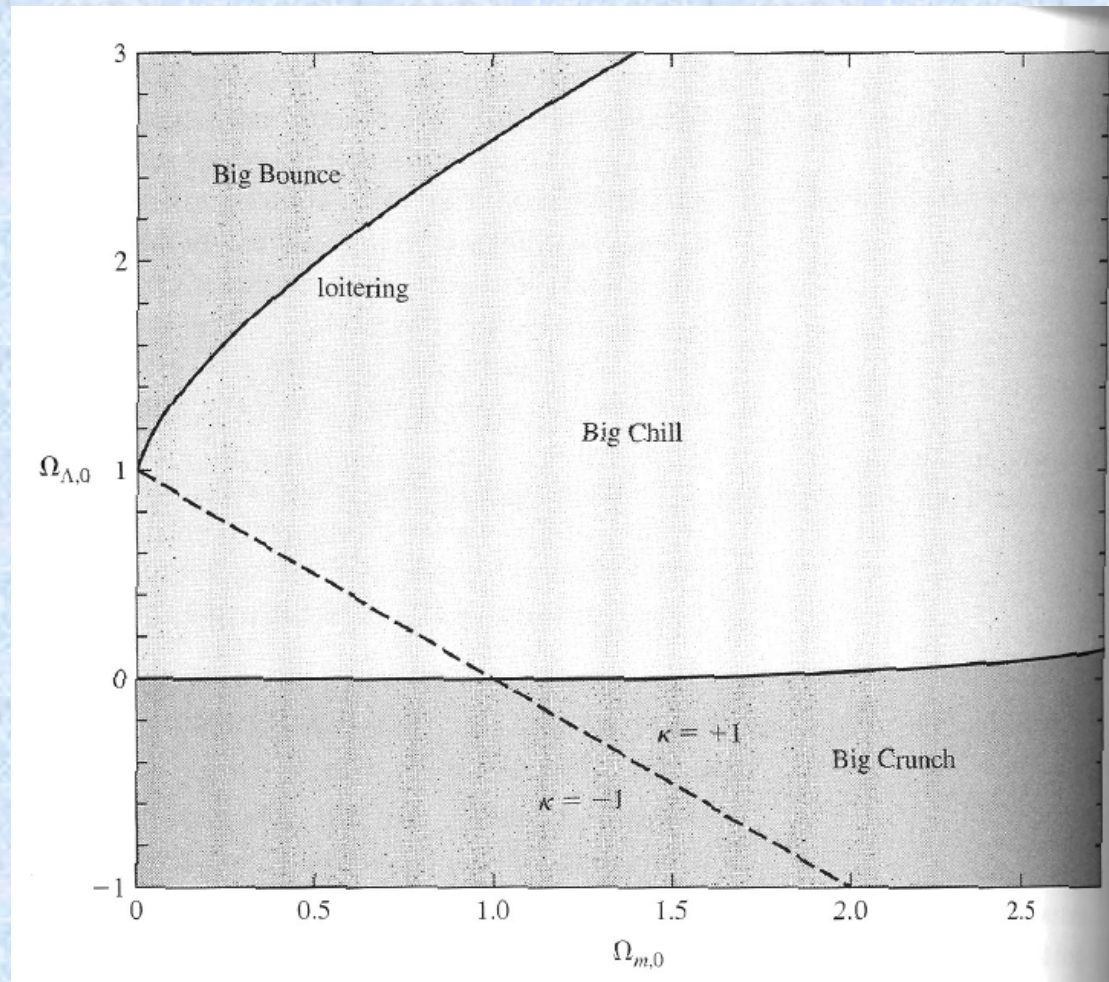


# Physics 133: Extragalactic Astronomy and Cosmology



Week 4

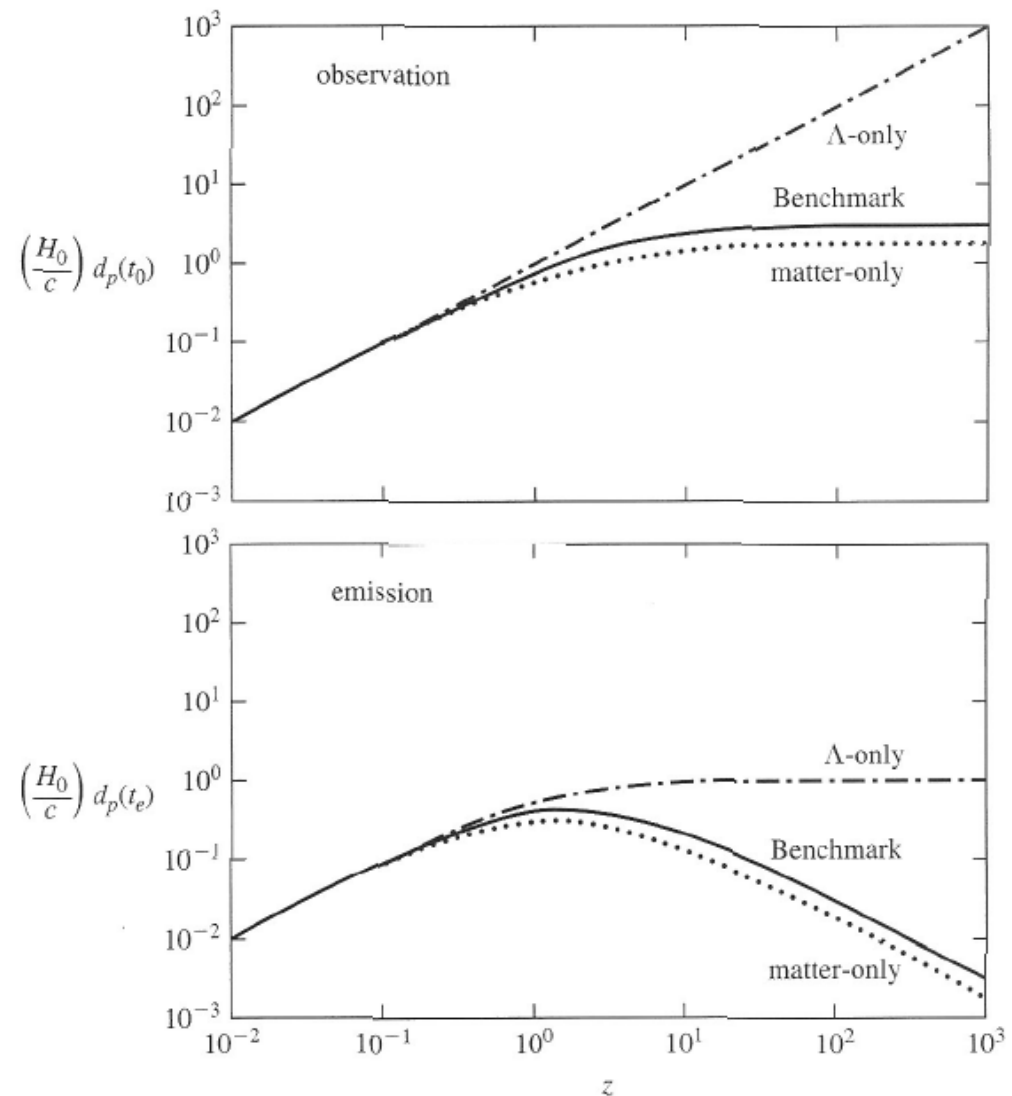
# Week 4 Outline

- Review proper distance
- Multi-component models [5.4]
- What kind of universe do we live in? [5.5]
- Distances that we can actually measure [6.1, 6.2, 6.3]
  - Luminosity distance
  - Angular diameter distance
- *Measuring the acceleration of the universe [6.4,6.5]*

**Midterm exam (April 28): Ryden chapters 1 – 6.3**

# Proper Distance – Redshift Relation

- Galaxies are not stamped with their lookback time.
- We need to express the proper distance in terms of the redshift of the galaxy.
- What's the relation between redshift and proper distance at low redshift?
- Are there other ways to define distance?  
*Yes, stay tuned.*

