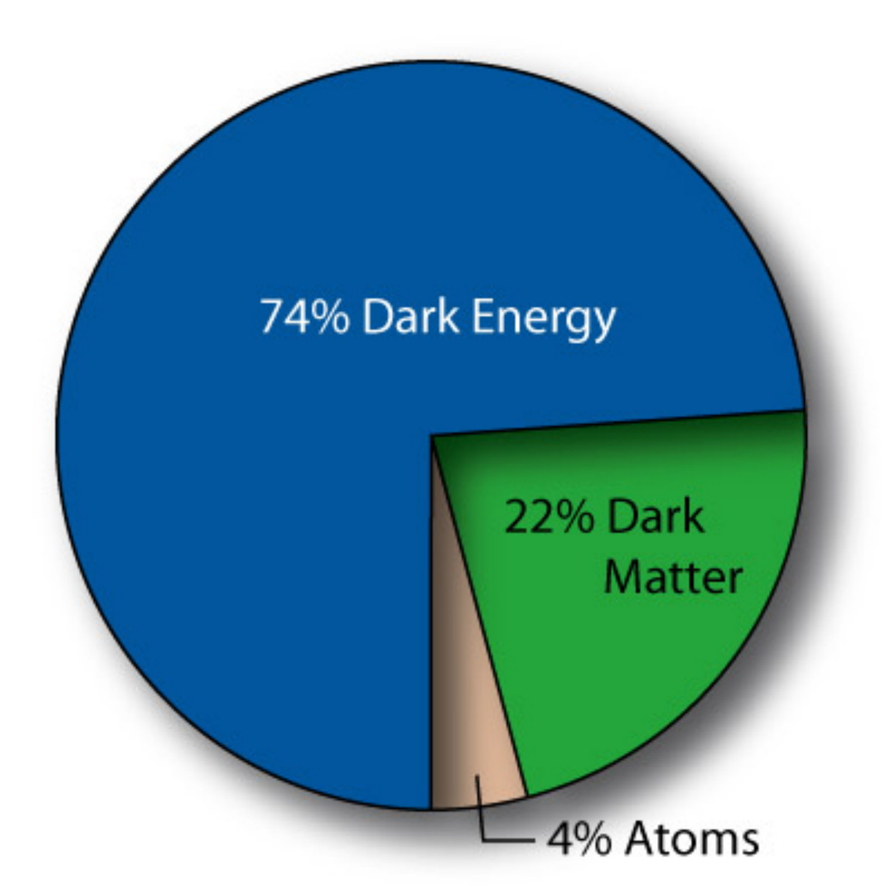
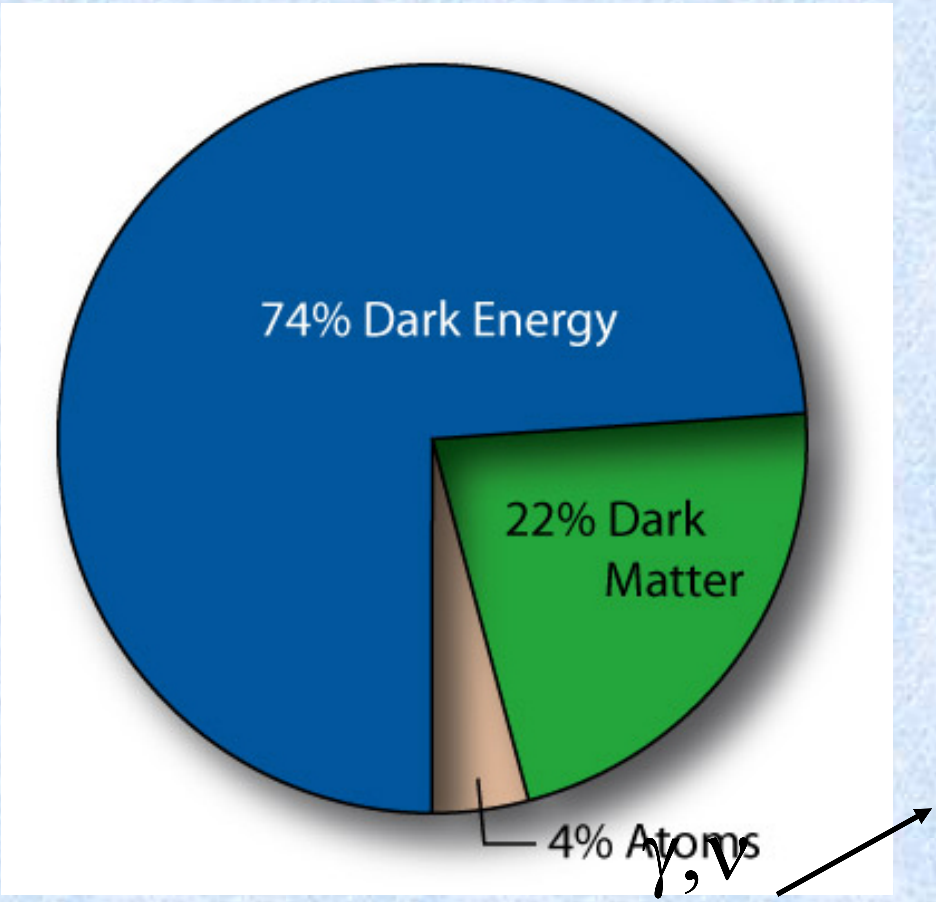
Physics 133: Extragalactic Astronomy and Cosmology



Week 3 – Where we solve the Friedmann Equation.

