

Physics 127A — Practice Exam Solutions

Spring 2026

Standard idealizations are assumed knowledge and are not printed on the exam: an active-region BJT has $V_{BE} \approx 0.6\text{ V}$ and a conducting diode drops 0.6 V ; a typical $\beta = 100$ is used where a problem calls for it (Problems 5 and 8 state it explicitly). Thermal voltage $V_T \approx 25\text{ mV}$ at room temperature, so $r_e = V_T/I_C \approx 25\text{ }\Omega/I_C(\text{mA})$. Where useful I give each number two ways as a consistency check.

Problem 1. Resistive divider, Thevenin equivalent, and power

- (a) Unloaded, no current flows except through the series pair, so the divider rule applies:

$$V_{\text{out}} = 12\text{ V} \cdot \frac{R_2}{R_1 + R_2} = 12 \cdot \frac{2\text{ k}\Omega}{3\text{ k}\Omega} = 8\text{ V}.$$

- (b) Killing the ideal source (short it) leaves R_1 and R_2 in parallel as seen from the output node:

$$R_{\text{th}} = R_1 \parallel R_2 = \frac{1 \cdot 2}{1 + 2}\text{ k}\Omega = \frac{2}{3}\text{ k}\Omega \approx 667\text{ }\Omega.$$

- (c) Model the divider as its Thevenin source ($V_{\text{th}} = 8\text{ V}$, $R_{\text{th}} = 667\text{ }\Omega$) feeding $R_L = 1\text{ k}\Omega$:

$$V_{\text{out}} = 8 \cdot \frac{1000}{1000 + 667} = 4.80\text{ V}.$$

Check by collapsing the load into the divider directly: $R_2 \parallel R_L = 1\text{ k}\Omega \parallel 2\text{ k}\Omega = 667\text{ }\Omega$, so $V_{\text{out}} = 12 \cdot 667/(1000 + 667) = 4.80\text{ V}$. The two routes agree.

- (d) With the output at 4.80 V , the current in R_1 is

$$I_{R_1} = \frac{12 - 4.80}{1\text{ k}\Omega} = 7.2\text{ mA},$$

so $P_{R_1} = I^2 R_1 = (7.2\text{ mA})^2 (1\text{ k}\Omega) = 51.8\text{ mW}$. Cross-check: $P = I V_{R_1} = (7.2\text{ mA})(7.2\text{ V}) = 51.8\text{ mW}$.

Problem 2. Non-ideal meters

- (a) The two $1\text{ M}\Omega$ resistors give an unloaded midpoint of 5 V . The voltmeter's $10\text{ M}\Omega$ sits in parallel with the lower resistor:

$$1\text{ M}\Omega \parallel 10\text{ M}\Omega = \frac{10}{11}\text{ M}\Omega = 0.909\text{ M}\Omega.$$

$$V_{\text{read}} = 10 \cdot \frac{0.909}{1 + 0.909} = 4.76\text{ V}.$$

Error = $(5 - 4.76)/5 = 4.76\%$ low. (Loading a source whose impedance is one-tenth of the meter's costs you a few percent — the usual rule of thumb.)

- (b) The ammeter's $1\ \Omega$ adds to the $100\ \Omega$:

$$I_{\text{read}} = \frac{10\ \text{V}}{101\ \Omega} = 99.0\ \text{mA},$$

versus the true $10\ \text{V}/100\ \Omega = 100\ \text{mA}$: about 1 % low.

- (c) Setting $|X_C| = 1/(\omega C) = 1\ \text{M}\Omega$:

$$f = \frac{1}{2\pi RC} = \frac{1}{2\pi(1\ \text{M}\Omega)(20\ \text{pF})} = 7.96\ \text{kHz}.$$

Above this frequency the $20\ \text{pF}$ reactance falls below the $1\ \text{M}\Omega$ resistance, so the probe loads the circuit more and more heavily and high-frequency signals are attenuated — the scope rolls off the measurement.