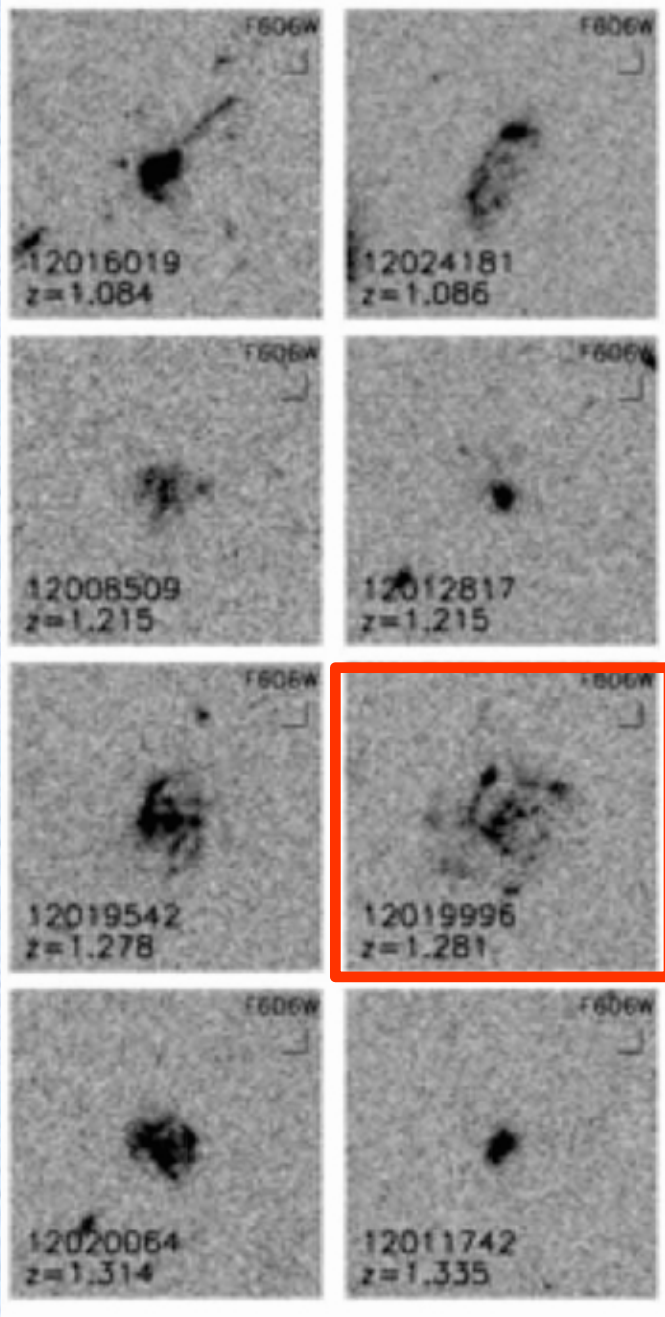


**FIGURE 6.6** The proper distance to a light source with observed redshift  $z$ . The upper panel shows the distance at the time of observation; the lower panel shows the distance at the time of emission. The bold solid line indicates the Benchmark Model, the dot-dash line a flat, lambda-only universe, and the dotted line a flat, matter-only universe.

# Quiz #7

The images show pictures of distant galaxies taken with the Hubble Space Telescope. Professor Martin took spectra of these galaxies and measured their redshifts. She found that galaxy 12019996 has a redshift  $z=1.2812$ .

- 1) What is the proper distance to this galaxy in a flat universe containing only matter? [Give your answer in units of the Hubble distance,  $c / H_0$ .]
- 2) Is the proper distance to this galaxy the same, larger, or smaller in a flat universe containing only cosmological constant?



# Generalized Friedmann Equation

$$H^2(t) = H_0^2 [\Omega_{\gamma,0} a^{-4} + \Omega_{m,0} a^{-3} + \Omega_{\kappa} a^{-2} + \Omega_{\Lambda}]$$

