Physics 133: Extragalactic Astronomy and Cosmology

"That wraps it up -- the mass of the universe."

Week 2 – Spring 2021

Week 1: Review

1. Cosmological models use the same physical laws that we study in the laboratory and are both motivated and tested by observations.
2. Copernican, or Cosmological, Principle.
   - The Universe is the same throughout.
   - **ONLY ON LARGE SCALES ≥ 100 MPC ON SMALL SCALES IT IS NOT!!!**
   - Observations of CMB and galaxy surveys are the same in every direction (isotropic). Since there is no special place, it is the same in every direction from every point, and therefore homogeneous.
3. The Universe is not time invariant. Only the weak form of the cosmological principle applies.

Week 1: Review

- Empirical foundations of the Big Bang theory. I:
  - The night sky is dark.
    - Inconsistent with an eternal, static and infinite Universe
  - The spectra of distant galaxies appear "redshifted" as if the distance between us and them was increasing with time.
    - In the Big Bang theory this is interpreted as due to the expansion of the Universe.
  - The ages of stars (and galaxies) are found to be less than $1/H_0$, consistent with the finite lifetime of the universe.

Week 1: Review

- Empirical foundations of the Big Bang theory. II:
  - The Universe is filled with an almost perfectly isotropic blackbody radiation at a temperature of 2.7 K. This is interpreted in the Big Bang theory as the remnant of an hot state when radiation and matter were in thermal equilibrium.
  - The baryonic matter in the universe is found to have a very regular chemical composition, mostly H, He and tiny amounts of heavier matter. Stars cannot produce all the He. Hence the universe must have been hot enough and dense enough for nucleosynthesis to occur before the first stars formed.

This Week: Theoretical Foundations of the Big Bang Theory

- Gravity is the dominant force on the scale of the Universe
  - Einstein's gravity
  - Generalized metric for isotropic and homogenous space
- The Robertson-Walker metric
  - Proper Distance
  - Cosmological Redshift
- Dynamics of the Universe:
  - Friedmann Equation & The Critical Density

Space-time in special relativity

- In special relativity, space and time are united in a (flat) 4D space-time.
- The metric of this space is?
  - Minkowski's:

    $$ds^2=-c^2dt^2+dr^2+r^2d\Omega^2$$
- A photon travels at the speed of light, $dr/dt = c$.
  - This means photons travel along null geodesics, $ds^2=0$.

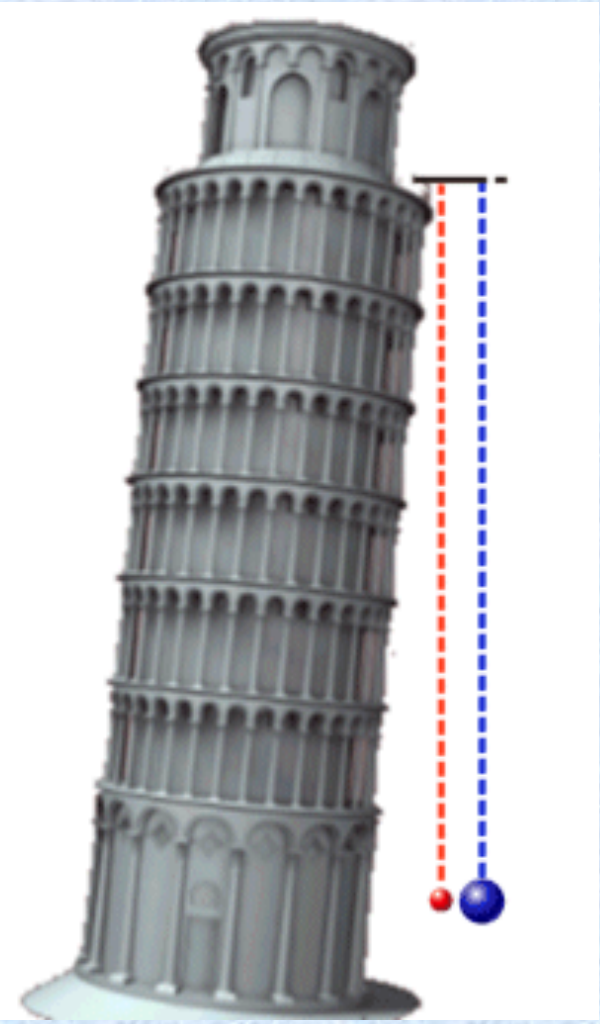
When gravity is added, the permissible space-times are more interesting.

Gravity

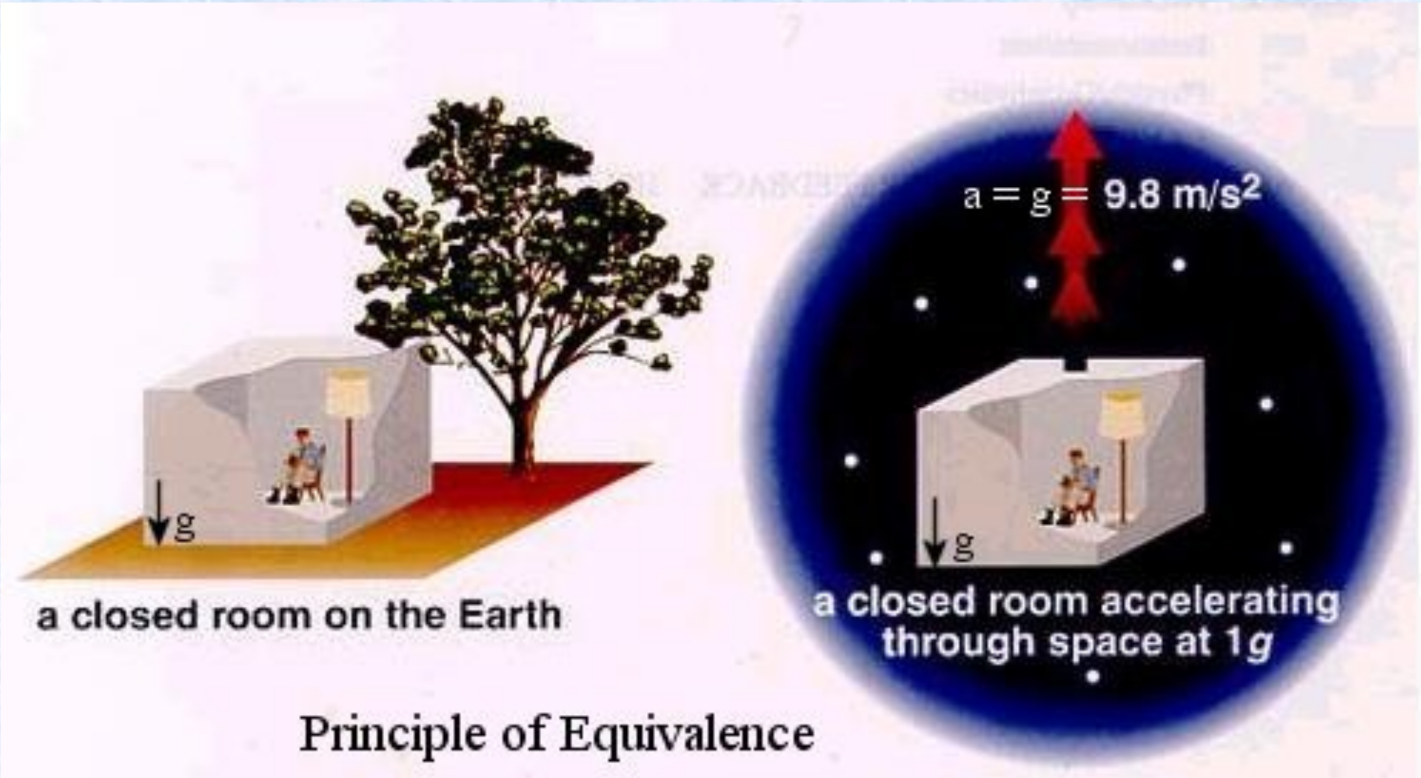
- Gravity is the weakest of the 4 fundamental forces.
  - Compare the ratio of the gravitational and electromagnetic forces between a proton and an electron:
  - $F_G = G\ m_p\ m_e / R^2$
  - $F_{EM} = k\ q^2 / R^2$
  - $F_{EM} / F_G = 10^{39}$
- Gravity is the dominant force on large scales
  - Example of systems dominated by gravity:
    - Universe
    - Black hole

