EXPERIMENT O-6

Michelson Interferometer

Abstract

A Michelson interferometer, constructed by the student, is used to measure the wavelength of He-Ne laser light and the index of refraction of a flat transparent sample.

References

Taylor, Zafiratos and Dubson, Modern Physics, second edition, Section 1.5.

Pre-Lab

Please do this section before coming to lab. Look at the Michelson interferometer diagram in one of the references, and compare to Figure 1 below. In texts, it is usually assumed that an observer looks into the interferometer, so that the eye receives light that originated at a relatively weak source. In this experiment a laser is used as the source, so the light leaving the interferometer is bright enough to be projected on a screen.

Suppose that in Figure 1, the lens between the laser and the interferometer is removed. The laser beam then follows the dotted path (length x) into the interferometer. When it hits the beamsplitter, half the light is reflected along path L_2 to a fixed mirror and the other half continues straight ahead, along path L_1, to a movable mirror. The mirrors reflect the light back to the beamsplitter, where each returning ray is split again. Half of the light returning from each mirror leaves the interferometer along the dotted path (length y) and travels to a screen mounted on the far wall. Since there are two light beams arriving at this point on the screen, the spot is bright or dark depending on whether they are in or out of phase. One beam travels a distance $x + 2L_1 + y$ from point S to the wall, while the second travels $x + 2L_2 + y$. The difference in these path lengths is therefore $2(L_2 - L_1)$, and the spot on the screen is bright only when this is an integer times the laser wavelength, i.e. when $(L_2 - L_1) = n\,(\lambda/2)$.

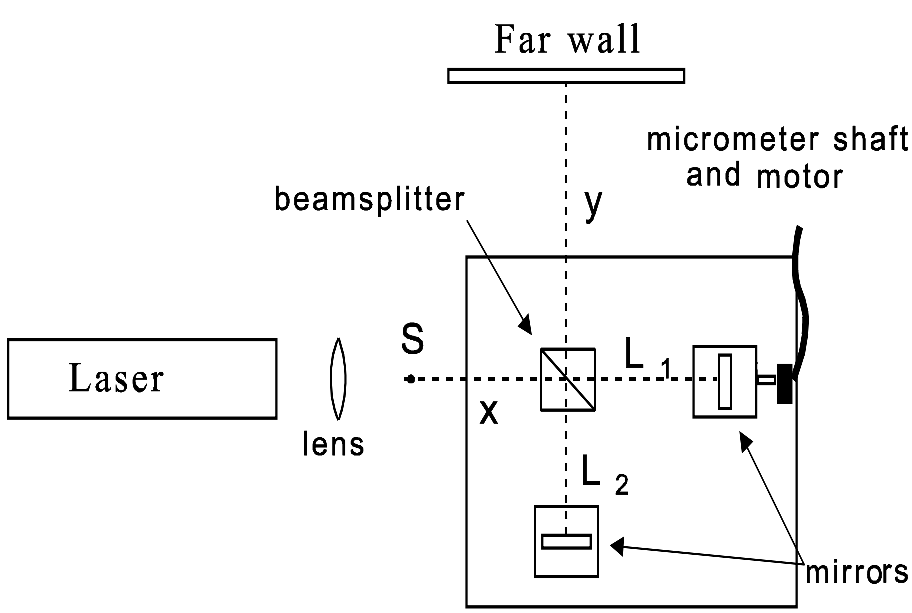

Figure 1: Michelson Interferometer

Now suppose the movable mirror's motor is running, so that L_1 changes at a constant rate. The spot where the dotted path meets the screen will therefore alternate between bright and dark. One complete cycle of this intensity variation is called a "fringe shift" and occurs each time L_1 changes by $\lambda/2$, i.e. when $\Delta L_1 = n(\lambda/2)$.

When the lens is in front of the laser, as shown in Figure 1, it focuses the laser beam to point S, which acts as a point source sending light into the interferometer. Light waves emerging from a point source are spherical, i.e. the light rays coming out of S diverge from one another. This means that the light passing through the interferometer illuminates the entire screen, not just the "central" point where dotted path y hits. To find out if a particular point P on the screen is bright or dark, we need to consider the difference in path lengths from S to P for light traveling along the two arms of the interferometer. The resulting pattern on the screen turns out to be a series of concentric circular bright and dark bands called "fringes", as illustrated in Figure 2 below.