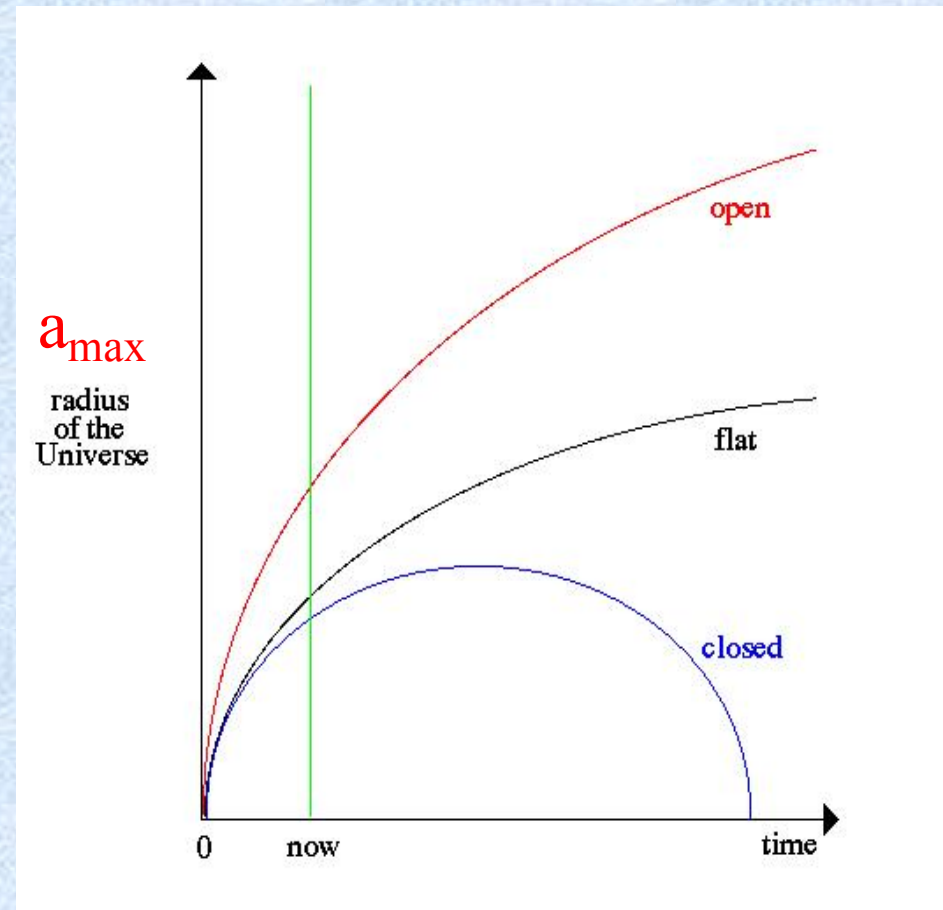
$$H^2(t) = H_0^2 [\Omega_{\gamma,0} (1+z)^4 + \Omega_{m,0} (1+z)^3 + \Omega_{\kappa} (1+z)^2 + \Omega_{\Lambda}]$$

- You work on some special cases that yield an analytic solution in HW #4.
- Let's examine some of the properties of these model universes.
- Why? Because these models make predictions that can be refuted or verified!