Review: Density Parameter



$$\Omega_x(t) = \varepsilon_x(t) / \varepsilon_{crit}(t)$$

Or using a(t), we have

$$\Omega_x(a) = \varepsilon_x(a) / \varepsilon_{crit}(a)$$

- The energy density has many components.
- $\Omega_{m,o} = 0.3$, $\Omega_{\Lambda,0} = 0.7$, and $\Omega_\gamma = 8 \times 10^{-5}$
- Dark energy dominates today.

Week 3 Outline

- Review: Why do Ω_0 and H_0 Determine Curvature, κ and R_0?
- Evolution for each component, $\varepsilon(a)$
  - Equation of State for the Components of the Universe
- The acceleration equation
  - Derive the Acceleration Equation
  - Motivation & Introduction to Dark Energy
  - Einstein's Static Universe & the Cosmological Constant
- Solutions to Friedmann's Equation.
- *Single-component models*
  - *Time - Redshift Relation*
  - *Proper Distances*
  - *Horizon*
- *Multi-component models*

Modeling the Universe: Friedmann Eqn., Fluid Eqn., and E.O.S.

- What is the connection between a(t) and ε(t) for any constant w?
- Given appropriate boundary conditions, we can **solve these equations for a(t), ε(t), and P(t)** for all times.
- Fluid equation tells us how **ε(t)** evolves with the expansion described by **a(t)**

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{\kappa c^2}{R_0^2 a^2}$$

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

$$P = w\epsilon$$

The Fluid Equation

$$\dot{\epsilon} + \frac{3\dot{a}}{a}(\epsilon + P) = 0$$

… Derive this statement of energy conservation (the Fluid Equation).

$$P = w\epsilon$$

… Show that this is a general equation of state.

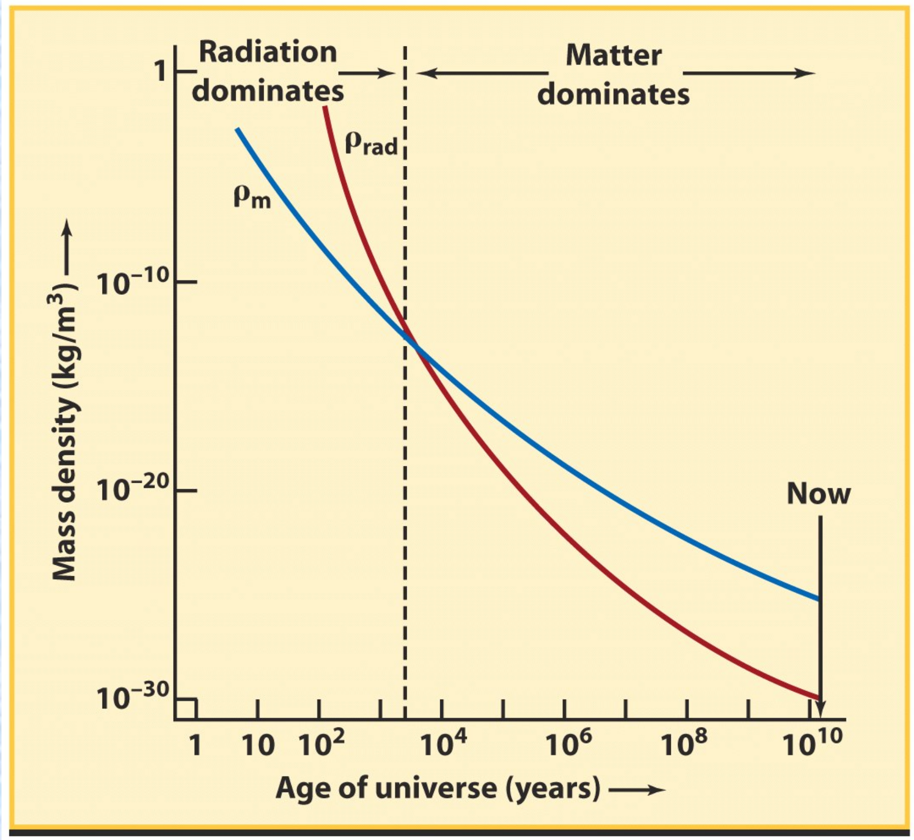
$$\frac{\dot{\epsilon}}{\epsilon} = -3(1+w)\frac{\dot{a}}{a}$$

$$\epsilon = \epsilon_0 a^{-3(1+w)}$$

Together, they tell us how the energy density of a specific component of the universe evolves as the scale factor changes.

What kind of Universe? Who rules?

- Energy density in matter is much larger today than that in CMB photons and neutrinos
- Depending on w, energy density evolves at different rates
- The dominant species changes with time. **Radiation was once the dominant component.**
- Set $\varepsilon_{rad}(a_{rm}) = \varepsilon_m(a_{rm})$, and solve for a_{rm}.



