.	.822.
Distance of the first pair of dark lines		1.165.	1.170. 0	r 1.160 :	mean.	1.165.
Interval of disappearance -	2	Continue and	and the -	124 154	.000	00104.
Distance of the second pair of dark lines	- 100	1.402.	1.305.0	or 1.400 :	mean.	1.200.
Interval of disappearance -	-	011.	dostio	e chis c	000	00137.
Distance of the third pair of dark lines	min	1.504.	1.580.0	r 1.585 :	mean.	1.586.
Interval of disappearance	1000	is ank	1 vei 1	Secter	.000	00128.

Dr. Young's Experiments and Calculations

It appears, from five of the six observations of the first Table, in which the distance of the shadow was varied from about 3 inches to 11 feet, and the breadth of the fringes was increased in the ratio of 7 to 1, that the difference of the routes constituting the interval of disappearance, varied but one-eleventh at most; and that, in three out of the five, it agreed with the mean, either exactly, or within $\frac{1}{160}$ part. Hence we are warranted in inferring, that the interval appropriate to the extinction of the brightest light, is either accurately or very nearly constant.

But it may be inferred, from a comparison of all the other observations, that when the obliquity of the reflection is very great, some circumstance takes place, which causes the interval thus calculated to be somewhat greater: thus, in the eleventh line of the third Table, it comes out one-sixth greater than the mean of the five already mentioned. On the other hand, the mean of two of NEWTON's experiments and one of mine, is a result about one-fourth less than the former. With respect to the nature of this circumstance, I cannot at present form a decided opinion; but I conjecture that it is a deviation of some of the light concerned, from the rectilinear direction assigned to it, arising either from its natural diffraction, by which the magnitude of the shadow is also enlarged, or from some other unknown cause. If we imagined the shadow of the wire, and the fringes nearest it, to be so contracted that the motion of the light bounding the shadow might be rectilinear, we should thus make a sufficient compensation for this deviation; but it is difficult to point out what precise track of the light would cause it to require this correction.

The mean of the three experiments which appear to have been least affected by this unknown deviation, gives .0000127

relative to physical Optics.

for the interval appropriate to the disappearance of the brightest light; and it may be inferred, that if they had been wholly exempted from its effects, the measure would have been somewhat smaller. Now the analogous interval, deduced from the experiments of NEWTON on thin plates, is .0000112, which is about one-eighth less than the former result; and this appears to be a coincidence fully sufficient to authorise us to attribute these two classes of phenomena to the same cause. It is very easily shown, with respect to the colours of thin plates, that each kind of light disappears and reappears, where the differences of the routes of two of its portions are in arithmetical progression; and we have seen, that the same law may be in general inferred from the phenomena of diffracted light, even independently of the analogy.

The distribution of the colours is also so similar in both cases, as to point immediately to a similarity in the causes. In the thirteenth observation of the second part of the first book, NEWTON relates, that the interval of the glasses where the rings appeared in red light, was to the interval where they appeared in violet light, as 14 to 9; and, in the eleventh observation of the third book, that the distances between the fringes, under the same circumstances, were the 22d and 27th of an inch. Hence, deducting the breadth of the hair, and taking the squares, in order to find the relation of the difference of the routes, we have the proportion of 14 to $9\frac{1}{4}$, which scarcely differs from the proportion observed in the colours of the thin plate.

We may readily determine, from this general principle, the form of the crested fringes of GRIMALDI, already described; for it will appear that, under the circumstances of the experiment related, the points in which the differences of the lengths of the

Dr. Young's Experiments and Calculations

8

paths described by the two portions of light are equal to a constant quantity, and in which, therefore, the same kinds of light ought to appear or disappear, are always found in equilateral hyperbolas, of which the axes coincide with the outlines of the shadow, and the asymptotes nearly with the diagonal line. Such, therefore, must be the direction of the fringes; and this conclusion agrees perfectly with the observation. But it must be remarked, that the parts near the outlines of the shadow, are so much shaded off, as to render the character of the curve somewhat less decidedly marked where it approaches to its axis. These fringes have a slight resemblance to the hyperbolic fringes observed by NEWTON; but the analogy is only distant.

III. APPLICATION TO THE SUPERNUMERARY RAINBOWS.

The repetitions of colours sometimes observed within the common rainbow, and described in the Philosophical Transactions, by Dr. LANGWITH and Mr. DAVAL, admit also a very easy and complete explanation from the same principles. Dr. PEM-BERTON has attempted to point out an analogy between these colours and those of thin plates; but the irregular reflection from the posterior surface of the drop, to which alone he attributes the appearance, must be far too weak to produce any visible effects. In order to understand the phenomenon, we have only to attend to the two portions of light which are exhibited in the common diagrams explanatory of the rainbow, regularly reflected from the posterior surface of the drop, and crossing each other in various directions, till, at the angle of the greatest deviation, they coincide with each other, so as to produce, by the greater intensity of this redoubled light, the common rainbow of 41 degrees. Other parts of these two portions will quit the drop

in directions parallel to each other; and these would exhibit a continued diffusion of fainter light, for 25° within the bright termination which forms the rainbow, but for the general law of interference, which, as in other similar cases, divides, the light into concentric rings; the magnitude of these rings depending on that of the drop, according to the difference of time occupied in the passage of the two portions, which thus proceed in parallel directions to the spectator's eye, after having been differently refracted and reflected within the drop. This difference varies at first, nearly as the square of the angular distance from the primitive rainbow : and, if the first additional red be at the distance of 2° from the red of the rainbow, so as to interfere a little with the primitive violet, the fourth additional red will be at a distance of nearly 2° more; and the intermediate colours will occupy a space nearly equal to the original rainbow. In order to produce this effect, the drops must be about $\frac{1}{76}$ of an inch, or .013, in diameter : it would be sufficient if they were between $\frac{1}{70}$ and $\frac{1}{80}$. The reason that such supernumerary colours are not often seen, must be, that it does not often happen that drops so nearly equal are found together : but, that this may sometimes happen, is not in itself at all improbable: we measure even medicines by dropping them from a phial, and it may easily be conceived that the drops formed by natural operations may sometimes be as uniform as any that can be produced by art. How accurately this theory coincides with the observation, may best be determined from Dr. LANGWITH's own words.