# Matter + Curvature Models. I.

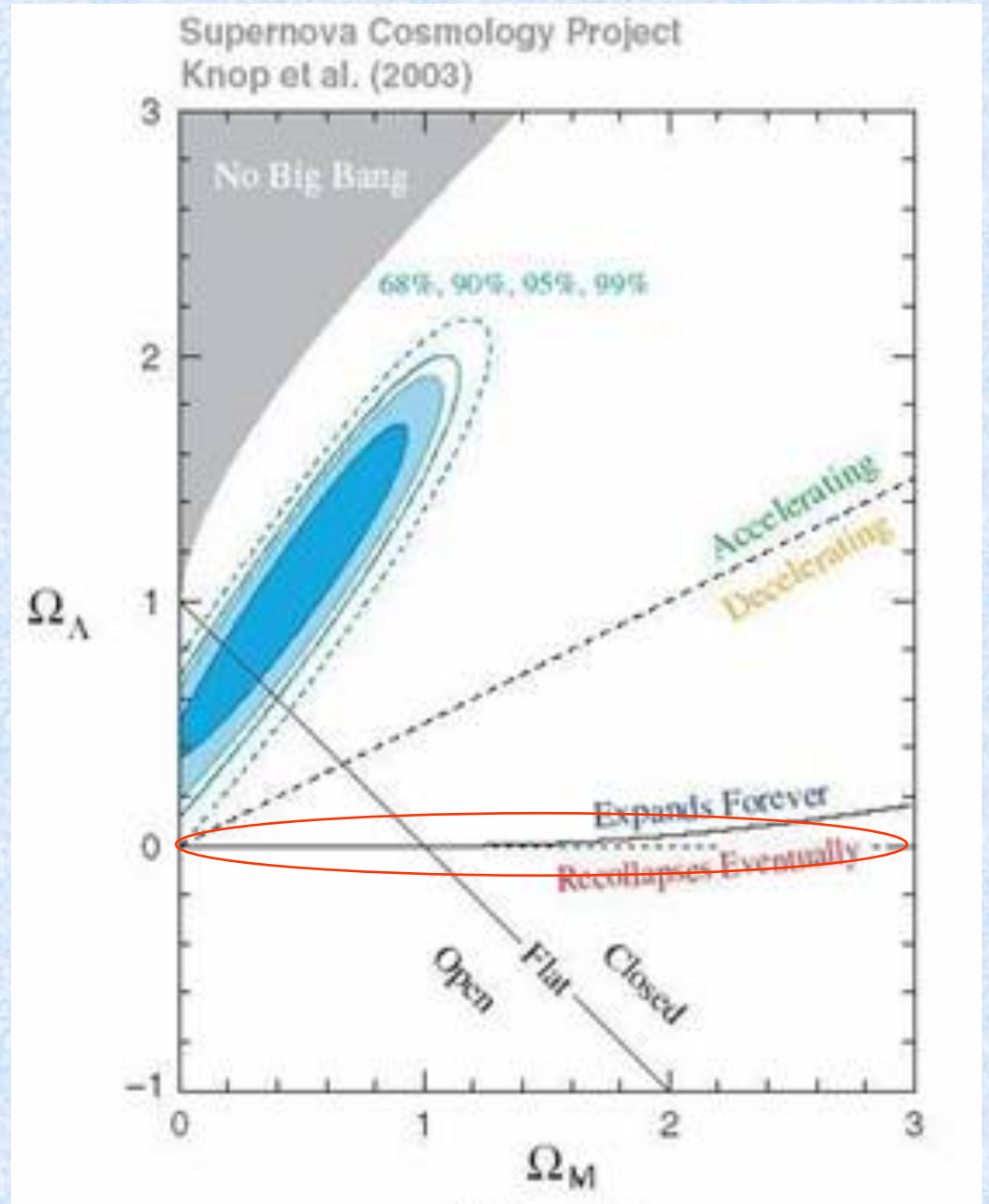
- Consider all values of  $\Omega_m$   
(  $\Omega_\Lambda = 0$  and  $\Omega_\gamma = 0$  )
- Allows closed, flat, or open geometry (i.e., positive, zero, or negative curvature,  $\kappa = +1, 0, \text{ or } -1$ )
- When do these solutions look like the single-component model for a flat matter only universe?
- New phenomena: **Maximum scale factor** when  $\kappa = +1$ .

## Curvature + Matter



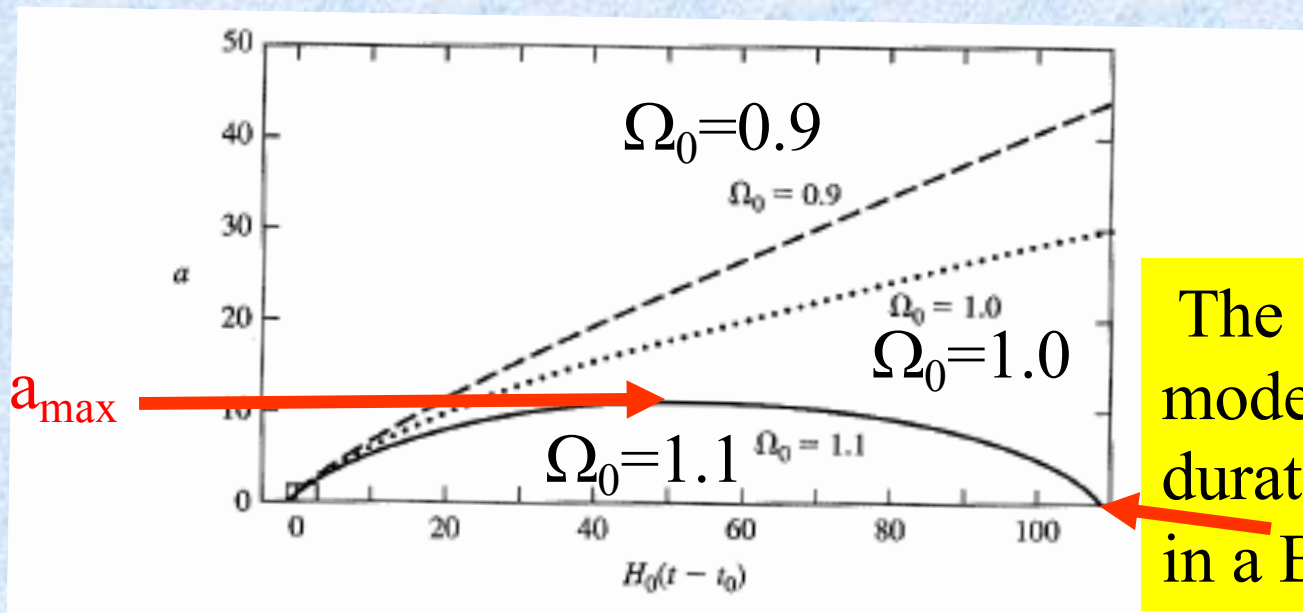
# Matter + Curvature Models. II.

