Experimental verification of the Heisenberg uncertainty principle—An advanced undergraduate laboratory

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The Heisenberg uncertainty principle can be experimentally demonstrated by combining a Mössbauer experiment with a measurement of a nuclear lifetime. The senior undergraduate students perform a Mössbauer experiment to measure the energy width of the 14.4 keV level of ^{57}Fe followed by measurements of coincident γ rays to determine the lifetime of the level. The experiments are designed to emphasize that the uncertainty principle is inherent in the wave function rather than resulting from the measurement process.

I. INTRODUCTION

The Heisenberg uncertainty principle is a cornerstone of modern physics. Usually students are convinced of its validity through numerous arguments in which it is used to explain phenomena and through exercises in which uncertainties in physical quantities are calculated from the wave function that describes a system.

Nevertheless, it is desirable to study this principle experimentally since this effort forces students to relate real measurements to fundamental concepts. Gedanken experiments, such as the attempt to locate an electron by scattering photons in a Heisenberg microscope, help students make this connection.[1] However, these efforts are often interpreted as experimental technicalities and do not emphasize the fundamental role of the uncertainty principle in nature.

In this report, we discuss two experiments that allow students to determine the uncertainty in energy, ΔE, and uncertainty in lifetime, Δt, of the 14.4 keV state in ^{57}Fe, and thereby verify the uncertainty principle. Since the experimental techniques are standard, Mössbauer spectroscopy[2] to give the width of the state and a coincidence measurement to determine the characteristic lifetime of the state, in this paper we will focus on the physics implications and less obvious details of the experiments. At Hope, senior students perform these experiments in two, six hour laboratory sessions. Both experiments are valuable individually since they introduce important concepts and experimental techniques. However, the combination is enhanced by the inclusion of rigorous analysis and interpretation of the results in terms of the uncertainty principle.

II. APPLICATION OF THE UNCERTAINTY PRINCIPLE TO THE DECAY OF THE 14.4 keV STATE IN ^{57}Fe

The two experiments discussed in this paper both deal with the decay of ^{57}Co nuclei into ^{57}Fe nuclei. The decay scheme is shown in Fig. 1. Of particular interest for these experiments in the $3/2^-$ state at 14.4 keV which has a 98 ns half-life. The properties of this level are well suited to both experiments. The Mössbauer experiment requires a

The key feature, as it relates to the uncertainty principle, is that the properties of the $3/2^-$ state are subject to the constraint $\Delta E\,\Delta t \geq \hbar/2$. That is, the wave function that describes this state must obey this restriction. The values realized for a particular measurement of the time delay between the formation and decay of the 14.4 keV state and for a particular measurement of the energy of the emitted photon are determined according to probability distributions implicit in the wave function. These distributions are not infinitely narrow but have characteristic widths, Γ and τ, which must obey the uncertainty principle. (The relations between ΔE and Γ and between Δt and τ are discussed below.) Of course, any single nucleus could decay very soon after formation and could also emit a photon with exactly the average energy. However, the widths of the fundamental distributions that characterize these variables cannot simultaneously be arbitrarily short and arbitrarily narrow.

The uncertainty principle manifests itself many times during these experiments. Notice, however, that we focus on the implications of the uncertainty principle for the quantum system composed of the excited nucleus alone. Obviously, the uncertainty principle also applies to the process of actually detecting the photons emitted from the nuclear state. One could not measure the detection time and energy of the emitted photon in a detector with infinite precision for both quantities. The uncertainty principle applies to the photon just as surely as it applies to the nuclear state. However, a focus on the measurement of the photons moves the students away from the fundamental nature of the principle as it exists in the basic wave function and moves them toward the idea that the uncertainty principle is only associated with one's ability to measure the energy and time. The restrictions placed on a particular measurement of an emitted photon by the uncertainty principle do not change the fact that the uncertainty principle is an integral part of the probability distributions that determine the particular decay properties of any single excited nucleus.