"August the 21st, 1722, about half an hour past five in the evening, weather temperate, wind at north-east, the appearance was as follows. The colours of the primary rainbow were as
" usual, only the purple very much inclining to red, and well MDCCCIV.

Dr. Young's Experiments and Calculations

10

" defined : under this was an arch of green, the upper part of " which inclined to a bright yellow, the lower to a more dusky " green : under this were alternately two arches of reddish " purple, and two of green: under all, a faint appearance of " another arch of purple, which vanished and returned several " times so quick, that we could not readily fix our eyes upon it. " Thus the order of the colours was, 1. Red, orange-colour, yel-" low, green, light blue, deep blue, purple. 11. Light green, dark " green, purple. 111. Green, purple. 1v. Green, faint vanishing " purple. You see we had here four orders of colours, and per-" haps the beginning of a fifth : for I make no question but that " what I call the purple, is a mixture of the purple of each of " the upper series with the red of the next below it, and the " green a mixture of the intermediate colours. I send you not " this account barely upon the credit of my own eyes; for there " was a clergyman and four other gentlemen in company, "whom I desired to view the colours attentively, who all " agreed, that they appeared in the manner that I have now de-" scribed. There are two things which well deserve to be taken " notice of, as they may perhaps direct us, in some measure, to " the solution of this curious phenomenon. The first is, that the " breadth of the first series so far exceeded that of any of the " rest, that, as near as I could judge, it was equal to them all " taken together. The second is, that I have never observed " these inner orders of colours in the lower parts of the rainbow, " though they have often been incomparably more vivid than " the upper parts, under which the colours have appeared. I " have taken notice of this so very often, that I can hardly look " upon it to be accidental; and, if it should prove true in general, " it will bring the disquisition into a narrow compass; for it will

relative to physical Optics.

" show that this effect depends upon some property which the " drops retain, whilst they are in the upper part of the air, but " lose as they come lower, and are more mixed with one ano-" ther." Phil. Trans. Vol. XXXII. p. 243.

From a consideration of the nature of the secondary rainbow, of 54° , it may be inferred, that if any such supernumerary colours were seen attending this rainbow, they would necessarily be external to it, instead of internal. The circles sometimes seen encompassing the observer's shadow in a mist, are perhaps more nearly related to the common colours of thin plates as seen by reflection.

IV. ARGUMENTATIVE INFERENCE RESPECTING THE NATURE OF LIGHT.

The experiment of GRIMALDI, on the crested fringes within the shadow, together with several others of his observations, equally important, has been left unnoticed by NEWTON. Those who are attached to the NEWTONIAN theory of light, or to the hypotheses of modern opticians, founded on views still less enlarged, would do well to endeavour to imagine any thing like an explanation of these experiments, derived from their own doctrines; and, if they fail in the attempt, to refrain at least from idle declamation against a system which is founded on the accuracy of its application to all these facts, and to a thousand others of a similar nature.

From the experiments and calculations which have been premised, we may be allowed to infer, that homogeneous light, at certain equal distances in the direction of its motion, is possessed of opposite qualities, capable of neutralising or destroying each

C 2

Dr. Young's Experiments and Calculations

other, and of extinguishing the light, where they happen to be united; that these qualities succeed each other alternately in successive concentric superficies, at distances which are constant for the same light, passing through the same medium. From the agreement of the measures, and from the similarity of the phenomena, we may conclude, that these intervals are the same as are concerned in the production of the colours of thin plates; but these are shown, by the experiments of NEWTON, to be the smaller, the denser the medium; and, since it may be presumed that their number must necessarily remain unaltered in a given quantity of light, it follows of course, that light moves more slowly in a denser, than in a rarer medium : and this being granted, it must be allowed, that refraction is not the effect of an attractive force directed to a denser medium. The advocates for the projectile hypothesis of light, must consider which link in this chain of reasoning they may judge to be the most feeble; for, hitherto, I have advanced in this Paper no general hypothesis whatever. But, since we know that sound diverges in concentric superficies, and that musical sounds consist of opposite qualities, capable of neutralising each other, and succeeding at certain equal intervals, which are different according to the difference of the note, we are fully authorised to conclude, that there must be some strong resemblance between the nature of sound and that of light.

I have not, in the course of these investigations, found any reason to suppose the presence of such an inflecting medium in the neighbourhood of dense substances as I was formerly inclined to attribute to them; and, upon considering the phenomena of the aberration of the stars, I am disposed to believe, that the luminiferous ether pervades the substance of all material

bodies with little or no resistance, as freely perhaps as the wind passes through a grove of trees.