Newton's gravity

- The gravitational force on a body depends on its gravitational mass
  $F = -G M \mathbf{m_g} / R^2$
- The acceleration of body depends on its inertial mass
  $F = \mathbf{m_i}\, a$
- Experiments that measure the accleration due to gravity,
  $a = G M_g / R^2 (\mathbf{m_g/m_i})$,
  find that the inertial and gravitational mass are the same to within 1 part in 10^{12}
- All objects accelerate in the same way on the Earth's surface (Galileo's experiment)
- Space is Euclidian.

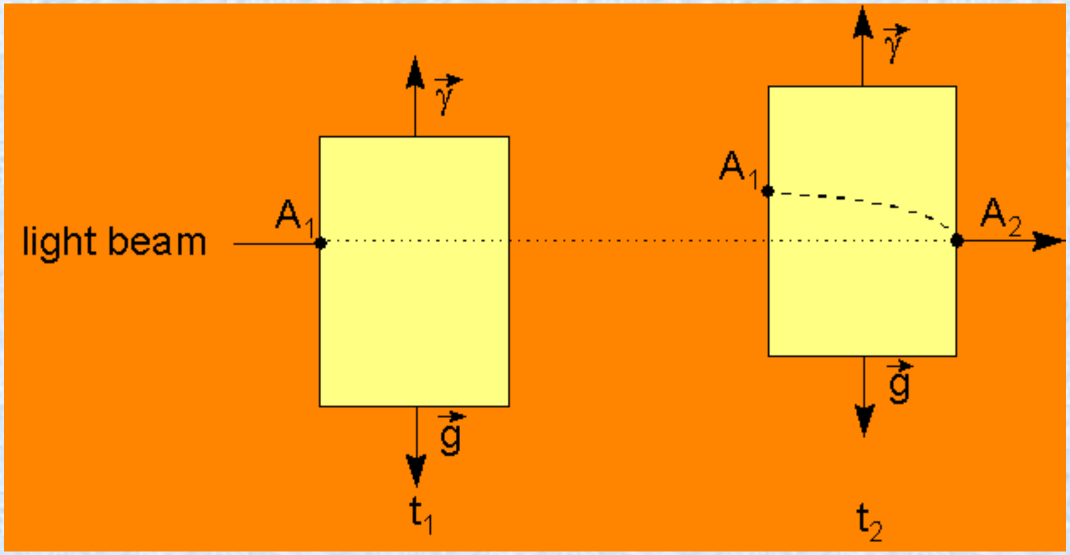


Einstein's gravity. Equivalence principle



Equivalence of inertial and gravitational mass is exact!

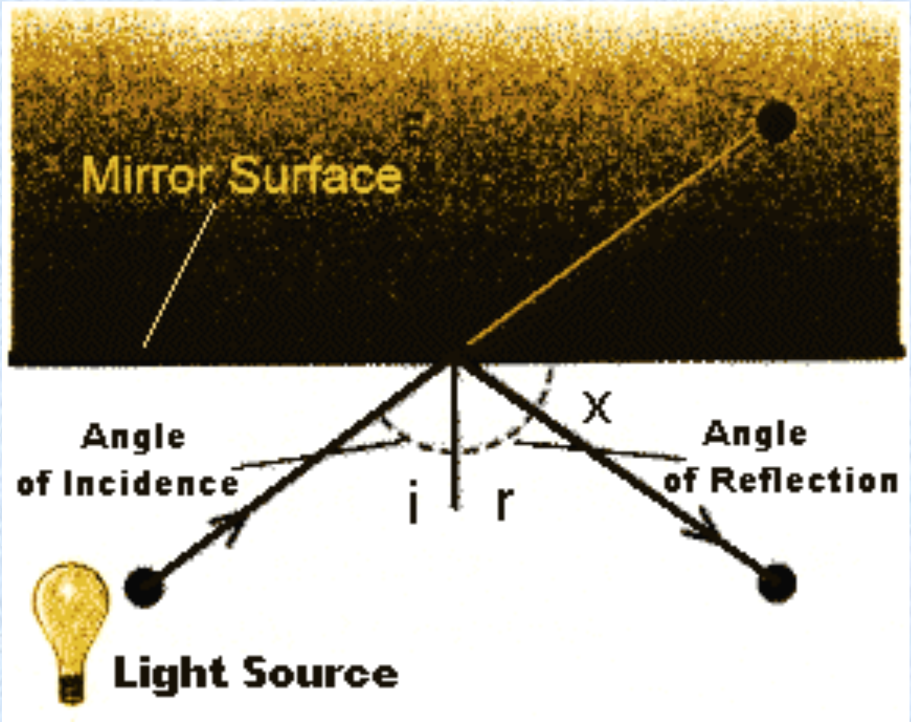
Einstein's gravity. Propagation of light.



•If the elevator is accelerating the beam of light appears to bend as it propagates through the elevator.
•The **equivalence principle** says we can replace the accelerated box by a box experiencing constant gravitational acceleration.
•This means the path of the photons is curved downward in the presence of a gravitational field.

Bonus: Fermat’s principle in Optics

- A fundamental principle of optics is that a light ray travels along the path that minimizes travel time (actually the extrema of the time delay surface)
- In a vacuum, this means light takes the shortest path between two points. In flat Euclidian space these are straight lines.
- What about the accelerating elevator? The path taken by the light is not a straight line.
- ***Aha! Space must not be Euclidean.***