While the full discussion of the uncertainty principle in all phases of the process, from decay to detection, is warranted with the best students, it tends to obscure the idea that the uncertainty principle is intimately associated with

The statement of the uncertainty principle as $\Gamma\tau = \hbar$ is

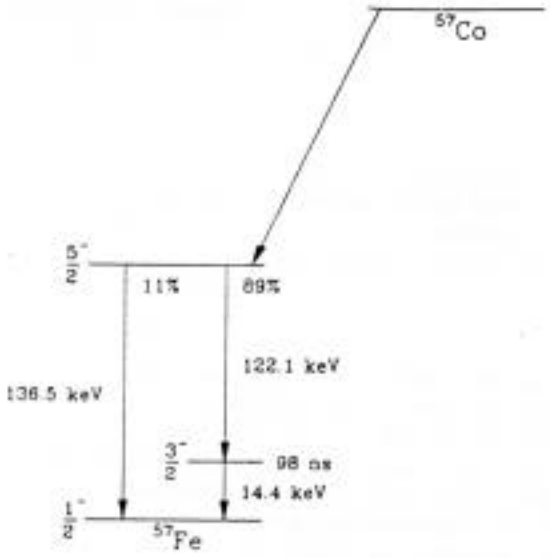


Fig. 1. Level diagram and decay scheme of ^{57}Co going to ^{57}Fe.

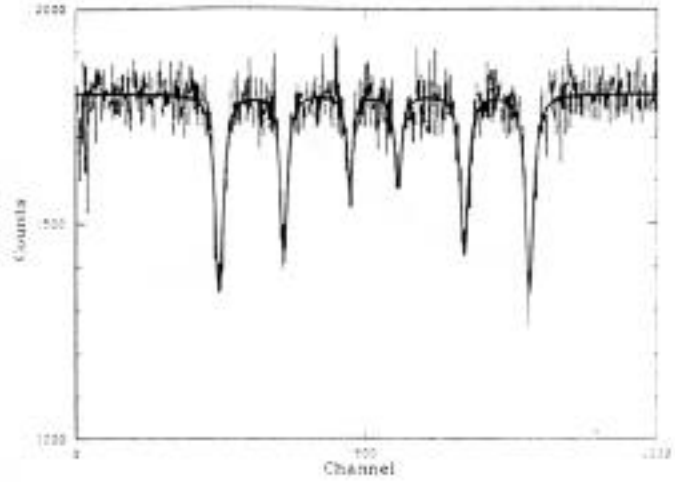


Fig. 2. Absorption spectrum for a natural iron absorber 25 μm thick. The line is a nonlinear least-squares fit to the data, assuming Poisson statistics, with Eq. (2). The fit has a weighted variance of 1.08, with the assumptions of strictly statistical weighting of the lines and Lorentzian shape.

often taken for granted. However, the relationship between this expression and the more traditional $\Delta E\,\Delta t \geq \hbar/2$ is revealing. A mechanical approach to the uncertainty principle would suggest that the use of the second moment or root mean square ($\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$) to characterize a particular distribution is part of the "fundamental" definition of the uncertainty principle.[3] For Δt this definition gives τ. However, with the present system, students are faced with a Lorentzian distribution of energies that has an infinite second moment. However, this fact is not fatal since the use of the second moments to characterize the distribution is not fundamental. According to Messiah,[4] Heisenberg's result essentially expresses the mathematical fact that the extension of the wave ψ and that of its Fourier transform ϕ cannot simultaneously be made arbitrarily small. The use of the second moments of the particular distributions (if they are defined) yields a particular constant such as $\hbar/2$. If, on the other hand, the full width at half-maximum Γ is used to characterize the width of the distribution, one merely gets a different constant that is proportional to $\hbar$. The fundamental fact is that variables that do not commute cannot both have narrow distributions.