The observations on the effects of diffraction and interference, may perhaps sometimes be applied to a practical purpose, in making us cautious in our conclusions respecting the appearances of minute bodies viewed in a microscope. The shadow of a fibre, however opaque, placed in a pencil of light admitted through a small aperture, is always somewhat less dark in the middle of its breadth than in the parts on each side. A similar effect may also take place, in some degree, with respect to the image on the retina, and impress the sense with an idea of a transparency which has no real existence: and, if a small portion of light be really transmitted through the substance, this may again be destroyed by its interference with the diffracted light, and produce an appearance of partial opacity, instead of uniform semitransparency. Thus, a central dark spot, and a light spot surrounded by a darker circle, may respectively be produced in the images of a semitransparent and an opaque corpuscle; and impress us with an idea of a complication of structure which does not exist. In order to detect the fallacy, we may make two or three fibres cross each other, and view a number of globules contiguous to each other; or we may obtain a still more effectual remedy by changing the magnifying power; and then, if the appearance remain constant in kind and in degree, we may be assured that it truly represents the nature of the substance to be examined. It is natural to inquire whether or no the figures of the globules of blood, delineated by Mr. HEWSON in the Phil. Trans. Vol. LXIII. for 1773, might not in some measure have been influenced by a deception of this kind : but, as far as I have hitherto been able to examine the globules, with a lens of one-fiftieth of an inch

Dr. Young's Experiments and Calculations

14

focus, I have found them nearly such as Mr. HEWSON has described them.

V. REMARKS ON THE COLOURS OF NATURAL BODIES.

Exper. 5. I have already adduced, in illustration of NEWTON'S comparison of the colours of natural bodies with those of thin plates, Dr. WOLLASTON'S observations on the blue light of the lower part of a candle, which appears, when viewed through a prism, to be divided into five portions. I have lately observed a similar instance, still more strongly marked, in the light transmitted by the blue glass sold by the opticians. This light is separated by the prism into seven distinct portions, nearly equal in magnitude, but somewhat broader, and less accurately defined, towards the violet end of the spectrum. The first two are red, the third is yellowish green, the fourth green, the fifth blue, the sixth bluish violet, and the seventh violet. This division agrees very nearly with that of the light reflected by a plate of air $\frac{1}{6840}$ of an inch in thickness, corresponding to the 11th series of red and the 18th of violet. A similar plate of a metallic oxide, would perhaps be about $\frac{1}{15000}$ of an inch in thickness. But it must be confessed, that there are strong reasons for believing the colouring particles of natural bodies in general to be incomparably smaller than this; and it is probable that the analogy, suggested by NEWTON, is somewhat less close than he imagined. The light reflected by a plate of air, at any thickness nearly corresponding to the 11th red, appears to the eye to be very nearly white; but, under favourable circumstances, the 11th red and the neighbouring colours may still be distinguished. The light of some kinds of coloured glass is pure red; that of others, red with a little green: some intercept all the light, except the extreme

COURSE OF LECTURES

ON

NATURAL PHILOSOPHY

AND THE

MECHANICAL ARTS.

BY THOMAS YOUNG, M.D.

FOR. SEC. R.S. F.L.S. MEMBER OF EMMANUEL COLLEGE, CAMBRIDGE, AND LATE PROFESSOR OF NATURAL PHILOSOPHY IN THE ROYAL INSTITUTION OF GREAT BRITAIN

IN TWO VOLUMES.

VOLUME I.



LONDON:

PRINTED FOR JOSEPH JOHNSON, ST. PAUL'S CHURCH YARD, BY WILLIAM SAVAGE, BEDFORD BURY.

1807.

451

ON THE NATURE OF LIGHT AND COLOURS.

THE nature of light is a subject of no material importance to the concerns of life or to the practice of the arts, but it is in many other respects extremely interesting, especially as it tends to assist our views both of the nature of our sensations, and of the constitution of the universe at large. The examination of the production of colours, in a variety of circumstances, is intimately connected with the theory of their essential properties, and their causes; and we shall find that many of these phenomena will afford us considerable assistance in forming our opinion respecting the nature and origin of light in general.

It is allowed on all sides, that light either consists in the emission of very minute particles from luminous substances, which are actually projected, and continue to move, with the velocity commonly attributed to light, or in the excitation of an undulatory motion, analogous to that which constitutes sound, in a highly light and elastic medium pervading the universe; but the judgments of philosophers of all ages have been much divided with respect to the preference of one or the other of these opinions. There are also some circumstances which induce those, who entertain the first hypothesis, either to believe, with Newton, that the emanation of the particles of light is always attended by the undulations of an etherial medium, accompanying it in its passage, or to suppose, with Boscovich, that the minute particles of light themselves receive, at the time of their emission, certain rotatory and vibratory motions, which they retain as long as their projectile motion continues. These additional suppositions, however necessary they may have been thought for explaining some particular phenomena, have never been very generally understood or admitted, although no attempt has been made to accommodate the theory in any other manner to those phenomena.

VOL. I.

3 N

We shall proceed to examine in detail the manner in which the two principal. hypotheses respecting light may be applied to its various properties and affections; and in the first place to the simple propagation of light in right lines through a vacuum, or a very rare homogeneous medium. In this circumstance there is nothing inconsistent with either hypothesis; but it undergoes some modifications, which require to be noticed, when a portion of light is admitted through an aperture, and spreads itself in a slight degree in every direction. In this case it is maintained by Newton that the margin of the aperture possesses an attractive force, which is capable of inflecting the rays: but there is some improbability in supposing that bodies of different forms and of various refractive powers should possess an equal force of inflection, as they appear to do in the production of these effects; and there is reason to conclude from experiments, that such a force, if it existed, must extend to a very considerable distance from the surfaces concerned, at least a quarter of an inch, and perhaps much more, which is a condition not easily reconciled with other phenomena. In the Huygenian system of undulation, this divergence or diffraction is illustrated by a comparison with the motions of waves of water and of sound, both of which diverge when they are admitted into a wide space through an aperture, so much indeed that it has usually been considered as an objection to this opinion, that the rays of light do not diverge in the degree that would be expected if they were analogous to the waves of water. But as it has been remarked by Newton, that the pulses of sound diverge less than the waves of water, so it may fairly be inferred, that in a still more highly elastic medium, the undulations, constituting light, must diverge much less considerably than either. (Plate XX. Fig. 266.)