To understand why the pattern is circular, remember that the lens focuses the parallel rays from the laser to the point, **S**, which can be considered as the actual "source" of light entering the interferometer. As described above, light from **S** gets to the screen by either of two routes through the interferometer and, for the center of the

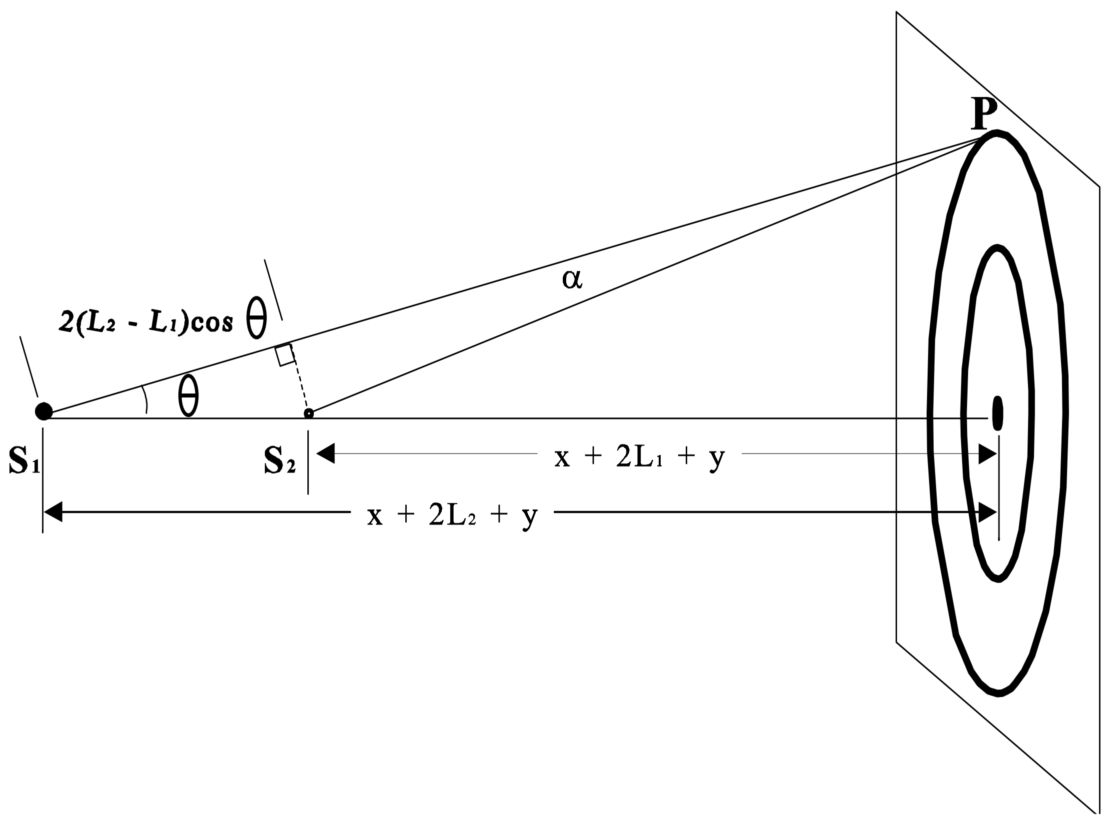

Figure 2: Two Source Representation

pattern, the lengths of these routes are $\mathbf{x + 2L_1 + y}$ and $\mathbf{x + 2L_2 + y}$, respectively. Therefore, what happens on the screen is the same as what *would* happen if there were *two* point sources located at these distances in front of the screen on a common axis, as shown in Figure 2, provided that wave crests leave each of the sources simultaneously. (Notice that if you "unfold" the paths in Figure 1 you get Figure 2.) Since $\mathbf{L_1 >> (L_2 - L_1)}$, the rays from $\mathbf{S_1}$ and $\mathbf{S_2}$ to **P** are nearly parallel. For a particular angle $\boldsymbol{\theta}$, the light from $\mathbf{S_2}$ travels a distance $\boldsymbol{\alpha}$ while the light from $\mathbf{S_1}$ travels $\boldsymbol{\alpha} + \mathbf{2(L_2 - L_1)\cos\theta}$. If the difference between these two path lengths is an integral number of wavelengths, constructive interference occurs and you have brightness at **P.** Since the same angle **θ** exists for all points **P** on a circle concentric with the center of the pattern, you see a bright circular ring. Finally, if you increase or decrease **θ** so that $\mathbf{2(L_2 - L_1)\cos\theta}$ changes by $\lambda/2$, you get a dark ring. The pattern is thus a series of concentric bright and dark fringes.

