ACOUSTIC WAVES IN A RESONANT CAVITY

In this experiment you have the opportunity to study the properties of standing acoustic waves in a cavity or resonator. By measuring the resonance frequencies f_p of standing waves, you can determine the velocity of sound v in the medium contained in the cavity. From v you can determine the molecular mass of the medium if you know the temperature and the gas constant. Alternatively you can use the device as a thermometer and determine the temperature if you know what kind of gas it contains. If the resonator contains a mixture of two known kinds of gases at a known temperature you can even determine the composition. Finally, if you know the temperature and the nature of the gas, you can determine the value of the gas constant. Thus, aside from being of interest for their own sake, resonant cavities have numerous useful applications.

Although the resonances usually occur at well defined frequencies, when you look closely you will see that they do have some width. The width gives information on the rate at which energy is lost from the standing wave of sound. There are several loss mechanisms. For the experiment which you will carry out, the loss at the sidewalls due to the viscosity of the gas dominates. So you can learn something about the viscosity of the gas from studying the resonance widths and, if you look carefully enough, you can also separate out the intrinsic losses of the sound wave traveling through the gas.

Stop! In your lab book, list the parameters of interest, defining their symbols, and the relationships between them (e.g., $w = f(x,y,z)$).

BACKGROUND:
PRINCIPLE OF OPERATION AND THEORY

Consider a cylindrical cavity with an electrostatic transducer at each end. Each transducer is a capacitor consisting of two parallel plates, one of which can move relative to the other. The moveable "plate" is actually a thin aluminum film evaporated onto the side of a mylar membrane that faces into the cavity. It is electrically grounded to the cavity body. The fixed plate is a metal piece mounted on top of the insulating mylar at the end of the cavity and, therefore, electrically isolated from the cavity body.

Stop! In your lab book, draw a picture of a transducer and its relationship to the cavity based on this description. Name the picture and label all the parts.

Applying a voltage across a transducer creates an attractive force between the plates which translates the aluminum film and compresses the mylar. For small displacements, the mylar acts like a spring. That is, its displacement is proportional to the applied voltage. If an alternating voltage of frequency f is applied, this motion will generate acoustic waves of frequency $2f$ in the gaseous medium in the cavity.

Stop! Do you understand why the acoustic wave frequency is twice the voltage frequency? Derive the relationship between force and voltage for a pair of charged plates and write the explanation in your lab book.

In principle, this transducer is the same as a loudspeaker, except that the plates are much smaller. Similarly, the other transducer is just like a microphone. When the acoustic waves arrive at the other end of the cavity, they displace the moveable plate of the other transducer. If a fixed charge is superimposed on the receiving transducer, its motion generates a small alternating voltage of the same frequency as the acoustic wave (twice the driving voltage frequency).

Think! How could you impose a fixed charge on the receiving transducer? Write your thoughts in your lab book.

The acoustic waves are reflected from the cavity ends. In general, they will destructively interfere with new waves emanating from the driven transducer. However, when an integer number p of half wavelengths is just equal to the cavity length, the interference will be constructive. At those resonant frequencies f_p, a large sound ampitude will build up in the cavity [1].

Stop! Superscripts cite references listed at the end of this Manual. Go read the citation. Later on you will be prompted to read the actual reference.

The resonant frequency depends on the dimensions of the cavity and on the properties of the medium it contains. The magnitude of the displacement of the mylar is very small, so the volume of the cavity may be accurately treated as constant.

Think! How accurately? Estimate the percent change in cavity volume due to the displacement of the mylar. Write your calculations in your lab book. Use common sense numbers. If you have no idea what numbers to use, guess!

The ability to make reasonable estimates on common sense alone is absolutely the single most useful skill you will ever have. Take every opportunity to hone this skill as a student and you will have a tremendous edge over the competition in whatever profession you choose.