With respect, however, to the transmission of light through perfectly transparent mediums of considerable density, the system of emanation labours under some difficulties. It is not to be supposed that the particles of ligh can perforate with freedom the ultimate atoms of matter, which compose a substance of any kind; they must, therefore, be admitted in all directions through the pores or interstices of those atoms : for if we allow such suppositions as Boscovich's, that matter itself is penetrable, that is, immaterial, it is almost useless to argue the question further. It is certain that some substances retain all their properties when they are reduced to the thickness of the ten millionth of an inch at most, and we cannot therefore suppose the distances

of the atoms of matter in general to be so great as the hundred millionth of an inch. Now if ten feet of the most transparent water transmits, without interruption, one half of the light that enters it, each section or stratum of the thickness of one of these pores of matter must intercept only about one twenty thousand millionth, and so much must the space or area occupied by the particles be smaller than the interstices between them, and the diameter of each atom must be less than the hundred and forty thousandth part of its distance from the neighbouring particles: so that the whole space occupied by the substance must be as little filled, as the whole of England would be filled by a hundred men, placed at the distance of about thirty miles from each other. This astonishing degree of porosity is not indeed absolutely inadmissible, and there are many reasons for believing the statement to agree in some measure with the actual constitution of material substances; but the Huygenian hypothesis does not require the disproportion to be by any means so great, since the general direction and even the intensity of an undulation would be very little affected by the interposition of the atoms of matter, while these atoms may at the same time be supposed to assist in the transmission of the impulse, by propagating it through their own substance. Euler indeed imagined that the undulations of light might be transmitted through the gross substance of material bodies alone, precisely . in the same manner as sound is propagated; but this supposition is for many reasons inadmissible."

A very striking circumstance, respecting the propagation of light, is the uniformity of its velocity in the same medium. According to the projectile hypothesis, the force employed in the free emission of light must be about a million million times as great as the force of gravity at the earth's surface; and it must either act with equal intensity on all the particles of light, or must impel some of them through a greater space than others, if its action be less powerful, since the velocity is the same in all cases; for example, if the projectile force is weaker with respect to red light than with respect to violet light, it must continue its action on the red rays to a greater distance than on the violet rays. There is no instance in nature besides of a simple projectile moving with a velocity uniform in all cases, whatever may be its cause, and it is extremely difficult to imagine that so immense a force of repulsion can reside in all substances capable of

2

becoming luminous, so that the light of decaying wood, or of two pebbles rubbed together, may be projected precisely with the same velocity, as the light emitted by iron burning in oxygen, gas, or by the reservoir of liquid fire on the surface of the sun. Another cause would also naturally interfere with the uniformity of the velocity of light, if it consisted merely in the motion of projected corpuscles of matter; Mr. Laplace has calculated, that if any of the stars were 250 times as great in diameter as the sun, its attraction would be so strong as to destroy the whole momentum of the corpuscles of light proceeding from it, and to render the star invisible at a great distance; and although there is no reason to imagine that any of the stars are actually of this magnitude, yet some of them are probably many times greater than our sun, and therefore large enough to produce such a retardation in the motion of their light as would materially alter its effects. It is almost unnecessary to observe that the uniformity of the velocity of light, in those spaces which are free from all material substances, is a necessary consequence of the Huygenian hypothesis, since the undulations of every homogeneous elastic medium are always propagated, like those of sound, with the same velocity, as long as the medium remains unaltered.

On either supposition, there is no difficulty in explaining the equality of the angles of incidence and reflection; for these angles are equal as well in the collision of common elastic bodies with others incomparably larger, as in the reflections of the waves of water and of the undulations of sound. And it is equally easy to demonstrate, that the sines of the angles of incidence and refraction must be always in the same proportion at the same surface, whether it be supposed to possess an attractive force, capable of acting on the particles of light, or to be the limit of a medium through which the undulations are propagated with a diminished velocity. There are, however, some cases of the production of colours, which lead us to suppose that the velocity of light must be smaller in a denser than in a rarer medium; and supposing this fact to be fully established, the existence of such an attractive force could no longer be allowed, nor could the system of emanation be maintained by any one.

The partial reflection from all refracting surfaces is supposed by Newton to arise from certain periodical retardations of the particles of light, caused

by undulations, propagated in all cases through an ethereal medium. The mechanism of these supposed undulations is so complicated, and attended by so many difficulties, that the few who have examined them have been in general entirely dissatisfied with them: and the internal vibrations of the particles of light themselves, which Boscovich has imagined, appear scarcely to require a serious discussion. It may, therefore, safely be asserted, that in the projectile hypothesis this separation of the rays of light of the same kind by a partial reflection at every refracting surface, remains wholly unexplained. In the undulatory system, on the contrary, this separation follows as a necessary consequence. It is simplest to consider the ethereal medium which pervades any transparent substance, together with the material atoms of the substance, as constituting together a compound medium denser than the pure ether, but not more elastic; and by comparing the contiguous particles of the rarer and the denser medium with common elastic bodies of different dimensions, we may easily determine not only in what manner, but almost in what degree, this reflection must take place in different circumstances. Thus, if one of two equal bodies strikes the other, it communicates to it its whole motion without any reflection; but a smaller body striking a larger one is reflected, with the more force as the difference of their magnitude is greater; and a larger body, striking a smaller one, still proceeds with a diminished velocity; the remaining motion constituting, in the case of an undulation falling on a rarer medium, a part of a new series of motions which necessarily returns backwards with the appropriate velocity: and we may observe a circumstance nearly similar to this last in a portion of mercury spread out on a horizontal table; if a wave be excited at any part, it will be reflected from the termination of the mercury almost in the same manner as from a solid obstacle.