III. MÖSSBAUER SPECTROSCOPY EXPERIMENT

The uncertainty in the energy of the 14.4 keV level is measured using the Mössbauer effect,[5,6] which involves recoilless gamma emission. If the nucleus that emits the γ ray is part of a crystalline material, the entire crystal can absorb the recoil energy. When the γ energy is very small, the recoil momentum is low and there is a high probability for exciting no phonons so that the gamma carries away all the decay energy. This probability is approximately 0.7 for the 14.4 keV gamma emission from ^{57}Fe.

$$P(E) = \frac{\Gamma}{2\pi}\,\frac{}{(\hbar\omega - \hbar\omega_0)^2 + (\Gamma/2)^2}, \qquad (1)$$

where $\hbar\omega_0$ is the average energy and Γ is the full width at half-maximum. For the 14.4 keV gamma in ^{57}Fe, $\Gamma = 4.67 \times 10^{-9}$ eV. The fractional width is extremely small, $\Gamma/\hbar\omega_0 = 3.24 \times 10^{-12}$.

This linewidth is measured using traditional Mössbauer spectroscopy; a ^{57}Co source is moved toward and away from an iron absorber at known velocity, which causes the energy of the gamma rays to be Doppler shifted. As this energy is varied, the degree to which the transition is resonant varies and the absorption profile can be measured.

Typical results for an iron absorber are shown in Fig. 2. The results with the Fe absorber show the expected six dips that result from the Zeeman effect caused by the local magnetic field in the Fe. The Zeeman effect is an additional bit of physics that emerges from the laboratory, but it is not the primary focus of the effort. However, the spacings of the six dips do provide a calibration of the velocity scale so that the level width can be accurately determined without relying on the numerical measurements of the source velocity.

The iron absorption spectrum was used to measure the width of the states rather than the single line spectrum of a stainless steel absorber because the latter line is broadened by unresolved quadrupole splitting and isomer shifts. The model used to fit (nonlinear least-squares fitting with errors[7,8]) the spectrum is

$$I = A + Bx - C \sum_{i=1}^{6} \frac{w_i}{(x - X_i)^2 + (\Gamma/2)^2}, \qquad (2)$$

where A and B characterize the unabsorbed background and C is the absorption amplitude that is modified for each dip by a statistical weighting, $w_i = 1$, 2, or 3. The dips are in channels X_i and have widths Γ_i. The parameters of interest Γ_i should be twice the natural width of the line Γ_N, as there is a contribution from the source and from the absorber weightings. These modifications arise because the ^{57}Fe is not infinitely dilute in the absorber. This results in the

preferential absorption at the center of the line changing the shape. An empirical correction[9-11] is

$$\frac{\Gamma}{\Gamma_N}=2.00+0.27T, \tag{3}$$

where the effective thickness T is determined by the fraction f of γ rays absorbed without energy loss, the number density n of iron nuclei, the isotopic fraction a of ^{57}Fe, the maximum resonance cross section σ_0, and the physical thickness t according to

$$T=fna\sigma_0 t. \tag{4}$$

In Eq. (4) the resonance cross section includes a factor $1/(1+\alpha)$, where α is the ratio of decay by internal conversion to γ decay. For the iron absorber the thickness is divided between the states according to $w_i/\Sigma w_i$, resulting in three slightly different widths. This effect in the absorber was corrected for since the experimentally determined parameters confirmed the effect quantitatively for the absorber. The equivalent effect in the source was not corrected for, as it depends upon manufacturing techniques and the age of the source. For our case, this effect is believed to be the reason the measured width is 25% larger than the established natural width. The extracted Γ for the data of Fig. 2, uncorrected for broadening in the source, is $5.93\pm0.19\times10^{-9}$ eV.