If the length of the cavity is much greater than its diameter, the lowest resonant frequency f_1 corresponds to sound propagation in the axial direction, and occurs when the frequency is such that a single half-wavelength fits into the cavity. It is given by $f_1 = v/(2\ell)$, where v is the velocity of sound in the medium and ℓ is the length of the cavity. Additional resonances occur when an integral number p of half wavelengths fits into the cavity, yielding

$$f_p = pv/(2\ell) \quad . \qquad (1)$$

A more complete analysis must include all three dimensions. Solving the wave equation with cylindrical boundary conditions [2], one finds that there are additional resonances due to modes which have pressure variations in the radial and azimuthal directions r and ϕ. These modes are described by Bessel functions of the first kind [3], and their frequencies are given by [2]

$$f_{pmn} = \frac{v}{2\pi}\sqrt{\left(\frac{B(m,n)}{r}\right)^2 + \left(\frac{p\pi}{\ell}\right)^2} \qquad (2)$$

where p, m and n are the mode numbers in the axial, azimuthal and radial directions respectively, r is the cavity radius and $B(m,n)$ are collected [2] in Table 1, below. From Eqs. 1 and 2 it is clear that one can determine the velocity of sound from the resonant frequency spectrum if the cavity dimensions are known.

M \ *n*	*0*	*1*	*2*
0	*0.0000*	*1.2197*	*2.2331*
1	*0.5861*	*1.6970*	*2.7140*
2	*0.9722*	*2.1346*	*3.1734*

Table 1. Values of $B(m,n)$ for m, $n = 0, 1, 2$.

Stop! By now you should have glanced at the reference list to see what superscripts 2 and 3 cite. There are probably a number of statements you're taking on faith at this point. You may be asking yourself, where did Eq. 1 come from? Why is there a resonance at every integer number of half-wavelengths? How does one solve the wave equation with cylindrical boundary conditions? What do these modes look like?

Throughout the manual, exercises will prompt you to stop reading and check into issues like these. As always, write your work in your lab book. Feel free to use the references but, if you copy something, credit the source.

Exercise 1: Derive Eq. 1.

Exercise 2: Draw the nodes in the r-φ plane of (p,0,1), (p,1,0), (p,1,1), (p,2,1), (p,1,2).

Think! Do you have lingering questions about what you've read or done so far? Write them in your lab book. This really does help keep confusion at bay. Try it!

The velocity of sound v, which enters into the resonance formula given by Eq. 1, is determined by the thermodynamic properties of the medium in the cavity. One can show that [3,3]

$$v^2 = (\partial P/\partial \rho)_S \ . \qquad (3)$$

The right hand side of Eq. 3 is related to the adiabatic compressibility [4] κ_S:

$$(\partial P/\partial \rho)_S = 1/\rho\, \kappa_S \ . \qquad (4)$$

For an ideal gas, v can be expressed in terms of the specific heat ratio $\gamma = C_P/C_V$, the molar mass M, and the absolute temperature T. One has [4]

$$V = (\gamma RT\ /M)^{1/2} \ . \qquad (5)$$

Exercise 3: For an ideal gas, $C_P - C_V = R$. Derive this relation from the equation of state of an ideal gas.

Exercise 4: Derive Eq. 5 from Eqs. 3 and 4 and the equation of state of an ideal gas.

Think! A simple sort of non-ideal gas is one made of diatomic molecules. Since the value of C_v depends upon the number of degrees of freedom of the molecule, in the monatomic case $C_v = (3/2)R$, but for diatomic molecules $C_v = (5/2)R$. Why is this true? What are the consequences for Eq. 5? Write your thoughts in your lab book.

The approximation of an ideal gas gets better as the pressure is reduced. If the approximation is valid, the resonance frequencies will be indpendent of pressure. If the resonance frequencies depend on the pressure, a version of Eq. 5 derived using the Virial equation of state must be used.

Exercise 5: Rederive Eq. 5 from Eqs. 3 and 4 and the Virial equation of state.