The total reflection of light, falling, with a certain obliquity, on the surface of a rarer medium, becomes, on both suppositions, a particular case of refraction. In the undulatory system, it is convenient to suppose the two mediums to be separated by a short space in which their densities approach by degrees to each other, in order that the undulation may be turned gradually round, so as to be reflected in an equal angle: but this supposition is not absolutely necessary, and the same effects may be expected at the surface of two mediums separated by an abrupt termination.

The chemical process of combustion may easily be imagined either to disengage the particles of light from their various combinations, or to agitate the elastic medium by the intestine motions 'attending it: but the operation of friction upon substances incapable of undergoing chemical changes, as well as the motions of the electric fluid through imperfect conductors, afford instances of the production of light in which there seems to be no easy way of supposing a decomposition of any kind. The phenomena of solar phosphori appear to resemble greatly the sympathetic sounds of musical instruments, which are agitated by other sounds conveyed to them through the air: it is difficult to understand in what state the corpuscles of light could be retained by these substances so as to be reemitted after a short space or time; and if it is true that diamonds are often found, which exhibit a red light after having received a violet light only, it seems impossible to explain this property, on the supposition of the retention and 'subsequent emission of the same corpuscles.

The phenomena of the aberration of light agree perfectly well with the system of emanation; and if the ethereal medium, supposed to pervade the earth and its atmosphere, were carried along before it, and partook materially in its motions, these phenomena could not easily be reconciled with the theory of undulation. But there is no kind of necessity for such a supposition: it will not be denied by the advocates of the Newtonian opinion that all material bodies are sufficiently porous to leave a medium pervading them almost absolutely at rest; and if this be granted, the effects of aberration will appear to be precisely the same in either hypothesis.

The unusual refraction of the Iceland spar has been most accurately and satisfactorily explained by Huygens, on the simple supposition that this crystal possesses the property of transmitting an impulse more rapidly in one direction than in another; whence he infers that the undulations constituting light must assume a spheroidical instead of a spherical form, and lays down such laws for the direction of its motion, as are incomparably more consistent with experiment than any attempts which have been made to accommodate the phenomena to other principles. It is true that nothing has yet been done to assist us in understanding the effects of a subsequent refraction by a second crystal, unless any person can be satisfied with the name of polarity

assigned by Newton to a property which he attributes to the particles of light, and which he supposes to direct them in the species of refraction which they are to undergo: but on any hypothesis, until we discover the reason why a part of the light is at first refracted in the usual manner, and another part in the unusual manner, we have no right to expect that we should understand how these dispositions are continued or modified, when the process is repeated.

In order to explain, in the system of emanation, the dispersion of the rays of different colours by means of refraction, it is necessary to suppose that all refractive mediums have an elective attraction, acting more powerfully on the violet rays, in proportion to their mass, than on the red.' But an elective attraction of this kind is a property foreign to mechanical philosophy, and when we use the term in chemistry, we only confess our incapacity to assign a mechanical cause for the effect, and refer to an analogy with other facts, of which the intimate nature is perfectly unknown to us. It is not indeed very easy to give a demonstrative theory of the dispersion of coloured light upon the supposition of undulatory motion; but we may derive a very satisfactory illustration from the well known effects of waves of different breadths. The simple calculation of the velocity of waves, propagated in a liquid perfectly elastic, or incompressible, and free from friction, assigns to them all precisely the same velocity, whatever their breadth may be: the compressibility of the fluids actually existing introduces, however, a necessity for a correction according to the breadth of the wave, and it is very easy to observe, in a river or a pond of considerable depth, that the wider waves proceed much more rapidly than the narrower. We may, therefore, consider the pure ethereal medium as analogous to an infinitely elastic fluid, in which undulations of all kinds move with equal velocity, and material transparent substances, on the contrary, tas resembling those fluids, in which we see the large waves advance beyond the smaller; and by supposing the red light to consist of larger or wider undulations and the violet of smaller, we may sufficiently elucidate the greater refrangibility of the red than of the violet light.

It is not, however, merely on the ground of this analogy that we may be induced to suppose the undulations constituting red light to be larger than those of violet light: a very extensive class of phenomena leads us still

more directly to the same conclusion; they consist chiefly of the production of colours by means of transparent plates, and by diffraction or inflection, none of which have been explained, upon the supposition of emanation, in a manner sufficiently minute or comprehensive to satisfy the most candid even of the advocates for the projectile system; while on the other hand all of them may be at once understood, from the effect of the interference of double lights, in a manner nearly similar to that which constitutes in sound the sensation of a beat, when two strings, forming an imperfect unison, are heard to vibrate together.

Supposing the light of any given colour to consist of undulations, of a given breadth, or of a given frequency, it follows that these undulations must be liable to those effects which we have already examined in the case of the waves of water, and the pulses of sound. It has been shown that two equal series of waves, proceeding from centres near each other, may be seen to destroy each other's effects at certain points, and other points at to redouble them; and the beating of two sounds has heen explained from a similar interference. We are now to apply the same principles to the alternate union and extinction of colours. (Plate XX. Fig. 267.)