Problem 3. Diodes and a Zener regulator

- (a) Circuit (i): the diode drops $0.6\ \text{V}$, leaving $5\ \text{V} - 0.6\ \text{V} = 4.4\ \text{V}$ across the $1\ \text{k}\Omega$:

$$I = \frac{4.4}{1\ \text{k}\Omega} = 4.4\ \text{mA}.$$

- (b) No load: all the series current flows through the Zener.

$$I = \frac{15 - 5.1}{470\ \Omega} = 21.06\ \text{mA}, \quad P_Z = (5.1\ \text{V})(21.06\ \text{mA}) = 107\ \text{mW}.$$

- (c) A $10\ \text{mA}$ load steals current from the Zener:

$$I_Z = 21.06 - 10 = 11.06\ \text{mA} > 0,$$

so the Zener still conducts and the output stays regulated at $5.1\ \text{V}$.

- (d) Regulation holds until the Zener current reaches zero, i.e. the load takes the entire series current:

$$I_{L,\text{max}} = 21.06\ \text{mA}.$$

Check: that load looks like $R_L = 5.1/21.06\ \text{mA} = 242\ \Omega$; the divider $15 \cdot 242/(242+470) = 5.1\ \text{V}$ sits right at the edge of dropout. Consistent.

Problem 4. RC low-pass filter

- (a) Output taken across the capacitor, whose impedance falls with frequency, so this is a *low-pass* filter. The $-3\ \text{dB}$ corner is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1\ \text{k}\Omega)(0.1\ \mu\text{F})} = 1.59\ \text{kHz}.$$

- (b) The transfer function is the capacitor's share of the divider:

$$H(\omega) = \frac{1/i\omega C}{R + 1/i\omega C} = \frac{1}{1 + i\omega RC}.$$

At $f = f_c$, $\omega RC = 1$, so $|H| = 1/\sqrt{2} = 0.707$, i.e. $-3.0\ \text{dB}$.

- (c) At $f = 10f_c$, $\omega RC = 10$:

$$|H| = \frac{1}{\sqrt{1+100}} = 0.0995 \approx -20.0 \text{ dB},$$

and the phase is $\angle H = -\arctan(10) = -84.3^\circ$. (A single-pole low-pass rolls off at -20 dB/decade and approaches -90° — both checks consistent.)

- (d) At $f = f_c$ the gain magnitude is 0.707 and the phase is $-\arctan(1) = -45^\circ$. A 2 V-amplitude input gives a $2(0.707) = 1.41 \text{ V}$ -amplitude output, lagging by 45° .

Problem 5. Emitter follower

- (a) The base divider ($R_1 = R_2 = 100 \text{ k}\Omega$) sets, to first order, $V_B = 15 \text{ V}/2 = 7.5 \text{ V}$, hence

$$V_E = V_B - V_{BE} = 6.9 \text{ V}, \quad I_E = \frac{6.9}{6.8 \text{ k}\Omega} = 1.01 \text{ mA} \approx I_C.$$

Consistency note: the divider current is $\sim 75 \mu\text{A}$ while the base current is $I_B = I_C/\beta \approx 10 \mu\text{A}$, a ratio near 7:1. That is close to the usual 10:1 stiffness target but not perfectly stiff; base loading pulls V_B down by roughly $I_B(R_1 \parallel R_2) = (10 \mu\text{A})(50 \text{ k}\Omega) \approx 0.5 \text{ V}$, to about 7.0 V. The simple model ignores this; either answer is acceptable if the assumption is stated. Using $I_C \approx 1 \text{ mA}$ gives $r_e \approx 25 \Omega$.

- (b) Looking into the base, the emitter resistor is multiplied by $\beta+1$:

$$Z_{\text{in,base}} = \beta(r_e + R_E) = 100(25 + 6800) \Omega \approx 683 \text{ k}\Omega.$$

In parallel with the divider:

$$Z_{\text{in}} = R_1 \parallel R_2 \parallel Z_{\text{in,base}} = 50 \text{ k}\Omega \parallel 683 \text{ k}\Omega \approx 46.6 \text{ k}\Omega.$$

The bias network, not the transistor, dominates the input impedance here.

- (c) Driven from $R_s = 1 \text{ k}\Omega$, the output impedance at the emitter is the source impedance divided down by β , plus r_e :

$$Z_{\text{out}} = r_e + \frac{R_s \parallel R_1 \parallel R_2}{\beta} = 25 + \frac{1 \text{ k}\Omega \parallel 50 \text{ k}\Omega}{100} \approx 25 + 9.8 \approx 35 \Omega$$

(R_E in parallel is negligible).