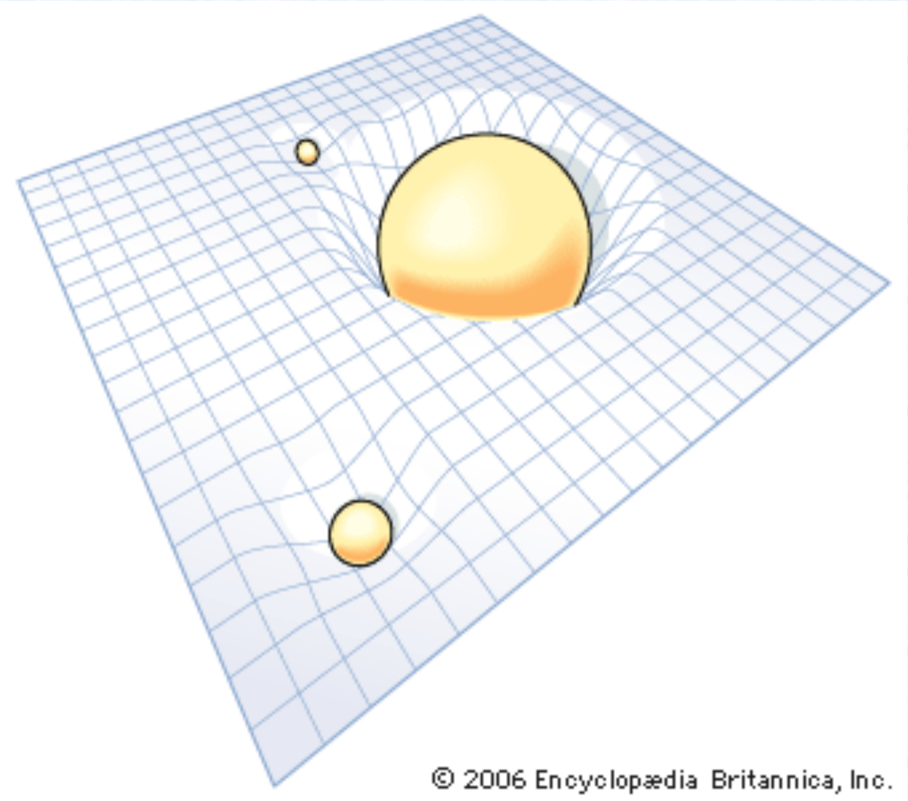


Newton vs Einstein:

- **Newton**: Gravitational mass tells gravity how to exert a force. Force tells inertial mass how to accelerate. Inertial and gravitational mass are the same just by coincidence.
- **Einstein**: Mass-energy tells space-time how to curve. Curved space time tells mass-energy how to move.

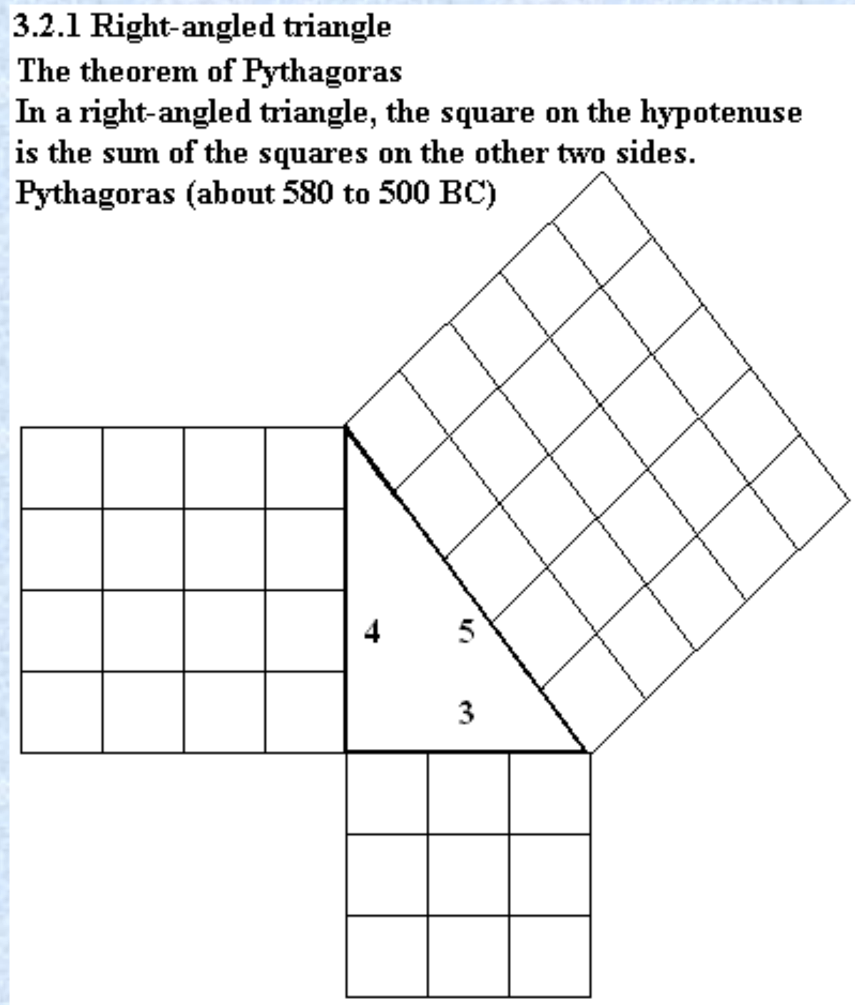
Einstein's gravity. The bottom line.

1. Since light does not travel along straight lines, space is not flat!
2. Light rays follow null geodesics in a curved space.
3. Energy and mass are equivalent ($E=mc^2$).
4. Energy and mass curve space time.



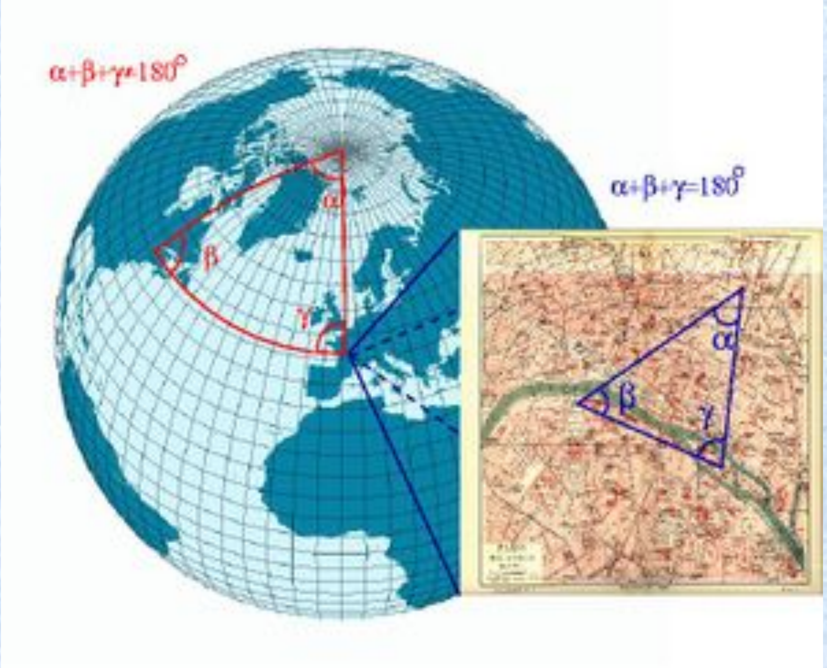
Flat Space or Euclidean Geometry: Examples in 2D Plane

1. In Euclidean geometry. The angles of a triangle add up to π
2. The distance between two points is given by: $ds^2=dx^2+dy^2$
3. In polar coordinates: $ds^2=dr^2+r^2d\theta^2$
4. These expressions are called the metric



Curved Space: Example of Non-Euclidean Geometry in 2D. Sphere.