IV. $\gamma-\gamma$ COINCIDENCE EXPERIMENT

To complete the verification of the uncertainty principle, the distribution of the nuclear lifetimes needs to be measured. With available fast NIM timing modules one can directly measure the lifetime of the same intermediate level in ^{57}Fe that was examined with the Mössbauer apparatus. As can be seen in Fig. 1, this intermediate level is populated by the emission of a 122.1 keV gamma ray, and subsequently decays to the ground state with the emission of a 14.4 keV gamma ray.

The mean lifetime of the intermediate state is 141 ns, making it very easy to measure directly.[12] The 122.1 keV gamma was measured in NaI(Tl) detector and the 14.4 keV γ ray in a Si(Li) detector. A histogram of the time difference between the two γ rays was recorded with a multichannel analyzer to obtain the exponential decay curve. This curve can easily extend over four or more lifetimes. The lifetime of the 14.4 keV level was found by fitting an exponential to the data (see below).

The time coincidence spectrum is shown in Fig. 3. The rise of the spectrum marks zero time difference between formation and decay, and is not in channel zero due to electronic delays. The delay occurs primarily because of the signal collection time in our Si(Li) detector, and also because the time signals were derived from the slow, microsecond, shaped energy signals. From the fact that the rise is not very steep, it is clear that there is considerable noise on the time signals.

The poor time resolution raises the question as to whether the lifetime of the 14.4 keV level can really be see Fig. 4. The only change in the electronics setup was to open the energy windows to increase the count rate. This

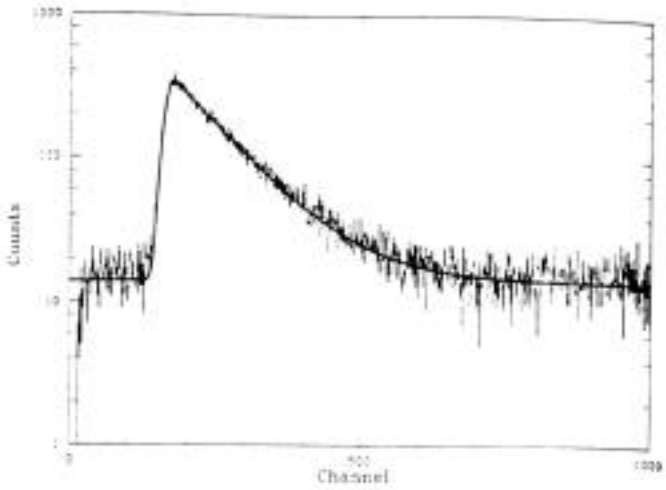


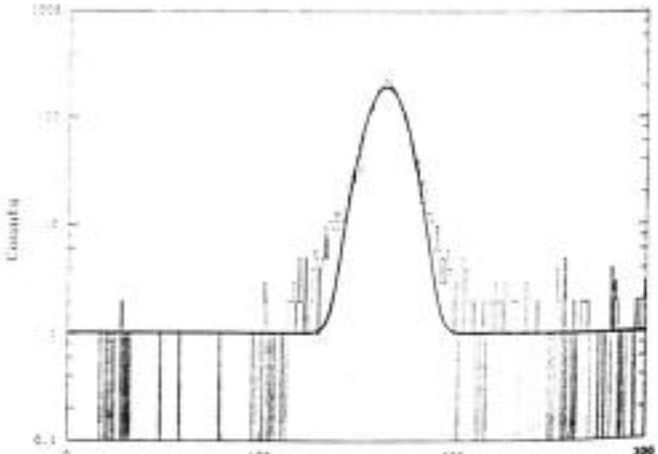
Fig. 3. Time coincidence spectrum for the formation and decay of the 14.4 keV state of ^{57}Fe. The line is a nonlinear least-squares fit to the data with Eq. (5) with a weighted variance of 1.01.

time spectrum was fitted with a Gaussian shape to determine the resolution and yielded $\sigma=12.8$ ns, corresponding to a FWHM = 33.9 ns. Since this time is a substantial fraction of the lifetime of the 14.4 keV state, the ^{57}Fe time spectrum was then fitted with the function

$$Y=A+\frac{B}{e^{(X_0-x)}+1}e^{-t/\tau}, \tag{5}$$

which was convoluted numerically with the Gaussian smearing function using the previously determined σ. The Heaviside function created the rise in the spectrum at channel X_0 corresponding to $t=0$ of the decay. The fit is shown in Fig. 3, and the resulting mean lifetime is $\tau=144\pm1$ ns, which is slightly greater than the accepted value of 141.4 ± 0.5 ns.