- (d) The follower gain is just below unity:

$$A_v = \frac{R_E}{R_E + r_e} = \frac{6800}{6825} = 0.996 \approx 1.$$

Problem 6. Common-emitter amplifier

- (a) With the base pinned at $V_B = 2.6 \text{ V}$, $V_E = 2.6 - 0.6 = 2.0 \text{ V}$, so

$$I_E = \frac{2.0}{2 \text{ k}\Omega} = 1.0 \text{ mA} \approx I_C, \quad V_C = 15 - (1 \text{ mA})(5 \text{ k}\Omega) = 10 \text{ V}.$$

$V_{CE} = V_C - V_E = 8 \text{ V} \gg 0.2 \text{ V}$: comfortably active. Also $r_e = 25 \Omega/1 = 25 \Omega$.

- (b) With the emitter fully bypassed at signal frequencies, the gain is set by R_C and r_e :

$$A_v = -\frac{R_C}{r_e} = -\frac{5000}{25} = -200.$$

- (c) In decibels, $20 \log_{10}(200) = 46.0 \text{ dB}$.

- (d) Leaving 100Ω unbypassed in the emitter adds emitter degeneration:

$$A_v = -\frac{R_C}{r_e + R'_E} = -\frac{5000}{25 + 100} = -40.$$

The gain drops fivefold but becomes far less sensitive to r_e (hence to temperature and bias) — the usual reason to degenerate.

Problem 7. Current source and current mirror

- (a) Ebers–Moll gives $I_C = I_S(e^{V_{BE}/V_T} - 1) \approx I_S e^{V_{BE}/V_T}$. Then

$$r_e \equiv \frac{dV_{BE}}{dI_C} = \left(\frac{dI_C}{dV_{BE}} \right)^{-1} = \frac{V_T}{I_C}.$$

At $I_C = 1 \text{ mA}$ and $V_T = 25 \text{ mV}$, $r_e = 25 \Omega$.

- (b) Circuit (i): the 2.0 V base sets $V_E = 2.0 - 0.6 = 1.4 \text{ V}$, so

$$I_{\text{out}} = I_E = \frac{1.4}{1.4 \text{ k}\Omega} = 1.0 \text{ mA}.$$

The collector current is essentially independent of the load voltage — that is the point of a current source.

- (c) The transistor stays in the active region as long as $V_{CE} \geq 0.2 \text{ V}$, i.e. the collector stays at least $V_E + 0.2 \text{ V} = 1.6 \text{ V}$. With a 15 V supply the load can therefore drop anywhere from 0 up to $15 - 1.6 = 13.4 \text{ V}$: a compliance range of about 13.4 V .
- (d) For the mirror, the diode-connected Q_1 carries the program current set by R_{prog} from the 15 V rail:

$$R_{\text{prog}} = \frac{15 - V_{BE}}{I_{\text{prog}}} = \frac{14.4}{1 \text{ mA}} = 14.4 \text{ k}\Omega.$$

Because Q_1 and Q_2 share the same V_{BE} and are matched, Ebers–Moll forces equal collector currents, so $I_{\text{out}} = I_{\text{prog}} = 1 \text{ mA}$. The output current is "mirrored" from the programmed side.

Problem 8. Differential amplifier

- (a) The 2 mA tail splits equally when the inputs are balanced: $I_C = 1 \text{ mA}$ per side, $r_e = 25 \Omega$. Each collector sits at

$$V_C = 15 - (1 \text{ mA})(7.5 \text{ k}\Omega) = 7.5 \text{ V}.$$

- (b) For a differential input, each emitter is a virtual signal ground through the other transistor, so the single-ended gain is

$$\frac{v_{o1}}{V_1 - V_2} = \frac{R_C}{2r_e} = \frac{7500}{50} = 150,$$

and the collector-to-collector (double-ended) gain is twice that, $R_C/r_e = 300$.

- (c) With a finite tail resistance $R_{\text{tail}} = 500 \text{ k}\Omega$, a common-mode input sees both emitters move together through $2R_{\text{tail}}$:

$$A_{\text{cm}} = -\frac{R_C}{2R_{\text{tail}}} = \frac{7500}{1,000,000} = 0.0075.$$

Then

$$\text{CMRR} = \left| \frac{A_{\text{diff}}}{A_{\text{cm}}} \right| = \frac{150}{0.0075} = 20000 = 86.0 \text{ dB}.$$

(Cross-check: $\text{CMRR} = R_{\text{tail}}/r_e$ in the single-ended form $= 500\text{k}/25 = 20000$. Consistent.)