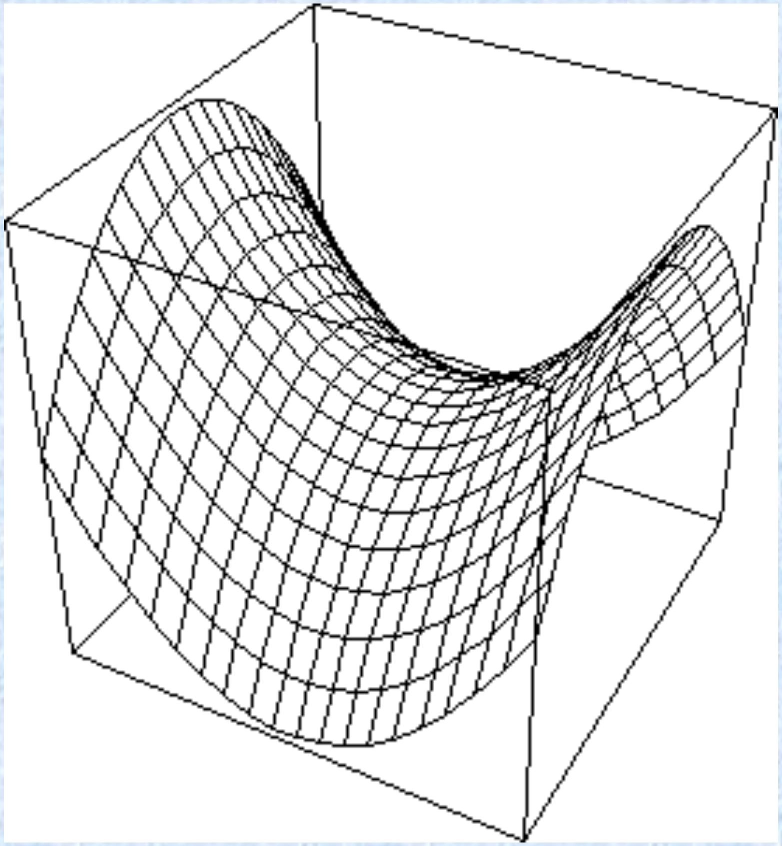
1. On the surface of a sphere, the angles of a triangle add up to $\pi + A/R^2 = \alpha + \beta + \gamma$.
   - Does not depend on the location on the sphere. The curvature is isotropic and homogeneous
   - All spaces where the angles of a triangle add up to more than π, are positively curved



2. On a sphere of radius R, the distance between two points is given by: $ds^2 = dr^2 + R^2 \sin^2(r/R) d\theta^2$. [Define coordinates r and theta.]
3. The surface is finite, and there is a maximum distance

Non-Euclidean Geometry: Example of 2D Space with Negative Curvature. Hyperboloid.

1. On the surface of a negatively curved surface the sum of the angles is $\pi - A/R^2$
2. The distance between two points is given by: $ds^2=dr^2+R^2\sinh^2(r/R)d\theta^2$
3. The surface is infinite, and there is no maximum distance



Let’s build an intuitive picture for the form of the metric that we will use in cosmology.

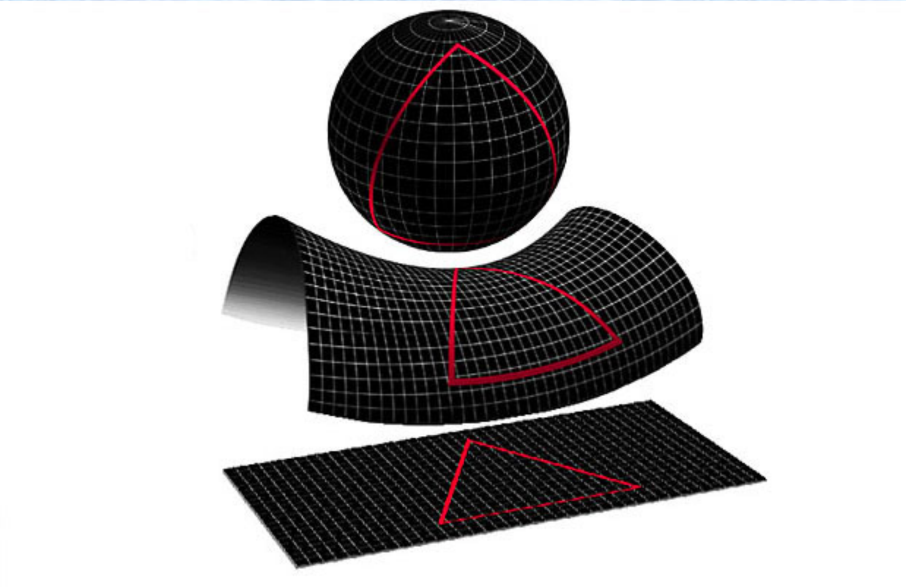
[Blackboard]

Non-Euclidean Geometry: 3D isotropic surfaces.

1. Curvature is a local property. Isotropic and homogenous spaces need to have a constant curvature. Call the curvature constant **κ.**
2. Curvature constant has only 3 possible values:
   - κ=+1 (positive curvature)
   - κ=0 (flat)
   - κ=-1 (negative curvature)
3. All the properties of the surface are described by the sign of κ and the radius of curvature **R**.
4. Distances on a 3D surface are given by **$ds^2=dr^2+S_k(r)^2\, d\Omega^2$, or**

   $ds^2 = dr^2+S_k(r)^2\,(d\theta^2 + \sin^2\theta\, d\phi^2)$.
   - *How does the spatial 'metric' for a 3-sphere (hyper-sphere) differ from a 2-sphere?*

2D analogs: for a sphere $ds^2=dr^2+R^2\sin^2(r/R)d\theta^2$.



$$S_\kappa[r] = \begin{cases} R\sin(r/R) & (\kappa=+1) \\ r & (\kappa=0) \\ R\sinh(r/R) & (\kappa=-1) \end{cases}$$

Space-time in general relativity. Homogeneous and isotropic universe

- **If the universe is homogenous and isotropic at all time, and distances are allowed to expand** (or contract) as a function of time, then we can separate the time part of the metric from the space part of the metric. [Again, on scales > 100 Mpc.]
- Howard Robertson and Arthur Walker first realized, independently in fact, that these conditions allow just **three possibilities for the curvature of space** -- flat everywhere, positive curvature everywhere, or negative curvature everywhere. The **Robertson-Walker** metric can be written in a concise form.

Space-time in general relativity. Properties of RW metric

$$ds^2 = -c^2dt^2 + a(t)^2\left(dr^2 + S_\kappa(r)^2(d\theta^2 + sin^2\theta d\phi^2)\right)$$

- The RW metric describes the universe over large scales (100 Mpc or more).
- κ describes the geometry
  - Positive curvature (hyper-sphere)
  - Negative curvature (hyper-hyperboloid)
  - Zero curvature (flat Euclidean space)
- The spatial variables (x, theta, phi) or (r, theta,phi) are called **comoving coordinates** of a point in space. They remain constant in time for an objects at rest, i.e. those not acted upon by perturbative forces. The distance between objects at rest increases as a(t) due to the Hubble flow.

Robertson-Walker Metric
Homogeneous and isotropic universe

$$ds^2 = -c^2dt^2 + a(t)^2\left(dr^2 + S_\kappa(r)^2(d\theta^2 + sin^2\theta d\phi^2)\right)$$

- Space component is a hyper-sphere, hyper-hyperboloid or a flat Euclidean space (κ and R) up to a constant scalar factor a(t) that depends on time.
- The time variable is **cosmological proper time, or cosmic time**, and it is measured by an observer who sees the universe expanding uniformly, i.e. an observer at rest with respect to the Hubble flow.
- The spatial variables (x, theta, phi) or (r, theta,phi) are called **comoving coordinates** of a point in space. They remain constant in time for an objects at rest, i.e. those not acted upon by perturbative forces.