It is not necessary to be this fancy. The spectrum resulting from the convolution of a Gaussian and an exponential



squares fit to the data with a Gaussian, $Y=A+Be^{-(x-X_0)^2/2\sigma^2}$, and determines the time resolution of the system.

decay retains its exponential character with the same decay constant if one ignores the channels near $t=0$, which are strongly affected by the resolution. Thus an exponential fit is adequate if one has the strength of character to ignore enough of the early channels, which have such tantalizingly good statistics. The accidental coincidence background must be included in the fit or removed by subtraction. The usual bias from using a linear fit to the logarithm of the data will arise if one chooses the simplified linear analysis after background subtraction.

It was also found that the high intensity ^{57}Co source (5 mCi) needed for the Mössbauer measurement was not appropriate for the coincidence measurement. Basically, the emission rate is so high from the 5 mCi source that it is much more likely that two unrelated gamma rays will be emitted and detected than two γ rays from a single decay. Simply placing the detectors farther from the source to reduce the count rate does not overcome the inherent problem that there is a high probability of random coincident emissions from the source. Ultimately a 10 μCi source was used for the lifetime measurement.

V. SYNTHESIS OF RESULTS AND THE UNCERTAINTY PRINCIPLE

The two numbers Γ and τ are now combined and compared to the accepted value of $\hbar$. For the results shown in Figs. 2 and 3, we obtain $5.93\pm0.19\times10^{-9}$ eV and 144 ± 1 ns, which, when multiplied together, result in $8.54\pm0.28\times10^{-16}$ eV s or $1.37\pm0.04\times10^{-34}$ J s. This result is to be compared with the accepted value of 6.582×10^{-16} eV s. The fact that the quoted error is small compared to the difference between the measured and accepted values is due to the systematic error introduced by not correcting for the broadening of the linewidths by the source (Sec. III). Once again, the key feature is that this uncertainty represents an inherent feature of the quantum nature of the state rather than resulting from the mechanics of measuring numbers. It also serves to reinforce key ideas about the transition from a wave function and probabilities to particular values for energies and times.

VI. PROVOCATIVE QUESTIONS

As the reader and the best students will realize, the discussion presented above is overly simplified. A thorough discussion and understanding of all the implications of the uncertainty principle is definitely an ambitious undertaking. However, as presented, the students are certainly led to a more insightful understanding than they had before the exercise.

To supplement the standard exercise it is stimulating and provocative to pose questions about the experiment as done or about different approaches to the measurements. For example, "What would be the measured energy width in an experiment which simultaneously measures the time and energy if only limited ranges of times were accepted?" "How does the uncertainty principle apply to the entire time-dependent wave function which includes the nucleus, final nucleus, photons, and detectors?" "How does the uncertainty principle relate to conservation of energy in these experiments?" This list is not exhaustive and is easily extended. Questions such as this challenge the students and also serve to illustrate that practicing physicists and teachers do not draw on a set of memorized answers but apply thought and creativity to any process of understanding.

VII. SUMMARY

Students have benefited greatly from the combination of the Mössbauer experiment to measure Γ and the coincidence experiment to measure τ. While the technology, experimental and analysis techniques, and physics insight (into Zeeman and decay processes) from the two experiments is worthwhile, the inclusion of a focus on the uncertainty principle provides an additional educational opportunity for students. The inclusion of open-ended questions also stimulates both the student and the faculty to critical thinking and hallway discussion.

ACKNOWLEDGMENT

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