In order that the effects of two portions of light may be thus combined, it is necessary that they be derived from the same origin, and that they arrive at the same point by different paths, in directions not much deviating from each other. This deviation may be produced in one or both of the portions by diffraction, by reflection, by refraction, or by any of these effects combined; but the simplest case appears to be, when a beam of homogeneous light falls on a screen in which there are two very small holes or slits, which may be considered as centres of divergence, from whence the light is diffracted in every direction. In this case, when the two newly formed beams are received on a surface placed so as to intercept them, their light is divided by dark stripes into portions nearly equal, but becoming wider as the surface is more remote from the apertures, so as to subten't very nearly equal angles from the apertures at all distances, and wider also in the same proportion as the apertures are closer to each other. The middle of the two portions is always light, and the bright stripes on each side are at such distances, that the light, coming to them from one of the apertures, must have passed through a

465

longer space than that which comes from the other, by an interval which is equal to the breadth of one, two, three, or more of the supposed undulations, while the intervening dark spaces correspond to a difference of half a supposed undulation, of one and a half, of two and a half, or more.

From a comparison of various experiments, it appears that the breadth of the undulations constituting 'the extreme red light must be supposed to be, in air, about one 36 thousandth of an inch, and those of the extreme violet about one 60 thousandth; the mean of the whole spectrum, with respect to the intensity of light, being about one 45 thousandth. From these dimensions it follows, calculating upon the known velocity of light, that almost 500 millions of millions of the slowest of such undulations must enter the eve in a single second. The combination of two portions of white or mixed light. when viewed at a great distance, exhibits a few white and black stripes, corresponding to this interval; although, upon closer inspection, the distinct effects of an infinite number of stripes of different breadths appear to be compounded together, so as to produce a beautiful diversity of tints, passing by degrees into each other. The central whiteness is first changed to a yellowish, and then to a tawny colour, succeeded by crimson, and by violet and blue, which together appear, when seen at a distance, as a dark stripe; after this a green light appears, and the dark space beyond it has a crimson hue: the subsequent lights are all more or less green, the dark spaces purple and reddish; and the red light appears so far to predominate in all these effects, that the red or purple stripes occupy nearly the same place in the mixed fringes as if their light were received separately.

The comparison of the results of this theory with experiments fully establishes their general coincidence; it indicates, however, a slight correction in some of the measures, on account of some unknown cause, perhaps connected with the intimate nature of diffraction, which uniformly occasions the portions of light, proceeding in a direction very nearly rectilinear, to be divided into stripes or fringes a little wider than the external stripes, formed by the light which is more bent. (Plate XXX. Fig. 442, 443.)

When the parallel slits are enlarged, and leave only the intervening substance to cast its shadow, the divergence from its opposite margins still con-

VOL. I.

tinues to produce the same fringes as before, but they are not easily visible." except within the extent of its shadow, being overpowered in other parts by a stronger light; but if the light thus diffracted be allowed to fall on the eye, either within the shadow, or in its neighbourhood, the stripes will still appear; and in this manner the colours of small fibres are probably formed. Hence if a collection of equal fibres, for example a lock of wool, be held before the eye when we look at a luminous object, the series of stripes belonging to each fibre combine their effects, in such a manner, as to be converted into circular fringes or coronae. This is probably the origin of the coloured circles or coronae sometimes seen round the sun and moon, two or three of them appearing together, nearly at equal distances from each other and from the luminary, the internal ones being, however, like the stripes, a little dilated. It is only necessary that the air should be loaded with globules of moisture, nearly of equal size among themselves, not much exceeding one two thousandth of an inch in diameter, in order that a series of such coronae, at the distance of two or three degrees from each other, may be exhibited. (Plate XXX. Fig. 444.) it is in the second s

If, on the other hand, we remove the portion of the screen which separates the parallel slits from each other, their external margins will still continue to exhibit the effects of diffracted light in the shadow on each side; and the experiment will assume the form of those which were made by Newton on the light passing between the edges of two knives, brought very nearly into contact; although some of these experiments appear to show the influence of a portion of light reflected by a remoter part of the polished edge of the knives, which indeed must unavoidably constitute a part of the light concerned in the appearance of fringes, wherever their whole breadth exceeds that of the aperture, or of the shadow of the fibre.

a provide a second a final second

The edges of two knives, placed very near each other, may represent the opposite margins of a minute furrow, cut in the surface of a polished substance of any kind, which, when viewed with different degrees of obliquity, present a series of colours nearly resembling those which are exhibited within the shadows of the knives: in this case, however, the paths of the two portions of light before their incidence are also to be considered, and the whole difference of these paths will be found to determine the appearance of

2

colour in the usual manner; thus when the surface is so situated, that the image of the luminous point would be seen in it by regular reflection, the difference will vanish, and the light will remain perfectly white, but in other cases various colours will appear, according to the degree of obliquity. These colours may easily be seen, in an irregular form, by looking at any metal, coarsely polished, in the sunshine; but they become more distinct and conspicuous, when a number of fine lines of equal strength are drawn parallel to each other, so as to conspire in their effects.

It sometimes happens that an object, of which a shadow is formed in a beam of light, admitted through a small aperture, is not terminated by parallel sides; thus the two portions of light, which are diffracted from two sides of an object, at right angles with each other, frequently form a short series of curved fringes within the shadow, situated on each side of the diagonal, which were first observed by Grimaldi, and which are completely explicable from the general principle, of the interference of the two portions encroaching perpendicularly on the shadow. (Plate XXX. Fig. 445.)