Whether or not the gas is ideal, frictional losses will dampen the acoustic standing waves, dissipating their energy. A system that is very lossy is highly damped and is said to have a low quality factor Q. For an oscillator driven at frequency f, the Q is defined as [5]

$$\boldsymbol{Q} \equiv f \frac{\text{average energy stored in a cycle}}{\text{average power dissipated in a cycle}} = \frac{f}{\Delta f} \qquad (6)$$

where f is the resonant frequency and Δf is the width of the peak at an amplitude that is $1/\sqrt{2}$ (down by $3dB$) of the peak amplitude (i.e., at a power that is half of the peak power). Thus, a high-Q resonator is one that has very low losses at the frequency of interest and produces a very narrow resonance line. It follows that a high-Q resonator can transmit a signal at its resonant frequency with very little attenuation. The attenuation, α, is defined as the fraction of the power lost (i.e., $1/Q$) per unit length of sound propagation, and thus is equal to

$$\alpha = f/vQ = \Delta f/v \qquad (7)$$

Stop! Take a little time to list your questions in your lab book. How does the right hand side of Eq. 6 follow from the definition? Do the definitions of Q and α make sense? How about Eq. 7?

A sharp resonance (i.e. one with a width much less than its frequency) due to the (p, m, n) mode of oscillation can be reasonably approximated by the Lorentzian line shape

$$A_{pmn}(f) = \frac{a}{[(\Delta f/2)^2 + (f - f_{pmn})^2]^{1/2}} \qquad (8)$$

where A is the amplitude of the oscillation, f is the driving frequency, f_{pmn} is the resonant frequency and a is an amplitude scale factor. The exact expression for the line-shape is slightly more complicated; you may read about it in Ref. 6 [6].

Think! How would you extract the parameters of interest from the line shape? How would you quantify the error with your method? Draw a sketch in your lab book.

In this experiment, there is the possibility of a background signal due to electronic noise. This background will have a phase which may differ from that of the resonance, and thus may beat against the signal of interest. This effect can be represented by adding a complex constant to the formula

$$A(f) = A_{pmn}(f) + B_0 + iB_1 . \qquad (9)$$

The voltmeter on the receiving transducer will pick up a signal equal to the modulus of *A*, i.e.

Exercise 6: Derive a formula for $V(f)$ in terms of a, f_{pmn}, Δf, B_0, and B_1. Show that Δf is the width of the peak at an amplitude of $f_{pmn}/\sqrt{2}$.

If two or more resonances overlap, their contributions can be represented by several additive terms like the one in Eq. 8; but, since they, too, can be out of phase, the coefficients a would have to be complex, i.e., $a = a_0 + ia_1$. Fitting measured line shapes to formulas which account for overlapping resonances, is sometimes necessary to extract valuable information from complicated situations. For an example of where this was done, see Ref. 7 [7].

Think! How close is too close? Given two resonances with Q_1 and Q_2,, write a criterion for no overlap in your lab book and explain your reasoning.

Measurements of f_{pmn} give the sound velotciy v from Eqs. 1 or 2. Eq. 5 (or its Virial analog) relates v to properties of the gas. Measurements of Δf give the attenuation α from Eq. 7. Relating α to thermodynamic properties of the gas is more difficult because there can be many different microscopic mechanisms at work. Consider

$$\alpha = \alpha_r + \alpha_w + \alpha_b$$

where α_r, α_w and α_b are due to the losses from reflection at the cavity ends, losses from friction at the sidewalls, and losses from intrinsic dissipation in the gas, respectively. In the acoustic resonator you will use, reflection losses are very close to zero and may be neglected.

Sidewall losses are caused by the fact that while the gas moves back and forth in the interior of the resonator, it does not move at all next to the sidewalls because of friction. The "boundary layer" in which the velocity of the gas is diminished due to the presence of the walls has thickness

$$t = \sqrt{\eta/\rho\omega}\ . \qquad (11)$$

here η is the shear viscosity, ρ the density and $\omega = 2\pi f$ the angular frequency of the sound. The resulting attenuation of the signal is

$$\alpha_w = \omega t/2rv = (1/2rv)\sqrt{\eta/\rho\omega}\ . \qquad (12)$$

Exercise 7: The cgs unit of viscosity is called the Poise. What is it in terms of cm, g, and s? What are the units of α_w? Calculate t and α_w for $f = 1, 10$ and 50 kHz. Assume the resonator contains air at 1 atm and 20°C with $\eta = 1.8\times10^{-4}$ Poise.