Space-time in general relativity. The RW metric and the Universe

$$ds^2 = -c^2dt^2 + a(t)^2\left(dr^2 + S_\kappa(r)^2(d\theta^2 + sin^2\theta d\phi^2)\right)$$

- The kinematics of the universe is described by a(t). As we will see, one can write dynamical equations for a(t) and solve them, thus reconstructing the past and future of the universe.
- The distance between objects at rest increases as a(t) due to the Hubble flow.
- We know the local derivative of a(t). What is it?

The Hubble constant!

RW Metric: Distances in the Universe. Proper distance

- The proper distance is the distance between two sets of comoving coordinates at a given cosmic time.
- This is given by the spatial part of the metric at fixed a(t)
- From the RW metric

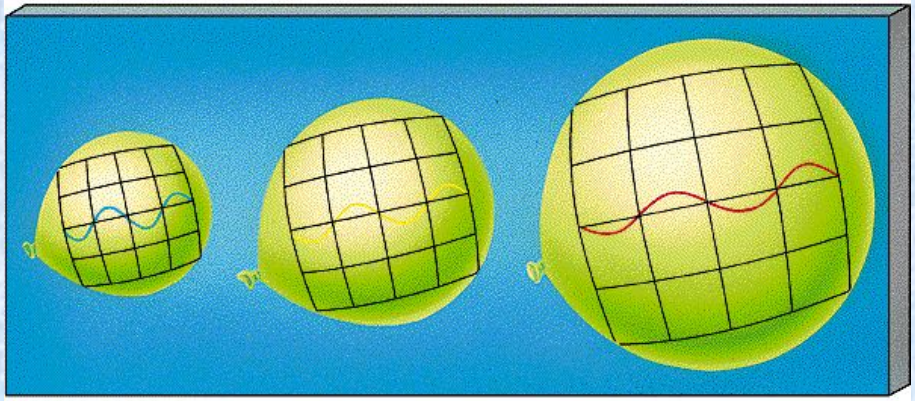
  $d_p = a(t)\, r$, where r is the comoving distance to the object
- When are proper distance and comoving distance equal?
- What is v_p? [Blackboard]

RW Metric: Propagation of Light. Scale Factor & Cosmological Redshift

- Light travels along null geodesics: $c^2dt^2=a(t)^2dr^2$

- Imagine a wave of light. As time goes by, between one crest and another, the universe expands. So that the distance between wave crests appears longer to the observer.

- Light is redshifted! [Blackboard]



Cosmological Redshift

$$\lambda_0/\lambda_e = a(t_o)/a(t_e) = 1 + z$$

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Summary: Robertson-Walker Metric

$$ds^2 = -c^2 dt^2 + a(t)^2 \left(dr^2 + S_\kappa(r)^2 (d\theta^2 + sin^2\theta d\phi^2) \right)$$

$$S_\kappa[r] = \begin{cases} R\sin(r/R) \ (\kappa=+1) \\ r \quad (\kappa=0) \\ R\sinh(r/R) \ (\kappa=-1) \end{cases}$$

- The curvature of the (homogeneous, isotropic, and expanding) universe is described completely by the curvature constant κ, the radius of curvature at the present moment $R_0 = R(t_0)$, and the scale factor a(t).
- We define $a(t_0) = 1$, so $R(t) = R_0\, a(t)$

Quiz 3

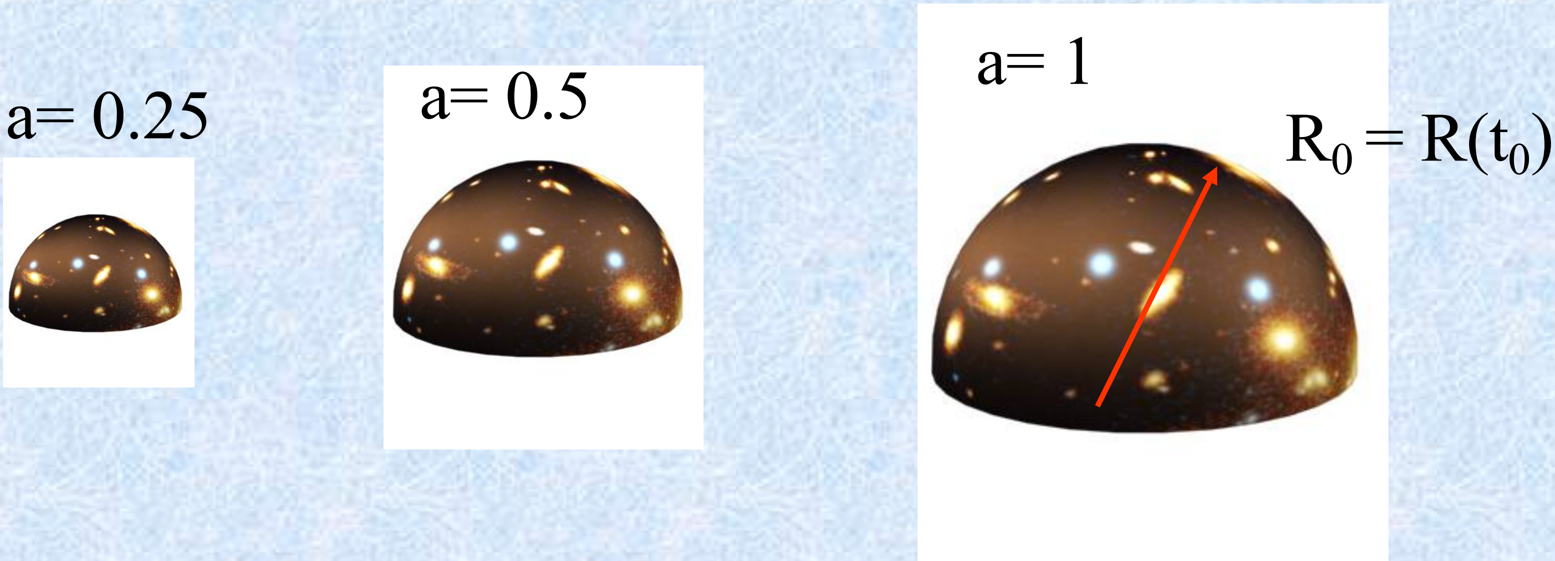
A team of astronomers find a quasar. They measure a redshift of $z = 2.0$ from its spectrum.

What was the scale factor $a(t_e)$ at the time t_e when the quasar emitted the photons that we observe today?

Recall:

We define the scale factor of the universe such that $a(t_0) = 1$ today.

Summary:
Relationship between scale factor and redshift



The past ⟶ Today is t_0

Cosmological Redshift:

$\lambda_0/\lambda_e = 1 + z$ **(Observational Definition)**

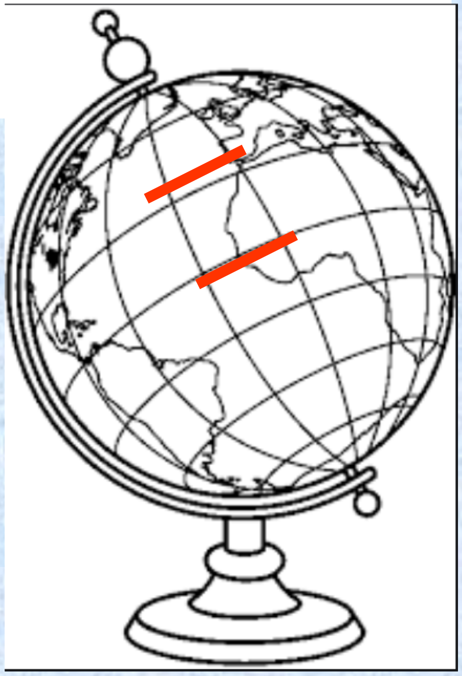
$\lambda_0/\lambda_e = a(t_o)/a(t_e)$ **(Follows from RW Metric)**

So we have $a(t_e) = (1 + z)^{-1}$ **because** $a(t_o) = 1$

RW Metric. Direct limits on the geometry of the Universe. 2.