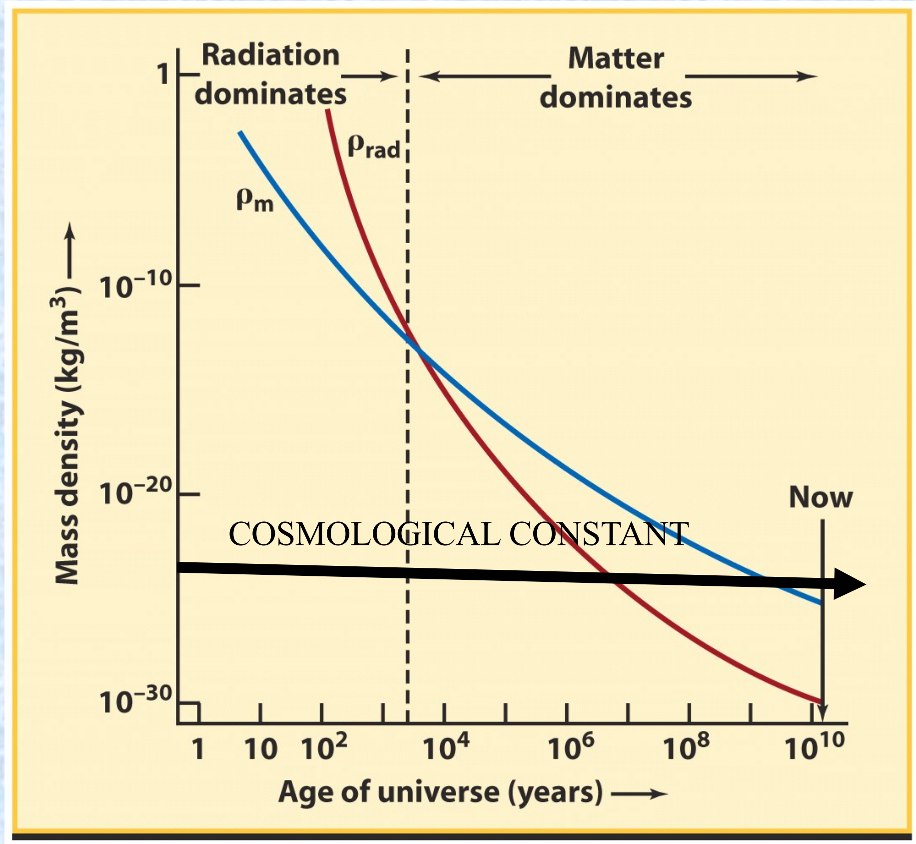
Could you find the redshift a_{rm} where these curves crosss?

When would the cosmological constant be the dominant energy density?

- The cosmological constant dominates the energy density in Friedmann's equation today. We have

  $\varepsilon_{\Lambda,0} / \varepsilon_{m,0} \sim 0.7 / 0.3 = 2.3.$

- The energy density of the cosmological constant is time independent.
- So dark matter dominated at some point in the past. Why?

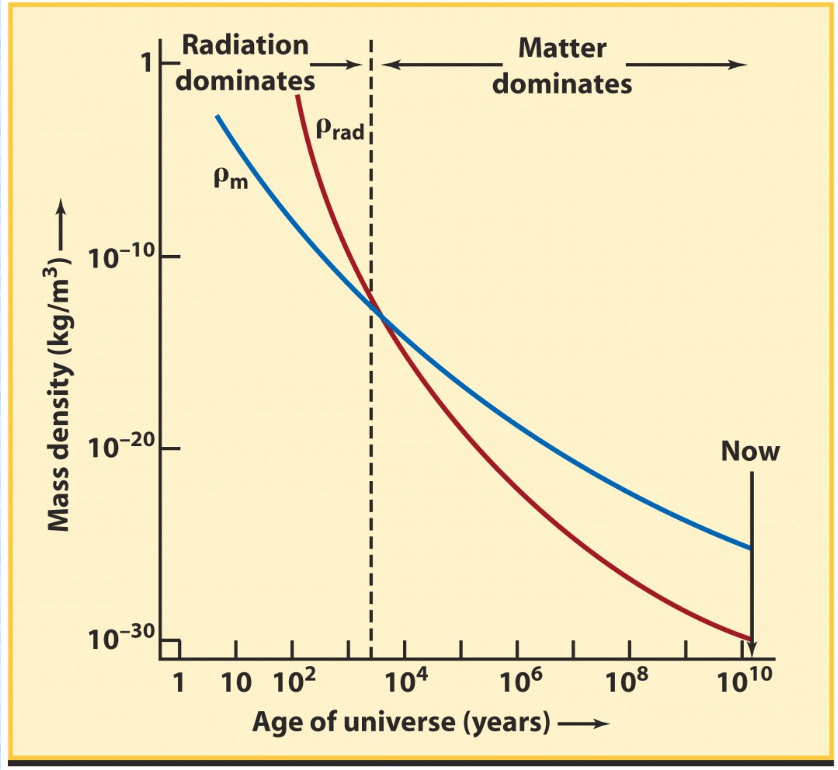


Could you calculate the scale factor, $a_{\Lambda m}$, when $\varepsilon_\Lambda(a_{\Lambda m}) = \varepsilon_m(a_{\Lambda m})$?

Quiz #5

- Using the following parameters for the concordance cosmology,
- $\Omega_{m,o} = 0.3$ **where** ($\varepsilon_{0,m} = \Omega_{m,o}\,\varepsilon_{crit,0}$)
- $\Omega_{\Lambda,0} = 0.7$, **and**
- $\Omega_{\gamma} = 8 \times 10^{-5}$,

answer the following questions.