But the most obvious of all the appearances of this kind is that of the fringes, which are usually seen beyond the termination of any shadow, formed in a beam of light, admitted through a small aperture: in white light three of these fringes are usually visible, and sometimes four; but in light of one colour only, their number is greater; and they are always much narrower as they are remoter from the shadow. Their origin is easily deduced from the interference of the direct light with a portion of light reflected from the margin of the object which produces them, the obliquity of its incidence causing a reflection so copious as to exhibit a visible effect, however narrow that margin may be; the fringes are, however, rendered-more obvious as the quantity of this reflected light is greater. Upon this theory it follows that the distance of the first dark fringe from the shadow should be half as great as that of the fourth, the difference of the lengths of the different paths of the light being as the squares of those distances; and the experiment precisely confirms this calculation, with the same slight correction only as is required in all other cases; the distances of the first fringes being always a little increased. It may also be observed, that the extent of the shadow itself is always augmented, and nearly in an equal degree with that of the fringes: the

LECTURE XXXIX.

the second secon

reason of this circumstance appears to be the gradual loss of light at the edges of every separate beam, which is so strongly analogous to the phenomena visible in waves of water. The same cause may also perhaps have some effect in producing the general modification or correction of the place of the first fringes, although it appears to be scarcely sufficient for explaining the whole of it. (Plate XXX. Fig. 446.)

A still more common and convenient method, of exhibiting the effects of the mutual interference of light, is afforded us by the colours of the thin plates of transparent substances. The lights are here derived from the successive partial reflections produced by the upper and under surface of the plate, or when the plate is viewed by transmitted light, from the direct beam which is simply refracted, and that portion of it which is twice reflected within the plate. The appearance in the latter case is much less striking than in the former, because the light thus affected is only a small portion of the whole beam, with which it is mixed; while in the former the two reflected portions are nearly of equal intensity, and may be separated from all other light tending to overpower them. In both cases, when the plate is gradually reduced in thickness to an extremely thin edge, the order of colours may be precisely the same as in the stripes and coronae already described; their distance only varying when the surfaces of the plate, instead of being plane, are concave, as it frequently happens in such experiments. The scale of an oxid, which is often formed by the effect of heat on the surface of a metal, in particular of iron, affords us an example of such a series formed in reflected light: this scale is at first inconceivably thin, and destroys none of the light reflected, it soon, however, begins to be of a dull yellow, which changes to red, and then to crimson and blue, after which the effect is destroyed by the opacity which the oxid acquires. Usually, however, the series of colours produced in reflected light follows an order somewhat different: the scale of oxid is denser than the air, and the iron below than the oxid; but where the mediums above and below the plate are either both rarer or both denser than itself, the different natures of the reflections at its different surfaces appear to produce a modification in the state of the undulations, and the infinitely thin edge of the plate becomes black instead of white, one of the portions of light at once destroying the other, instead of cooperating with it. Thus when a film of soapy water is stretched over a

wine glass, and placed in a vertical position, its upper edge becomes extremely thin, and appears nearly black, while the parts below are divided by horizontal lines into a series of coloured bands; and when two glasses, one of which is slightly convex, are pressed together with some force, the plate of air between them exhibits the appearance of coloured rings, beginning from a black spot at the centre, and becoming narrower and narrower, as the curved figure of the glass causes the thickness of the plate of air to increase more and more rapidly. The black is succeeded by a violet, so faint as to be scarcely perceptible; next to this is an orange yellow, and then crimson and blue. When water, or any other fluid, is substituted for the air between the glasses, the rings appear where the thickness is as much less than that of the plate of air, as the refractive density of the fluid is greater; a circumstance which necessarily follows from the proportion of the velocities with which light must, upon the Huygenian hypothesis, be supposed to move in different mediums. It is also a consequence equally necessary in this theory, and equally inconsistent with all others, that when the direction of the light is oblique, the effect of a thicker plate must be the same as that of a thinner plate, when the light falls perpendicularly upon it; the difference of the paths described by the different portions of light precisely corresponding with the observed phenomena. (Plate XXX. Fig. 447...449.)

Sir Isaac Newton supposes the colours of natural bodies in general to be similar to these colours of thin plates, and to be governed by the magnitude of their particles. If this opinion were universally true, we might always separate the colours of natural bodies by refraction into a number of different portions, with dark spaces intervening; for every part of a thin plate, which exhibits the appearance of colour, affords such a divided spectrum, when viewed through a prism. There are accordingly many natural colours in which such a separation may be observed; one of the most remarkable of them is that of blue glass, probably coloured with cobalt, which becomes divided into seven distinct portions. It seems, however, impossible to suppose the production of natural colours perfectly identical with those of thin plates, on account of the known minuteness of the particles of colouring bodies, unless the refractive density of these particles be at least 20 or 30 times as great as that of glass or water; which is indeed not at all improbable with respect to the ultimate atoms of bodies;

but difficult to believe with respect to any of their arrangements constituting the diversities of material substances.

The colours of mixed plates constitute a distinct variety of the colours of thin plates, which has not been commonly observed. They appear when the interstice between two glasses, nearly in contact, is filled with a great number of minute portions of two different substances, as water and air, oil and air, or oil and water: the light, which passes through one of the mediums, moving with a greater velocity, anticipates the light passing through the other; and their effects on the eye being confounded and combined; their interference produces an appearance of colours nearly similar to those of the colours of simple thin plates, seen by transmission; but at much greater thicknesses, depending on the difference of the refractive densities of the substances employed. The effect is observed by holding the glasses between the eye and the termination of a bright object, and it is most conspicuous in the portion which is seen on the dark part beyond the object, being produced by the light scattered irregularly from the surfaces of the fluid. Here, however, the effects are inverted, the colours resembling those of the common thin plates. seen by reflection; and the same considerations on the nature of the reflec-. tions are applicable to both cases. (Plate XXX. Fig. 450.)