- Consider all values of  $\Omega_m$  ( $\Omega_\Lambda = 0$  and  $\Omega_\gamma = 0$ )
- This is still a small subset of all possible models!



# Matter + Curvature Models. III.

- *Density determines destiny (in these models only).*
  - The matter density alone ( $\Omega_m$ ) determines whether the expansion stops.
  - It is equivalent to think in terms of curvature or matter density in these models.
  - Notice that in matter + curvature models,  $\Omega_0$  is  $\Omega_m$ .
- When  $\Omega_m > 1$ , the contraction time is equal to the expansion time.
  - You can easily compute  $t_{\text{crunch}}$ .

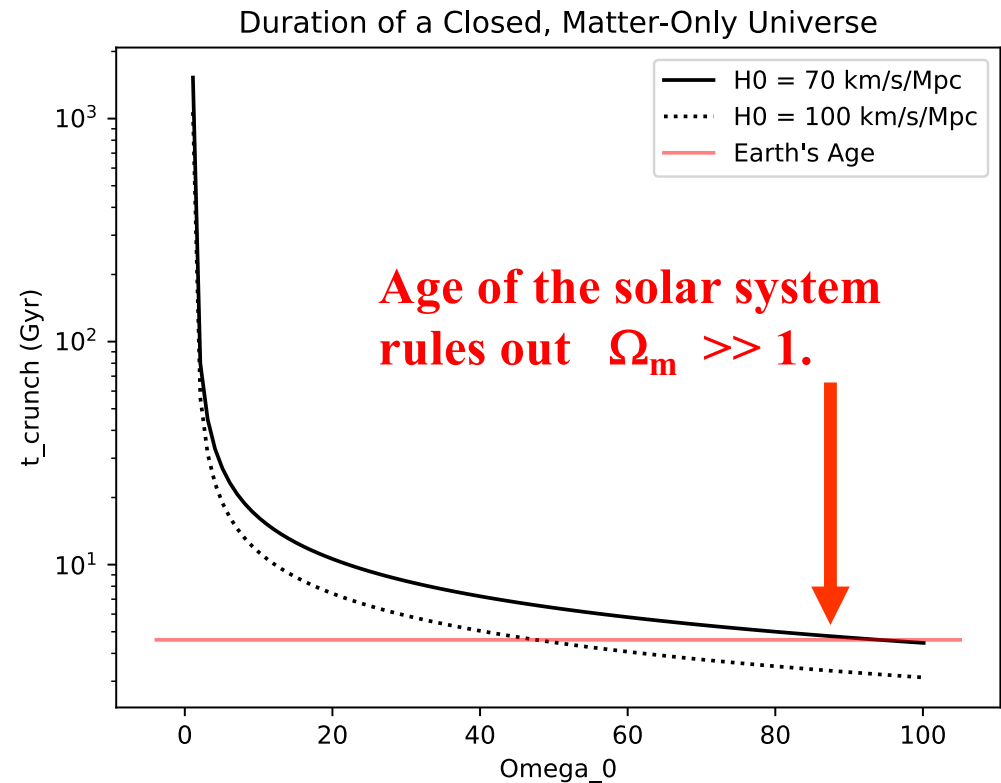


The  $\Omega_m > 1$  models have finite duration; they end in a Big Crunch!