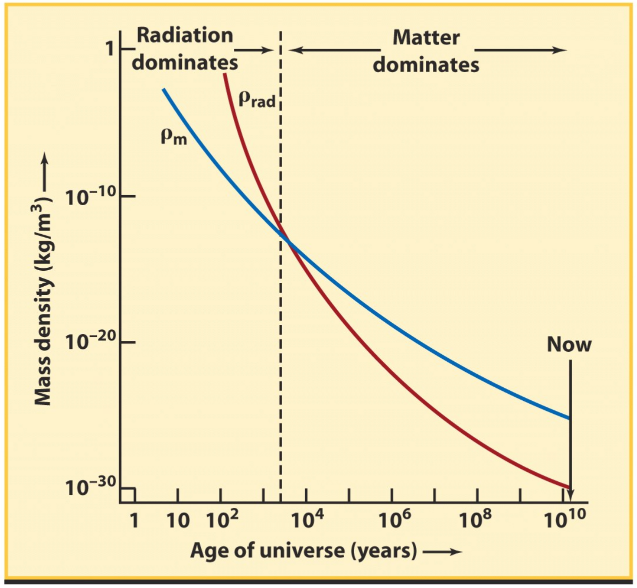
1. Find the redshift of matter-radiation equality.

2. Find the redshift where dark energy became the dominate form of the energy density.

Quiz #5

- Using the following parameters for the concordance cosmology,
- $\Omega_m = 0.3$,
- $\Omega_\Lambda = 0.7$, and
- $\Omega_\gamma = 8 \times 10^{-5}$,

answer the following questions.



1. Find the redshift of matter-radiation equality.

   (Check Yourself)
   Answer: $1 + z_{m\gamma} = 3600$

2. Find the redshift where dark energy became the dominate form of the energy density.

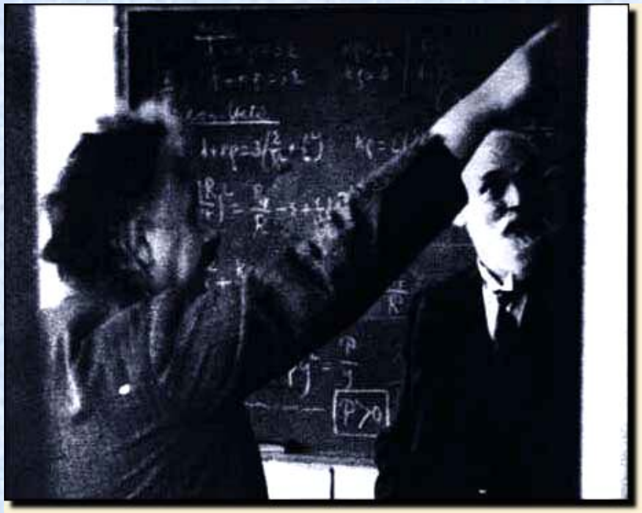
   (Check Yourself)
   Answer: $z_{m\Lambda} = 0.32$

Einstein's Dilema & The Cosmological Constant

- Soon after the completion of general relativity (1916) people used it to describe the universe.
- However, with only matter there was no way to obtain a static solution, which at that time was the prejudice.
- Einstein added the cosmological constant …

  $\Lambda = 4\pi G\rho$

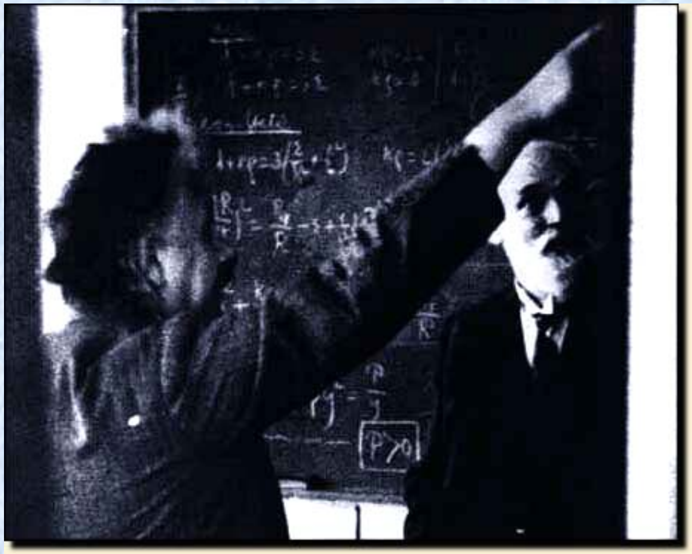
  … to his equations to find a static solution.



$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3c^2}\epsilon(t) = -\frac{\kappa c^2}{a(t)^2 R_0^2} + \frac{\Lambda}{3}$$

Even Einstein Makes Mistakes

- The static solution is unstable.
- The required value of the cosmological constant in a static universe is difficult to understand.
- And, when Hubble announced his discovery of the expansion, Einstein's cosmological constant became unnecessary



- *So the cosmological constant remained on the outskirts of cosmology for a long time.*
- *Now its back because astronomers measure a positive acceleration.*

Dynamics of the Universe. Acceleration equation

- Combine the Friedmann equation and the fluid equation to find out how a(t) changes with time. This convenient form (not independent) is the equation of motion with the second derivative.
- [Blackboard]

Announcements

- We have covered through [R] 5.1.
- Read 5.2 end 5.3 for Wednesday.
- Make sure you answer the questions from the 2nd edition; some people did the wrong problems on HW 1.