To make sure you are prepared for lab and read the **Procedure** section below. Think about ***QUESTIONS 1 and 2***, so you can quickly answer when you come to them. As always, feel free to ask questions!

Apparatus

Steel plate on an inner tube	Short focal length positive lens
He-Ne laser and stand	Rotational mount for transparent sample
Optically flat reflectors on magnetic bases	Micrometer
Beam splitting cube on magnetic base	

Procedure

Taking care not to touch any optical surfaces, examine the components on magnetic bases. In contrast to ordinary mirrors, the optically flat reflectors have their reflective coating on the front surface of the glass. This results in higher optical quality but makes it much easier to damage the coatings. **Please avoid contact with these coatings, since they scratch easily and are severely corroded by skin oils**. The tilt of the mirrors can be adjusted by turning the two screws that attach it to the base. Set the screws near the middle of their travel range. Note that the beamsplitter cube has a "half-silvered" interior diagonal surface, causing a beam of light entering any side to be half transmitted (straight through) and half reflected (at 90 degrees).

Do not turn the micrometer shaft by hand. Movement causes "backlash" that can last 10 minutes! Note that the motor turns the shaft very slowly, so you can read the micrometer even with the motor on. Make sure you know how to read the scale (refer to the *Commonly Used Lab Equipment* link on the Physics 108 web page) before starting.

Place the steel slab on top of the inner tube and, with the levers on the magnetic bases in their "**off**" positions, arrange the optical components on the slab as shown in Figure 1. Place the laser opposite the movable mirror and adjust the components until the axes of the laser and mirrors are perpendicular to the beamsplitter faces. Lock the magnetic bases to the steel slab. *(Hint: It helps if* $\boldsymbol{L_1}$ *and* $\boldsymbol{L_2}$ *are nearly, but not quite, identical. Also, you may want to position the mirror with the motor in a location where you may read the micrometer scale easily.)* With the laser on, you should see a pattern of bright dots on the wall opposite the bench. Observe the behavior of these dots when the mirror tilt adjustments are varied, returning the screws to the middle of their travel range. ***QUESTION 1****: Why are there more than two dots? To determine why, observe what happens when an index card or piece of paper is inserted between the beamsplitter and either of the mirrors. Next, try blocking both mirrors.*

The first thing to do is to make the two brightest dots overlap. As a coarse adjustment, align the magnetic base first. For fine adjustment only, use the screws to modify the tilt of one of the mirrors. When the dots overlap, place the lens in front of the laser such that a broad spot of light appears on the wall opposite the bench. In reduced room light you should now see a fringe pattern in this spot. The pattern can be centered by making fine adjustments in the tilt of either mirror.

Wavelength Measurement

One "fringe shift" corresponds to a change in "arm length", $\mathbf{\Delta L_1}$, equal to $\boldsymbol{\lambda/2}$. You get $\mathbf{\Delta L_1}$ by taking the difference between two-micrometer readings. The goal in this lab, as in all labs is to minimize uncertainty in your measurements. If the uncertainty of the micrometer is known, then the change in 'arm length' necessary to keep the uncertainty in our calculation as small as possible can be calculated. (ie: If the uncertainty of the instrument you are using is 1mm, and you take a measurement that is 4mm, then you are only certain to 25%, which is not great. However if the measurement you are taking is 100mm, then your result improves to 1% uncertainty.) ***QUESTION 2****: Assuming you can read the micrometer to one-tenth of its smallest scale division, and can count fringe shifts with complete certainty, how much should the arm length be changed to guarantee a wavelength result with less than 5 percent uncertainty?* **Note:** the rest of this experiment is based on the correctness of your answer to this question. <u>Check your answer with one of us before proceeding!</u>

Using the micrometer scale, determine the number of fringe shifts that result from a known change in the length of one interferometer arm. Be sure to count enough fringes to guarantee a final wavelength uncertainty smaller than 5 percent. Repeat (alternating lab partners) the measurements at least two more times.

Index of Refraction Measurement

First, turn off the motor. Measure the thickness of your flat transparent sample, remembering to justify uncertainty, and use the rotational mount to position the sample in one arm of your interferometer. Observe that the fringes shift as the sample is tilted, and that the shifting fringes reverse direction when the sample is perpendicular to the interferometer axis. Determine the angle reading at which this reversal occurs (referred to as 0°). Starting with the sample at an angle (>15°), slowly rotate it to the 0° point while counting the corresponding number of fringe shifts. Test and record the reproducibility of your result by doing at least two more trials.