Attenuation of signal due to dissipation in bulk α_b is given by

$$\alpha_b = \frac{\omega^2}{2v^3} D \qquad (13)$$

where D is the damping coefficient and is independent of ω.

Exercise 8: What are the units of D in cgs?

Exercise 9: If these are the only two significant contributions to α, and therefore to Δf, then, since α_w is proportional to $\omega^{1/2}$ and α_b is proportional to ω^2, a plot of $\alpha/\omega^{1/2}$ vs. $\omega^{3/2}$ should yield a straight line. Write down a formula for the intercept and slope of this line in terms of D, v, r, η and ρ.

Stop! Write your questions down in your lab book, if you haven't already. Put a star next to the ones you consider most important.

INSTRUCTIONS:
EXPERIMENTAL PROTOCOL AND DESIGN

THE RESONATOR

Go! First, read all the way through the step by step instructions below. Then, take apart and re-assemble the resonant cavity. Note important features and dimensions in your lab book as you go along.

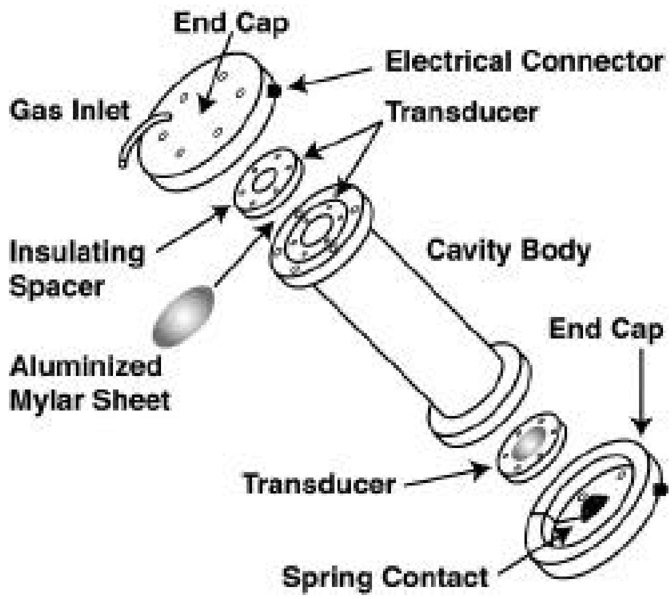

Figure 1. Schematic drawing of the resonator, not to scale. The two ends are identical, except for the gas inlet capillary.

To take apart and re-assemble the resonant cavity (Fig. 1), the following tools and materials are needed: 9/64" allen wrench, 1/2" allen wrench, razor blade knife, scissors, scribe (or some other sharp pointed instrument), screw driver, 2 O-rings, 2 3"×3"×0.00025" aluminized mylar sheets, stretch assembly (Fig. 2), micrometer caliper.

1. Take the resonator apart by removing the allen screws from one of the brass end pieces. Take care not to bend the gas inlet capillary.

2. Next, remove the screws from the transducer element. The mylar disk beneath each transducer may be thrown away. You will replace it with a new one when you re-assemble the resonator (step 5).

3. Repeat for the other end of the cavity.

4. IMPORTANT! While the cavity is apart, accurately measure the inside dimensions of the cavity. Write your measurements and the limits of your accuracy in your lab book.

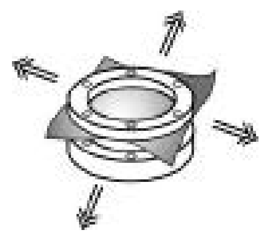

Figure 2. Schematic drawing of the stretcher assembly. Aluminized side of mylar faces the thicker ring.

5. Place a mylar sheet with the aluminized side down between the two rings of the stretcher assembly. (The thinner ring goes on top.) Use the scribe to poke holes in the mylar to accommodate the screws. (Aligning the first one is the hardest.) Four scres are sufficient to keep the sheet from slipping. The sheet does **not** need to be tightly stretched after the screws are in place, but it **must not** have any wrinkles in it.