- Start HW 3
  - Problem 1 – [R] 4.2
  - Problem 3 - [R] 5.6
    - typo in textbook - should say $-1 < w_q < -1/3$

- A note on academic integrity.
  - Looking at solutions from a previous year is <u>cheating</u>.
  - *Cheating will be reported to the administration as well as resulting in a failing grade for the class.*

Week 3 Outline

- *Review: Why do Ω_0 and H_0 Determine Curvature, κ and R_0?*
- *Evolution for each component, $\varepsilon(a)$*
  - *Equation of State for the Components of the Universe*
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  - *Motivation & Introduction to Dark Energy*
  - *Einstein's Static Universe & the Cosmological Constant*
- **Solutions to Friedmann's Equation.**
- **Single-component models**
  - **Time - Redshift Relation**
  - **Proper Distances**
  - **Horizon**
- **Multi-component models**

Dark Energy

[Note: Includes cosmological constant w = -1]

- **Inspection of the Acceleration Equation**
  - Increasing the mass-energy density slows down the expansion rate of the universe.
  - A positive pressure also slows down the expansion.
  - A universe with $P < - \varepsilon / 3$ will accelerate
- **A component with $w < - 1 / 3$ is called dark energy.**
  - It causes the universe to accelerate (see acceleration eqn.).
  - It has constant energy density during the expansion (see the fluid eqn.).
- A cosmological constant has $P = - \varepsilon$ and is one type of dark energy, which means $w = -1$.

Cosmologists talk in terms of dark energy, instead of cosmological constant, because we don't know exactly how close w is to -1.

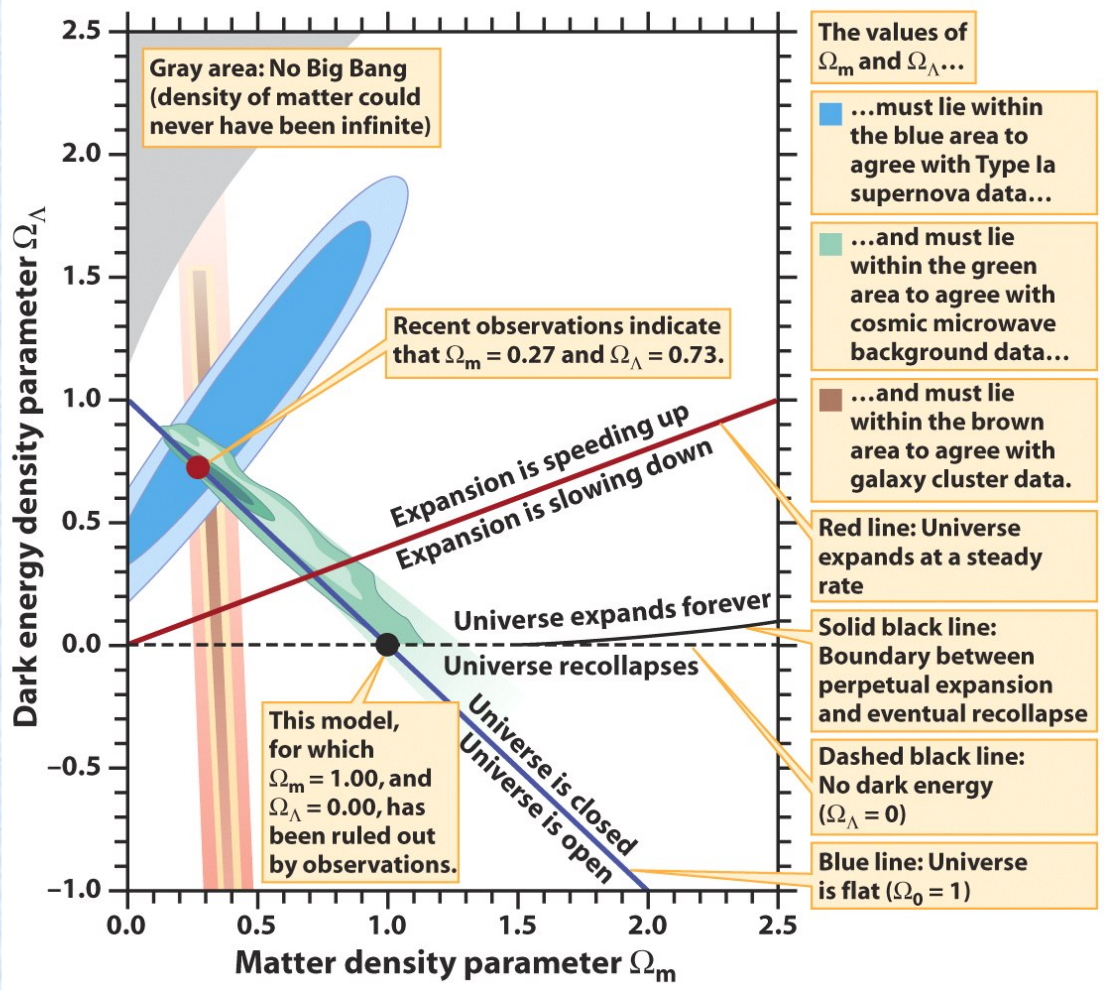
What is Dark Energy?

A negative pressure is permissible by the laws of physics. e.g. compress/stretch a piece of rubber.