- (d) Looking into one base for a differential signal, $Z_{\text{in}} = 2\beta r_e = 2(100)(25) = 5 \text{ k}\Omega$. The output impedance at one collector is essentially $R_C = 7.5 \text{ k}\Omega$.

Problem 9. Schmitt trigger

- (a) Returning the feedback resistor R_2 to the non-inverting (+) input makes any output change reinforce itself: a small rise in V_{out} raises V_+ , which (since the signal enters the inverting input) drives the output still higher until it slams to a rail. That regenerative action is positive feedback. The source driving V_{in} connects only to the op amp's inverting input terminal, which (ideal op amp) draws no current, so the input impedance is essentially infinite — the high-impedance feature of this configuration. (Contrast the alternative in which the signal is injected through a resistor into the feedback node, which presents a far lower impedance.)
- (b) There is no negative feedback, so $V_- = V_{\text{in}}$ tracks the input directly and the + input sits at a fraction of the output set by the R_1 – R_2 divider:

$$V_+ = V_{\text{out}} \frac{R_1}{R_1 + R_2} = V_{\text{out}} \frac{10}{30} = \frac{V_{\text{out}}}{3}.$$

With the two saturated output states $V_{\text{out}} = \pm 12 \text{ V}$,

$$V_+ = +4 \text{ V (when } V_{\text{out}} = +12), \quad V_+ = -4 \text{ V (when } V_{\text{out}} = -12).$$

The output switches when V_{in} crosses the current V_+ , so the thresholds are

$$V_{\text{th}}^+ = +4 \text{ V}, \quad V_{\text{th}}^- = -4 \text{ V}.$$

(Concretely: starting with $V_{\text{out}} = +12$, the input must rise past $+4 \text{ V}$ to trip it low; it then stays low until the input falls past -4 V .)

- (c) The hysteresis is the gap between the thresholds,

$$\Delta V = V_{\text{th}}^+ - V_{\text{th}}^- = 4 - (-4) = 8 \text{ V}.$$

- (d) The width scales with $R_1/(R_1 + R_2)$, namely $\Delta V = 2V_{\text{sat}} R_1/(R_1 + R_2)$. Halving it to 4 V requires $R_1/(R_1 + R_2) = 1/6$, i.e. $R_2 = 5R_1 = 50 \text{ k}\Omega$. Check: $V_+ = \pm 12(10/60) = \pm 2 \text{ V}$, giving thresholds $\pm 2 \text{ V}$ and a width of 4 V. Consistent.

Problem 10. Operational amplifiers

- (a) Golden rules for an op amp with negative feedback: (1) the output does whatever it must to drive the voltage difference between the inputs to zero ($V_+ = V_-$); (2) the inputs draw no current.
- (b) Inverting amplifier: $A_v = -R_f/R_{\text{in}} = -100/10 = -10$. The source sees the input resistor terminating on a virtual ground, so $Z_{\text{in}} = R_{\text{in}} = 10 \text{ k}\Omega$.
- (c) Non-inverting amplifier: $A_v = 1 + R_f/R_1 = 1 + 100/10 = 11$. Its input impedance is the op amp's own (ideally infinite), enormously larger than the $10 \text{ k}\Omega$ of the inverting circuit — the main reason to pick the non-inverting topology when you must not load the source.
- (d) Replacing R_f with C_f makes an integrator:

$$V_{\text{out}}(t) = -\frac{1}{R_{\text{in}}C_f} \int V_{\text{in}}(t) dt.$$

Its gain magnitude is $|A| = 1/(\omega R_{\text{in}}C_f)$, which equals unity when

$$\omega = \frac{1}{R_{\text{in}}C_f} = \frac{1}{(10 \text{ k}\Omega)(0.01 \text{ }\mu\text{F})} = 10^4 \text{ rad/s},$$

i.e. $f = \omega/2\pi = 1.59 \text{ kHz}$. (Same number as the Problem 4 corner, since RC is identical — a nice consistency check.)

- (e) Slew-rate limit: the maximum sinusoidal slope is $\omega V_p = 2\pi f V_p$. Setting this to the slew rate,

$$f_{\text{max}} = \frac{\text{SR}}{2\pi V_p} = \frac{1 \text{ V}/\mu\text{s}}{2\pi(10 \text{ V})} = 15.9 \text{ kHz}.$$