6. Stand the cavity body on end. Be sure the end facing up is clean. Center the prepared mylar (in the stretcher assembly) on top, aluminized side **down**. The mylar should now be stretched over the end of the cavity.

7. Place the transducer on top of the mylar with the smooth side down as shown. Again, use the scribe to put a pinhole through the mylar to accommodate the allen screws. Insert the first screw **but** don't tighten all the way. Insert the next screw in the hole opposite the first. Tighten them down evenly. Insert the rest of the screws similarly.

8. After all the screws are in place and tightened, guide the razor blade knife around the edge of the transducer, cutting away the excess foil. If you were successful, a view from the other end will reveal a smooth sheet at the bottom. (It's okay if the sheet is slightly wrinkled.)

9. Position one of the O-rings on the top, so that it skirts the transducer edge.

10. Re-attach the brass end piece without the gas inlet capillary with the 9/64" screws.

11. Repeat the same steps for the other end of the cavity. When mounting the bottom brass end piece, it is best to orient it so that the electrical connectors point in the same radial direction. Note that, except for the gas inlet capillary, each transducer is identical. The end with the capillary will be referred to as the DRIVER and the other end as the RECEIVER. The choice is arbitrary, but it is necessary to maintain the distinction so that a residual charge will not be left on the mylar sheet of the driver.

Stop! Look at your lab book. Does it document what you have done? Don't go on until it does. Remember, the idea isn't to copy what's above. It's to log your progress and help you remember anything unusual. If something goes wrong later, your log should save you hours of painstaking troubleshooting.

THE CIRCUIT

The heart of the electrical circuit is a wave analyzer (see Fig. 3, below). It does two things. It puts out a sine wave of frequency f (from the connector labeled "f out") with which to drive one transducer and it also takes in a signal of the same frequency ("f in") (e.g., from the other transducer) and puts out a dc current proportional to its amplitude ("Sig. Out"). It is important to understand that the waveanalyzer only responds to an input signal of a frequency equal to that of the signal it generates. It is not sensitive to signals with frequencies outside of a very narrow band near f.

This presents a problem. If you use the frequency output of the waveanalyzer to drive one transducer, it will oscillate at twice that frequency. So the signal from the receiving transducer will oscillate at $2f$, and be undetectable to the waveanalyzer. To overcome this problem, we have an $f/2$-circuit. This divides the wave analyzer output frequency in half so that the signal generated at the microphone will once again be equal to f and therefore detectable to the wave analyzer.

Think! Why do you suppose we don't just use two wave analyzers? (No, it's not the cost.) Write your thoughts in your lab book.

Unfortunately it is not so easy to generate a sine wave of half the frequency of another sine wave. But it is easy to generate such a square wave. So the $f/2$-circuit puts out a square wave of $f/2$. Now there are oscillators which can lock onto one of the Fourier components of a square wave and then put out a waveshape of your choice (e.g., a sine wave) at the frequency of that Fourier component. Usually, this phase-locked oscillator will lock onto the strongest Fourier component, which is the fundamental one of frequency $f/2$; but sometimes it gets lost and locks onto a different component. One of the tricky parts of this experiment is to make sure that the oscillator is locked onto the right frequency. Thus it is important to monitor both the frequency f coming from the wave analyzer and the frequency $f/2$ coming from the oscillator on the oscilloscope during the experiment.

Stop! If you don't know what a Fourier component is, now is the time to look it up [3].

The receiving transducer has a dc bias voltage applied to it. There are large capacitors in the bias box, which lead to a time constant for any changes of the transducer voltage that is large compared to a typical period of the sound wave, so that the charge on the transducer is held constant. The motion of the aluminum film due to the varying sound pressure will induce an ac voltage across the transducer. This voltage is quite small and has to be amplified by a pre-amplifier before it is sent back to the wave analyzer.