- Dark energy is interpreted as something with negative pressure filling space.
- Quantum field theory predicts a value of ε_{vac} 124 orders of magnitude larger than that measured by astrophysicists.
- Is it some sort of vacuum energy?
- *We really don't know. But whatever it is, it dominates the dynamics of the universe today!*

"Concordance cosmology" or "Benckmark model"

- Our current best guess
- Photons and neutrinos
- Matter (baryonic and dark)
- Cosmological Constant
- Spatially Flat



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Generalized Friedmann Equation

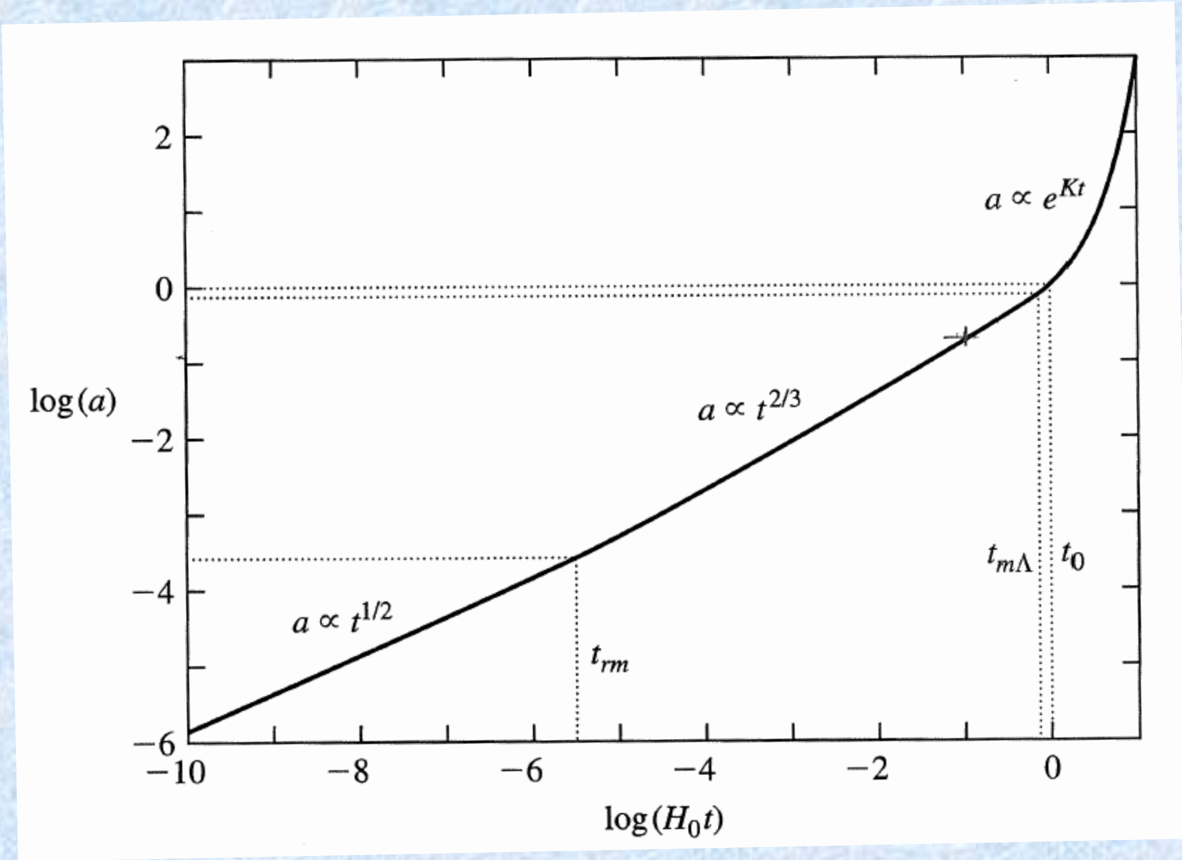
$$H^2(t) = H_0^2\left[\Omega_{\gamma,0}a^{-4} + \Omega_{m,0}a^{-3} + \Omega_{\kappa}a^{-2} + \Omega_{\Lambda}\right]$$

$$H^2(t) = H_0^2\left[\Omega_{\gamma,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{\kappa}(1+z)^2 + \Omega_{\Lambda}\right]$$

- Solution is straightforward by numerical integration.
- Let's examine properties of the concordance model.

Concordance cosmology. a(t) from numerical integration

- Evolved from radiation dominated to matter dominated, and is now entering a phase dominated by dark energy
- Current age is 13.5 Gyr



Single component Universes. Examples

- Let' s start simple… an empty Universe..
- Solve the Friedmann Equation..

  [Black board]

- Let's compare a flat universe which contains only a cosmological constant.
- Solve the Friedmann Equation..
  [Black board]