The production of the supernumerary rainbows, which are sometimes seen within the primary and without the secondary bow, appears to be intimately connected with that of the colours of thin plates. We have already seen that the light producing the ordinary rainbow is double, its intensity being only greatest at its termination, where the common bow appears, while the whole light is extended much more widely. The two portions concerned in its production must divide this light into fringes; but unless almost all the drops of a shower happen to be of the same magnitude, the effects of these fringes must be confounded and destroyed: in general, however, they must at least cooperate more or less in producing one dark fringe, which must cut off the common rainbow much more abruptly than it would otherwise have been terminated, and consequently assist the distinctness of its colours. The magnitude of the drops of rain, required for producing such of these rainbows as are usually observed, is between the 50th and the 100th of an inch: they become gradually narrower as they are more remote from the common

rainbows, nearly in the same proportions as the external fringes of a shadow, or the rings seen in a concave plate. (Plate XXX. Fig. 451.)

The last species of the colours of double lights, which it will be necessary to notice, constitutes those which have been denominated, from Newton's experiments, the colours of thick plates, but which may be called, with more propriety, the colours of concave mirrors. The anterior surface of a mirror of glass, or any other transparent surface placed before a speculum of metal, dissipates irregularly in every direction two portions of light, one before, and the other after its reflection. When the light falls obliquely on the mirror, being admitted through an aperture near the centre of its curvature, it is easy to show, from the laws of reflection, that the two portions, thus dissipated, will conspire in their effects, throughout the circumference of a circle, passing through the aperture; this circle will consequently be white, and it will be surrounded with circles of colours very nearly at equal distances, resembling the stripes produced by diffraction. The analogy between these colours and those of thin plates is by no means so close as Newton supposed it; since the effect of a plate of any considerable thickness must be absolutely lost in white light, after ten or twelve alternations of colours at most; while these effects would require the whole process to remain unaltered, or rather to be renewed, after many thousands or millions of changes. (Plate XXX. Fig. . 452.) J17, 111

It is presumed, that the accuracy, with which the general law of the interference of light has been shown to be applicable to so great a variety of facts, in circumstances the most dissimilar, will be allowed to establish its validity in the most satisfactory manner. The full confirmation or decided rejection of the theory, by which this law was first suggested, can be expected from time and experience alone; if it be confuted, our prospects will again be confined within their ancient limits, but if it be fully established, we may expect an ample extension of our views of the operations of nature, by means of our acquaintance with a medium, so powerful and so universal, as that to which the propagation of light must be attributed.

PLATE XXX



PLATE XXX.

787

Fig. 436. A section of the human cyc. A is the cornea; B the aqocous humour, in which the uvea hangs; C the crystalline lens; the ciliary processes being between it and the uvea; D the vitreous lumour; E F G is the choroid coat, lined by the retina; H I K the sclerotica, and L the optic nerve. P. 447.

Fig. 437. A picture painted on the retina in an inverted position, seen by dissecting off the sclerotica and choroid behind it. P. 448.

Fig. 438. The apparent figure of the heavens being nearly like the curve ABC, the sun or moon at A or C appears to be much larger than at B. P. 454.

Fig. 439. The red square A, inclosing a green square, produces, if viewed attentively, in a strong light, a spectrom resembling B, which is red within and green without, and which appears when we look soon after on any white object. P. 456.

Fig. 440. The spot, which is tinted with black lines only, appears, upon the yellow ground, of a purple hue. P. 456.

Fig. 441. A grey spot on a purple ground appears of a greenish yellow or olive hue. P. 456.

Fig. 442. The manner in which two portions of coloured light, admitted through two small apertures, produce light and dark stripes or fringes by their interference, proceeding in the form of hyperbolas; the middle ones are however usually a little dilated, as at A. P. 465.

Fig. 443. A series of stripes of all colours, of their appropriate breadths, placed side by side in the manner in which they would be separated by refraction, and combined together so as to form the fringes of colours below them, beginning from white. P. 465. Fig 444. A series of coronae, seen round the sun or moon. P 466.

Fig. 445. The internal hyperbolic fringes of a rectangular shadow. P. 467.

Fig. 446. The external fringes seen on each side of the shadow of a hair or wire, which is also divided by its internal fringes. The dotted lines show the natural magnitude of the shadow, independently of diffraction. P. 468.

Fig. 447. Analysis of the colours of thin plates seen by reflection, beginning from black. A line drawn across the curved fringes would show the portions into which the light of any part is divided when viewed through a prism. P. 469.

Fig. 448. The coloured stripes of a film of soapy water, covering a wine glass. P. 469.

Fig. 449. The colours of a thin plate of air or water, contained between a convex and a plane glass, as seen by reflection. P. 469.

Fig. 450. The colours of a mixed plate; as seen by partially greasing a lens a little convex, and a flat glass, and holding them together between the eye and the edge of a dark object. One half of the series begins from white, the other from black, and each colour is the contrast to that of the opposite half of the ring. P. 470.

Fig. 451. The composition of the colours of the primary rainbow, when attended by supernumerary bows. P. 471.

Fig. 452. The colours of concave mirrors. The small circles in the middle white ring represent the . aperture by which the light is admitted, and its image; the coloured rings are formed by the light irregularly dissipated, before and after reflection. P. 471.