Think! Why it is necessary to place a bias voltage on the transducer? Why does the motion of the film induce an ac voltage across the transducer? Write your thoughts in your lab book

Think! Estimate the time constant of the bias circuit. Estimate the typical period of a sound wave. Estimate the amplitude of the ac voltage induced. Write your calculations, algebra and all, in your lab book. Write your calculations, algebra and all, in your lab book.

The current put out by the wave analyzer (which is proportional to the amplitude of the pressure oscillation at the receiving transducer) is sent to a digital multimeter and converted to a voltage. The frequency output of the wave analyzer is sent also to a frequency counter and converted into a voltage. Finally, the voltages put out by the multimeter and the frequency counter can be interfaced to a computer or aq chart recorder for data acquisition.

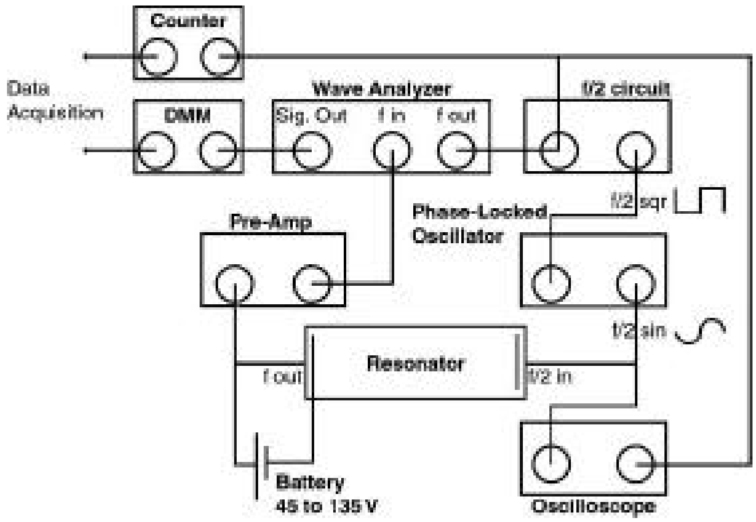

Figure 3. Diagram of the electrical circuit.

Go! It's time to connect the electronic components with co-axial cables. What follows are some tips on setting up the equipment for measurement. Don't forget to note any difficulties or surprises in your lab book as you go along.

To be sure that everything is working as expected, it is essential to use the oscilliscope to look at the signal at various locations in the circuit while making connections. Table 2 below provides initial settings for the different electrical components. Some (highlighted in grey) need to be varied in order to take data. The rest can be varied out of curiosity.

When connecting the bias supply to the cavity, be sure to connect it to the RECEIVER end (the one without the capillary) so that a residual charge will not be left on the mylar sheet of the driver. A bias of 90V usually works well.

Set up the experiment so that the frequency is displayed by the counter and that the oscilloscope shows both the fundamental frequency from the wave analyzer and the fundamental sine-wave component of the f/*2* signal which should come from the phase-locked oscillator. Go through a resonance by hand, turning the frequency wheel of the wave analyzer and watching the DMM display, to confirm that there is a signal to record.

Stop! Did you see a signal? Describe what you saw in your lab book. Write down any questions you may have that you can't resolve immediately, then go on.

Table 2. Initial settings for electrical components. Shading denotes main experimental variables.

Phase-Locked Oscillator	
Power:	X1
Frequency:	1 kHz
Cal:	Clockwise
Mode:	Phaselock
Trigger:	1.0 degrees
Attenuation:	Fully clockwise
Waveform:	Sine

Frequency Counter	
Range:	1.0 sec
Function:	FREQ A
Mode:	Continuous
Channel A:	Slope +
	AC
	Atten x1
	COM

Pre-Amplifier	
LP rolloff:	1
Gain:	2k (cal position)
HP rolloff:	100K
Channel A:	AC coupled

Wave Analyzer		
Meter Speed:	Medium	
Reading:	Absolute	
Mode:	tracking generator output	
Level:	full clockwise	
Frequency:	near resonant frequency	
Delta F:	0 Hz	
Meter FS:	1 mV	
Input Gain:	20 dB	
Sweep rate:	Setting	(~Hz/sec)
	10	12.5
	9	6.2
	8	4.2
	5	1.67
	3	0.625
	1	0.333

Exercise 10: From the resonance you just observed, estimate the value in Volts of the receiver signal at a typical resonance frequency (`i.e.,` before the pre-amp). Use this to estimate the transducer displacement. From the noise level of your data, estimate the smallest transducer displacement you can resolve.