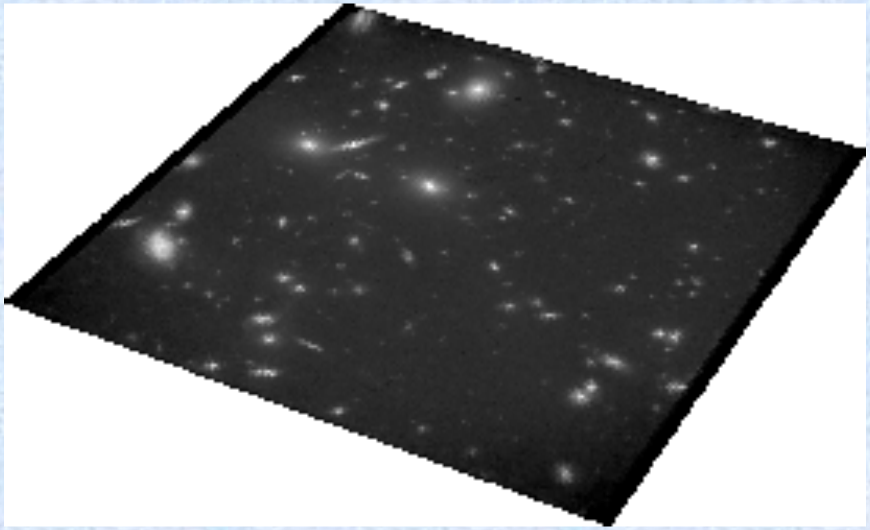
Quiz 6 – Answer in Gaucho Space
Problem 1:
What is the value of w in this model?
Problem 2:
What is the value of κ in this model?

- Let's get more general. Solve for all flat models with a single component, excluding w = -1.
- Solve the Friedmann Equation..
  [Black board]

Solutions to the Friedmann Eqn. for "Flat" Universe with a single "fluid"

- When curvature constant = 0, the universe has Infinite Volume

- Solution is power law in time: $a(t) = (t/t_0)^{2/(3+3w)}$, where $t_0 = 1/(1+w)\ \{ c^2/(6\pi G\varepsilon_0) \}^{0.5}$



- This solution is not valid when $w = -1$.

- Plot of $a(t)$ to build intuition.
  - –Matter only: $a(t)$ scales as t to the power of 2/3
  - –Radiation only: $a(t)$ scales as t to the power of 1/2
  - –Λ only: $a(t)$ exponential in t

- Will these universes expand forever?

Properties of Flat, Single Component Universes.

1. The Empty Universe

- Expansion or contraction rate is constant; this means the age is equal to the Hubble time.
- This universe has negative curvature. What is the radius of curvature?
- Redshift-time relation is linear.
- Redshift-distance relation; can see arbitrarily far.
    - Why can you see further than c/H_0?
    - Objects with high redshifts are seen as they were when the universe was very young, and their proper distance was small

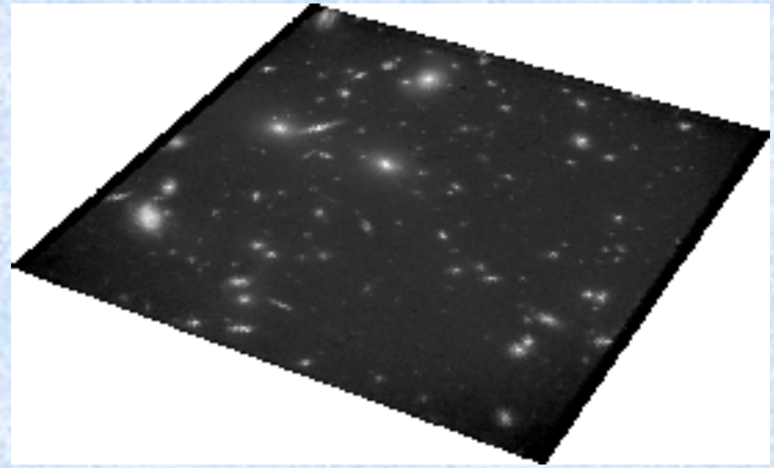
Properties of Flat, Single Component Universes.

2. Cosmological Constant (w = -1)

- The cosmological constant is one candidate for the dark energy that people talk about today. In the Benchmark Model, the cosmological constant dominates the energy density at late times.
- What happens for a pure cosmological constant?
  - The age is infinite.
  - Expansion rate is constant.
  - Horizon is infinite.

"Flat" Universe with a single "fluid." Redshift - Time Relationship

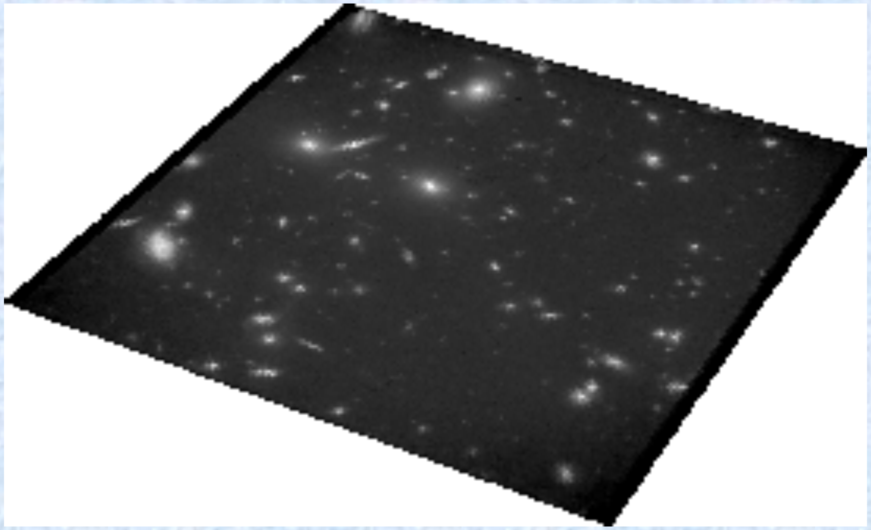
- **$1 + z = a(t_0) / a(t_e) = (t_0 / t_e)^{2/(3+3w)}$**
- Age **$t_0 = 2 / 3\, H_0^{-1} / (1 + w)$**
  - $t_0H_0=2/3$ (matter only)
  - $t_0H_0=1/2$ (radiation only)
- **Lookback time = $t_0 - t_e$**



- The age of the universe is less than a Hubble time for w > -1/3.
- Is the matter-only or radiation-only model older?