- If the universe is positively curved, the radius cannot be much smaller than the Hubble Length, otherwise photons would have had time to go around the surface in circles and we would see periodical images, i.e. $c\,(1/H_0) < 2\pi R$

- We could in principle measure the curvature constant by drawing a big triangle and measuring the angles, and the radius of curvature by measuring the area of the triangle.



- **HW#2 R 3.3 – Explores measuring curvature by comparing the circumference and radius of circles.**

- **HW#2 R 3.2 – Explores how curvature changes the angular size of a ruler.**

Please Read 4.1 and 4.2 for Wednesday.

Redshift tells you distance.

But how does that work?

- What is the distance to the objects in the Hubble Ultra-Deep Field?
- Given the redshift, which is easy to measure, we can infer $a(t_e)$ at the time the light was emitted.
- Once we find a model for $a(t)$, the redshift will give us the distance(s)

Distances in the Universe: A number of definitions are useful

- What is the distance between two objects in a RW metric?

$$ds^2 = -c^2dt^2 + a(t)^2\left(dr^2 + S_\kappa(r)^2(d\theta^2 + sin^2\theta d\phi^2)\right)$$

- In an expanding universe, the distance between objects is increasing with time. If we want to assign a proper distance between two objects, we must specify the time at which the distance is the correct one.
- There are several kinds of distance. e.g.:
  - Proper distance
  - Luminosity distance
  - Angular size distance

Dynamics of the Universe. Newtonian Analogy to the Friedmann Equation

- **The dynamical evolution of the universe is described by a(t).**
- Einstein's field equations link the geometry of the universe **(κ,R_0,a(t))** to the contents of the universe **(ε(t) and P(t))**.

- **Newtonian analog [blackboard]**
  - Poisson's equation is the closest Newtonian analogy.

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho(t) = -\frac{2C}{a(t)^2 R_s^2}$$

  - **What determines the fate of these model universes?**

- What are the shortcomings of this approach?

Dynamics of the Universe. Friedmann Equation

- **Friedmann Equation** (1922) provided the relativistic form. This was a full 7 years before Hubble's Law was discovered. Einstein did not accept the expanding universe until Hubble made his discovery.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{\kappa c^2}{R_0^2 a^2}$$

vs.

Newtonian Friedmann Equation:

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho(t) = -\frac{2C}{a(t)^2 R_s^2}$$

(1) Energy instead of mass.

Rest-mass contributes most of the energy-density of non-relativistic particles

$\rho \Longrightarrow \varepsilon / c^2$

(2) Curvature instead of internal energy.

$2\,C / r_s^2 \Longrightarrow \kappa c^2 / R_0^2$

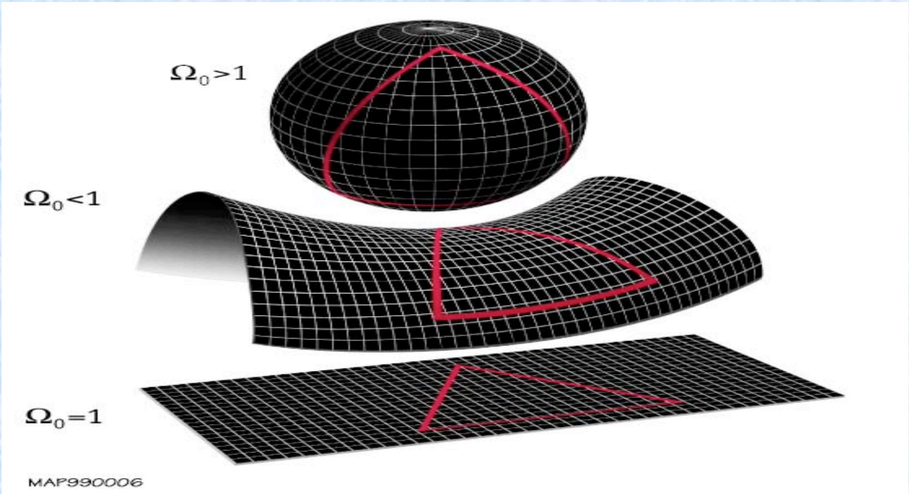
(3) Expansion rate: $H(t) = da(t)/dt\ /\ a(t)$

Dynamics of the Universe: Critical Density

- The Friedmann equation tells us how the curvature of space at any time is related to the mass-energy density of the contents of the universe at that time.
- **The critical density is the amount of stuff that makes the geometry of the universe flat.**
- By convention, a subscript "0" indicates the value of a time-varying quantity at the present.

$H_0 = H(t_0)$ = 71 km/s/Mpc

$\rho_{0,c}(t) = \rho_c(t_0)$ = 9e-27 kg/m3 = 9e-30 g/cm3 = 1.4e11 Msun/Mpc3



$$H^2 = \frac{8\pi G}{3}\rho_{crit}$$

Dynamics of the Universe: Density Parameter

$$\Omega \equiv \frac{\rho}{\rho_c} = \frac{8\pi G}{3H^2}\rho$$

- It is often convenient to use not the energy density, but the ratio of the energy density to the critical density. **This ratio is called the density parameter, $\Omega = \varepsilon(t) / \varepsilon_c(t)$**
- In a negatively curved universe, an empty universe produces minimum curvature, i.e. $R_0 = c / H_0$; so R_0 must be greater than the Hubble distance.
- The Friedman equation can be written in terms of the density parameter. **Notice that the curvature can't change sign!** If you know the density parameter at any time, you know the sign of the curvature.

Quiz #4: What is the Radius of Curvature for an Empty Universe? **[Assume H_0 = 70 km/s/Mpc; express your answer in Mpc.]**

Why is the Friedmann equation the starting point?

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{\kappa c^2}{R_0^2 a^2}$$

How can you simplify this for an *empty* universe?

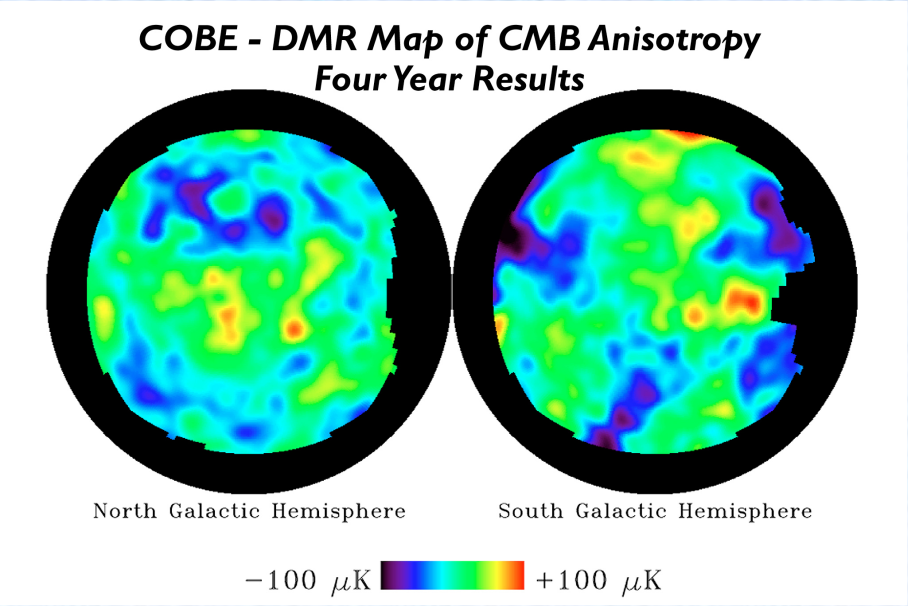
If the universe is *empty* that means $\varepsilon = 0$.