DATA ACQUISITION AND PROCESSING

The data consist of measurements of acoustic wave amplitude as a function of driving frequency. The measurements can be recorded by hand, off of the displays of the DMM and the frequency counter, or by a machine (such as a chart recorder or a computer), from the voltage outputs of both devices.

The interpretation of the data requires extracting the resonant frequencies and the widths of the resonances from the data. For greatest accuracy, it is necessary to fit the resonances to Lorentzian line-shapes. Taking the data directly into the computer requires investing some time to master the associated data acquisition routines, but eliminates the tedium and time lost in manual data entry, leading to more and better measurements in the long run.

EXPERIMENT AND ANALYSIS

There should be a thermometer in the insulator which houses the cavity. Temperature is needed in the caluclation of results, so don't forget to record it in your lab book. The resonator should be connected to the 1/8" copper tube leading to the bottom of the manifold on the rack at the point labelled CAVITY. The mechanical pump hose should be connected to VACUUM PUMP.

<u>Prepare to use air at ambient pressure for a test run.</u>

Exercise 11: Using Eq. 5, estimate v for air. Assume that air is composed of $x = 20\%$ O_2 and $1\text{-}x = 80\%$ N_2. Use a value of $M = xM_{O_2} +$

$(1-x)M_{N_2}$.

Exercise 12: Calculate the expected resonant frequencies of your air-filled cavity for the (p,m,n) modes with m, n = 0, 1, 2 and for p equal to all integers which yield a frequency less than 25 kHz.

To be sure there is air in the resonator (and not some gas left over from a previous run), pump out the cavity and let air into it a few times. In the end, leave it vented to atmosphere.

Go! Acquire data with air in the resonator. Describe the raw data in your lab book. The resonances at low f should be "clean" and well separated from each other. For $p > 4$ you may notice "sidebands", i.e. other resonances close to the (p,0,0) modes. Analyze the (p,0,0) modes by fitting the data near each peak to Eq. 10. Explain the small peak just above the (5,0,0) mode. For $p \geq 5$, be careful not to include data which are influenced by such peaks. From each (p,0,0) frequnecy, calculate v.

Stop! Is all your data and analysis clearly documented in your lab book? Remember to include units in your table headings, to label the axes on your graphs, and to emphasize the most important parts (with boxes, arrows, stars, or the like.)

Think! v should be independent of frequency. Is it?

Exercise 13: Make a plot of $\Delta f/f^{1/2}$ vs. $f^{3/2}$. Determine the best-fit straight line through the lower-frequency points. Use the slope and intercept to calculate D and η. Explain why the high frequency data deviate from the straight line.

Prepare to repeat the measurement using Helium gas.

Exercise 14: Using Eq. 5, estimate v for Helium.

Exercise 15: Calculate the expected resonant frequencies.

Flush the cell a few times and then fill it with Helium from the tank labeled C.

Go! Acquire data and analyze as you did for air.

Exercise 16: Determine the gas constant. Estimate the accuracy of the measurement.

Think! What are your main sources of error?

Think! At the National Institute of Standards and Technology (NIST) they use acoustic resonance measurements to measure the gas constant with a resolution of about 1 ppm. How do you suppose they do that?

<u>Prepare to repeat the measurement on an Unknown gas.</u>

Flush the cell a few times and then fill it with gas from the tank labeled C.

Go! Scan as many (p,0,0) resonances as possible. Analyze as before. Determine v from each resonance frequency. From v, determine what gas you are dealing with. From the line widths, determine η and D.

Exercise 17: Find values in the literature for comparison.

Go! For one or more of the (p,0,0) modes, measure the dependence of Δf on pressure.

Think! Explain your observations.

References

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