"Flat" Universe with a single "fluid." Redshift - Distance Relationship

- $d_p(t_0) = c/H_0\ 2 / (1 + 3w)\ [1 - (1+z)^{-(1+3w)/2}]$



- Flat, Matter only universe has a maximum $d_p(t_e) = (8/27)\ c/H_0$ @ $z = 5/4$
- Flat, Radiation only has a maximum at $z = 1$
- Empty universe had a maximum at $z = 1.7$

Review Distances

- Benchmark Model
  - Has a finite horizon of 14000 Mpc
  - $d_p(t_e)$ has a maximum at z=1.6
- Practice with single-component models on HW3 problem #4.

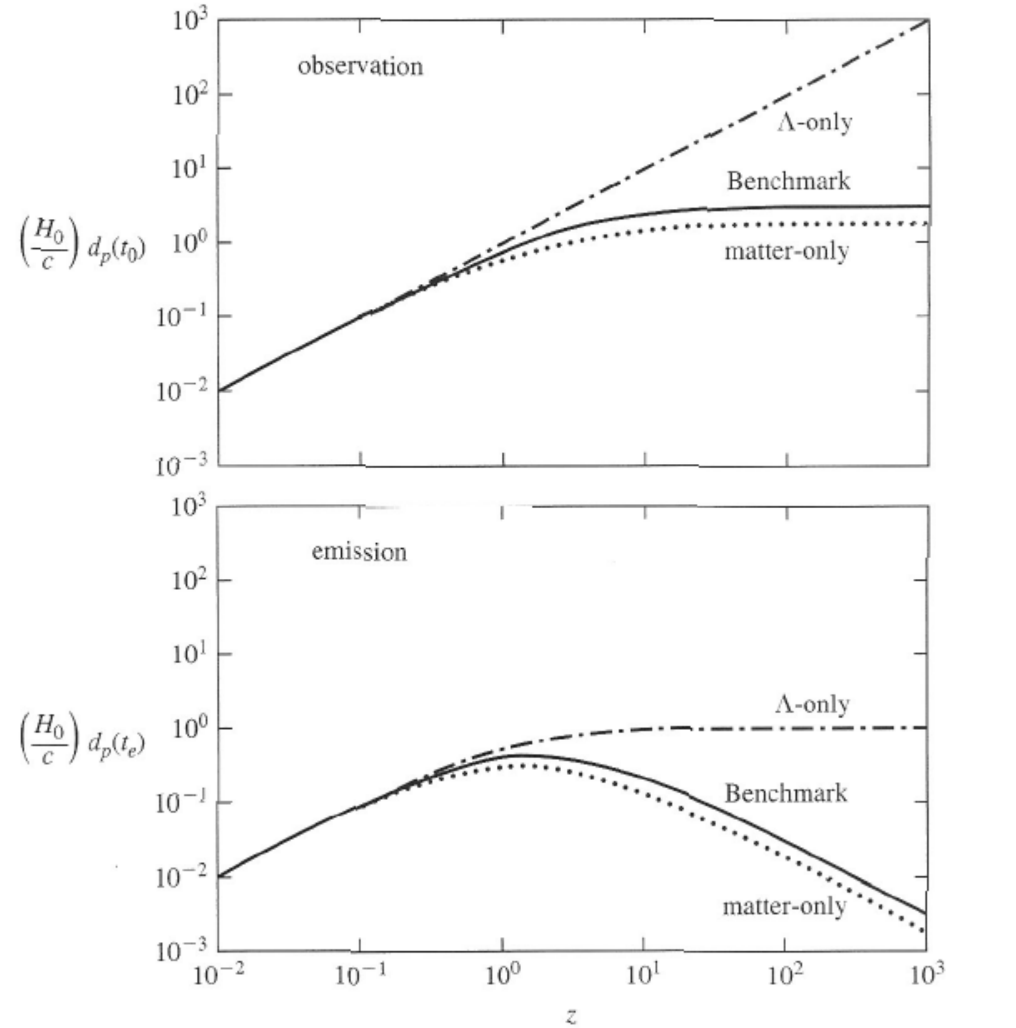


FIGURE 6.6 The proper distance to a light source with observed redshift z. The upper panel shows the distance at the time of observation; the lower panel shows the distance at the time of emission. The bold solid line indicates the Benchmark Model, the dot-dash line a flat, lambda-only universe, and the dotted line a flat, matter-only universe.

Week 3: Things you need to be able to do.

- Use Ω_0 and H_0 to determine curvature, (κ, R_0).
- Understand the evolution of the energy density $\varepsilon(a)$ in each component. Calculate the redshift of the transitions (radiation to matter to cosmological constant).
- Know the equation of state for each component of the Universe.
- Understand the implications of the acceleration equation.
- Be able to solve Friedmann's Equation for any single-component model.
- Be able to use a solution a(t) to compute the following:
  - Time - Redshift Relation
  - Proper Distances
  - Horizon