Solve for the radius of curvature of an empty universe.

We know the universe must have some mass. If space is negatively curved, then the radius of curvature must be greater than the Hubble distance.

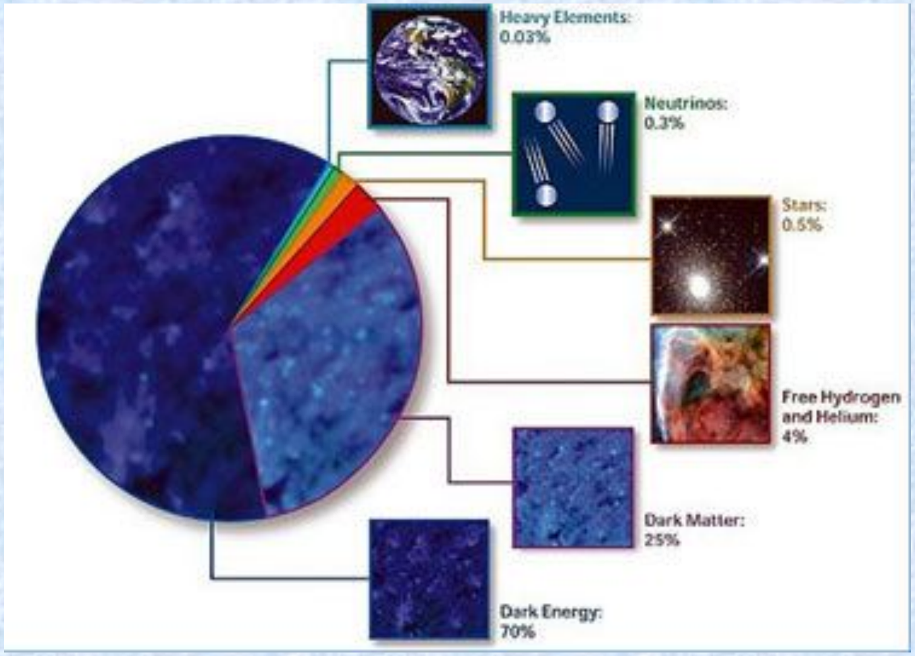
Matter density of the Universe.
1: Radiation

- Blackbody: ρ_{rad}= $a_{rad}\, T^4/c^2 = 4\, \sigma\, T^4/c^3$
- Where c is the speed of light,T is the temperature, σ is the Stefan-Boltzmann constant), 5.67e-8 W m^{-2} K^{-4}
- So ρ_{rad} = 4.6e-31 $(T/2.725K)^4$ kg/m^3 = 4.6e-34 g/cm^3



density of the Universe.
1: Radiation in critical units

- It is convenient to write this down in terms of the critical density, the amount of energy/matter needed to “close” the universe
- Defined as:
  - $\rho_{crit} = 3H_0^2/8\pi G$
  - $= 9.5e\text{-}27\ kg/m^3$ or $9.5e\text{-}30\ g/cm^3$
- The density of radiation is $4.8e\text{-}5\ \rho_{crit}$
- This can be written as $\Omega_{rad} \sim 5e\text{-}5$

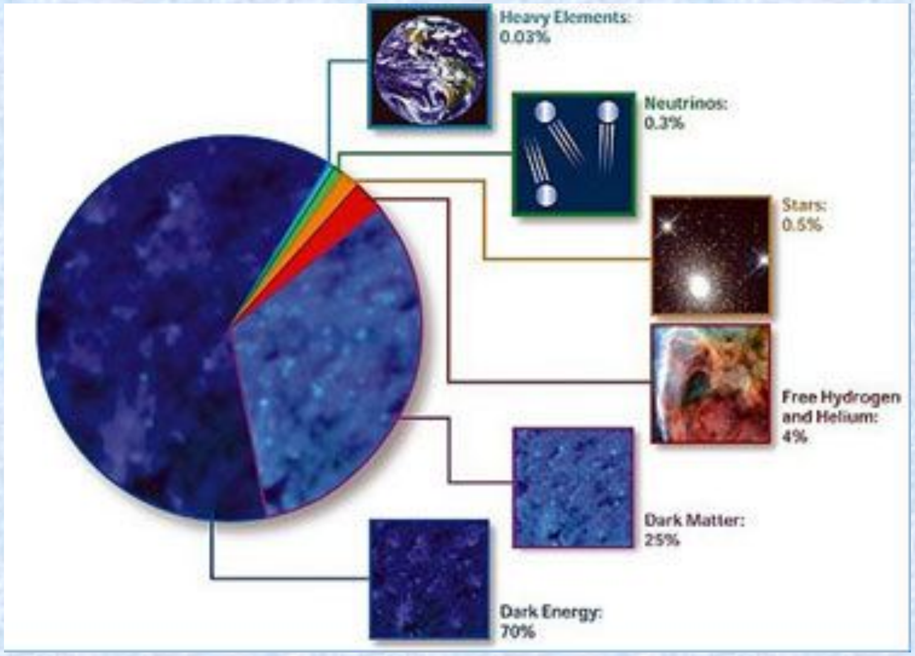


density of the Universe. 2: Neutrinos

- Limits on neutrino mass density come from:
  - -Oscillations (lower limit; superkamiokande)
  - -large scale structures (upper limits; CMB+2dF; Sanchez et al. 2006)
  - - cosmic rays striking atmosphere

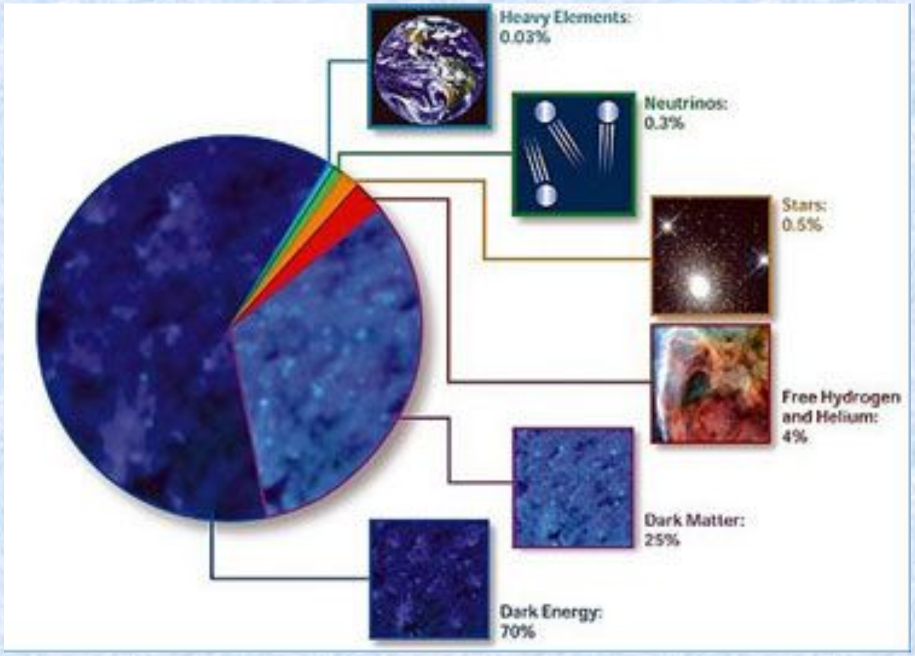
- In critical units neutrino mass density is between:

  $0.0010 < \Omega_\nu < 0.0025$



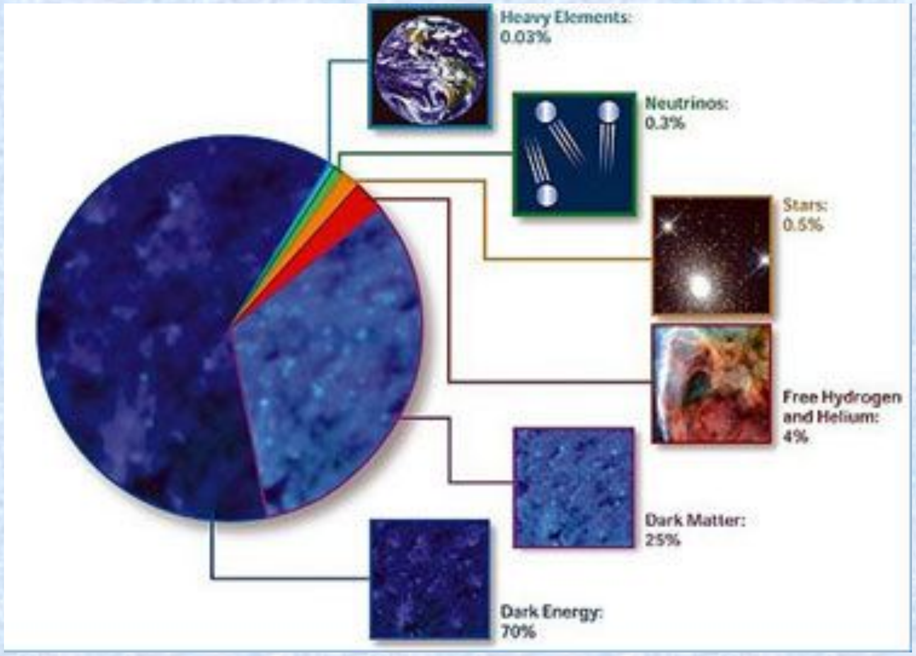
density of the Universe.
3: Baryons

- Count the amount of mass in visible baryons; it is small.
- Baryonic inventory, Ω_b=0.045 (0.003), from primordial nucleosynthesis:
  – Stars Ω_*=0.0024+-0.0007 (comparable mass in neutrinos and stars!)
  – Planets Ω_{planet}~10^{-6}
  – Warm intergalactic gas 0.040+-0.003
- Most of baryons are in intergalactic medium, filaments in the cosmic web.



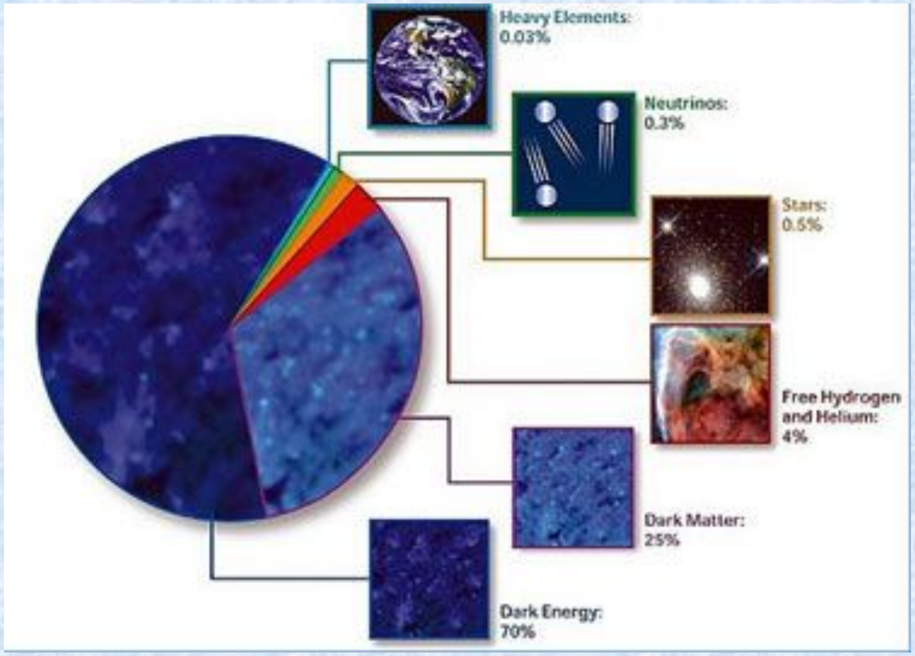
Matter density of the Universe. 4: Dark matter

- Dark matter is harder to count, because we can only "see" it via its gravitational effects
- One way to count it is for example is to measure the dark matter to baryon ratio in clusters
- Assume that this number is representative of the Universe because the collapsed volume is large
- Take the fraction of baryons (from BBN) and multiply
- This and other methods give $\Omega_{dm}=0.23$
- The total amount of matter is given by: $\Omega_m=\Omega_{dm}+\Omega_b=0.27$



density of the Universe.
5: Dark energy (or Λ)

- As we will see most of the energy in the universe appears to be of a mysterious form called dark energy
- Dark energy repels instead of attracting, and therefore causes the expansion of the universe to accelerate.
- One form of dark energy is the cosmological constant (Λ), introduced by Einstein a long time ago, and this is a purely geometrical term… We will explain this later
- According to current measurements Ω_{de}~0.72 or Ω_{Λ}~0.72.



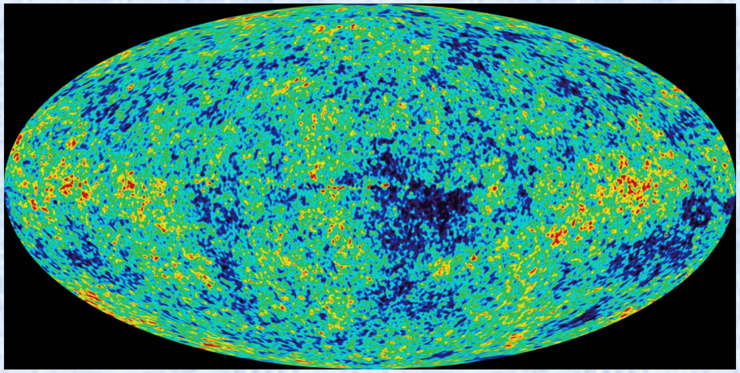
Measuring Ω_0 and H_0 Determines Curvature

[blackboard]

1) Measure Ω, then you know κ.
2) Measure Ω_0 and H_0, then you know R_0.
3) So you can write the Friedmann equation without R_0 and κ.

$H_0^2\,(1-\Omega_0) = -\kappa\, c^2 / R_0^2$

Curvature (R_0, κ) or equivalently Ω_0 and H_0 can be measured from the angular size of 'standard rulers.'



End Week 2

- You should have read through the end of chapter 4.
- Homework due Friday by 5 pm.
- Start reading chapter 5.