Sample Calculations

Compute the best wavelength for one of your trials from the first part of the lab. Then, using the method outlined in the Analysis section, find the best index of refraction of your glass sample from your data from one of your trials in the second part of the lab.

Dismantle the apparatus, unplug any equipment, and return the lab to its original state.

Analysis

From the data taken in the first part of the lab, find the wavelength, $\boldsymbol{\lambda}$, of the laser light and its associated uncertainty in Excel. Find the uncertainty first, by finding the wavelength for each trial and using (max-min)/2, and then by using partial uncertainty analysis wrt $\mathbf{\Delta L_1}$ and **N** (the # of fringe shifts). A sample spreadsheet for this experiment is included at the *Laboratory Handouts* link on the Physics 108 web page.

To analyze the second part, recall that a "fringe shift" corresponds to a unit change in the total number of wavelengths contained in the round trip path between beamsplitter and mirror. This number changes when you tilt the sample because of changes in the distances traveled by the light both inside and outside the sample. This is shown in the Figure 3. When the light enters the glass at normal incidence, the light travels a distance **t** through the glass. When the glass is tilted, the light bends according to Snell's Law and travels a distance **d** through the glass, where **d** > **t**.

The above analysis leads to the following expression for the index of refraction as a function of the number, **N**, of observed fringe shifts, the tilt angle, $\boldsymbol{\theta}$ and the thickness, **t**, of the slab:

$$n = \frac{(2t - N\lambda)(1 - \cos\theta)}{2t(1 - \cos\theta) - N\lambda} = 1 + \frac{N\lambda \cos\theta}{2t(1 - \cos\theta) - N\lambda}$$

For a complete derivation, see the reference by Englund. Using this equation and the wavelength determined, calculate the index of refraction of your transparent sample and its associated uncertainty by finding **n** for each trial, taking (max-min)/2 for the uncertainty, and by doing a partial uncertainty analysis wrt **N**, **λ**, **t**, and **θ**. See the sample spreadsheet for guidance. **Caution:** Excel assumes angles are in radians. Refer to "Useful Excel Formulas" in the *Excel Tutorial*, linked to from the Physics 108 web page, for guidance.

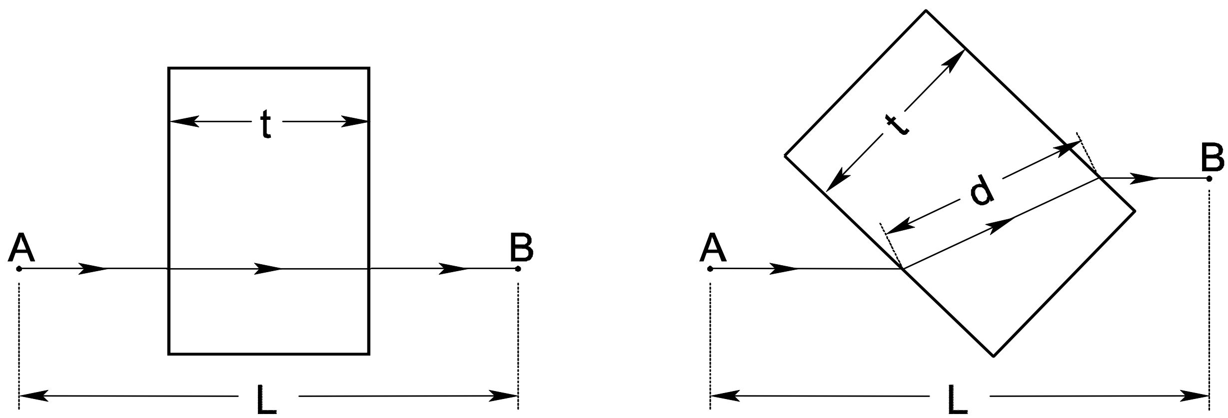

Figure 3: Effect of Glass Slide on Path Length

Questions

3. Suppose that in Figures 1 and 2, x = y = L_1 = 5.00 cm and L_2 = 5.15 cm. What will be the diameter of the smallest dark ring on the screen, if the center of the pattern is a intensity maximum, and if the wavelength of the light is 600 nm?

Write a conclusion that summarizes and interprets your results. Suggest ways you could improve the results if you were to repeat the experiment, mention problems you had in lab, etc...