# Matter + Curvature IV.

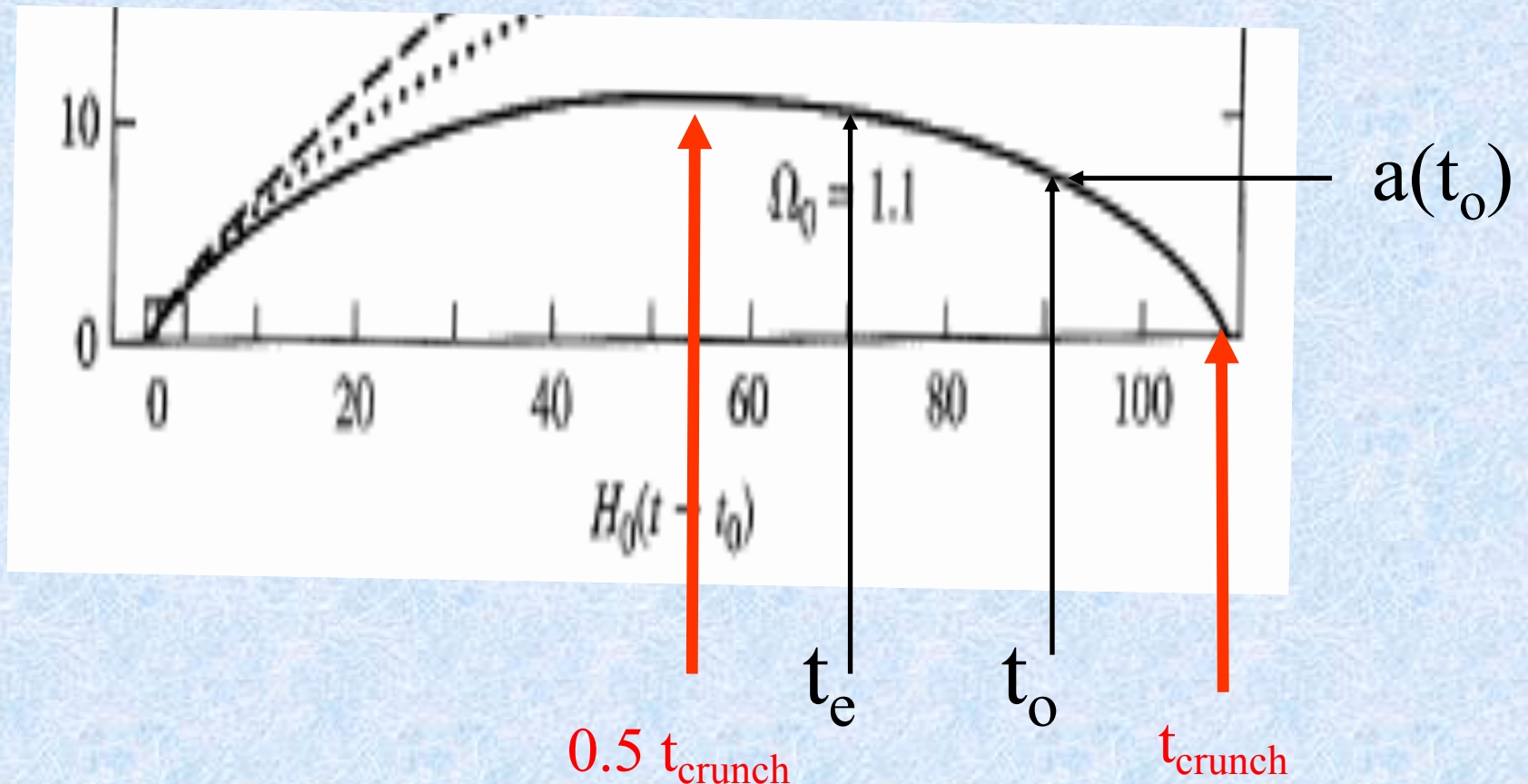
## Duration of the Universe

- The more matter there is, the shorter the timescale for  $\Omega_m > 1$  models collapse; [R 5.92,  $t_{\text{crunch}}$ ].
- Timescale depends on  $H_0^{-1}$ .
  - $H_0^{-1} = 13.98$  Gyr (for  $H_0 = 70$  km/s/Mpc)
  - *What is age  $t_0$  when  $\Omega_m = 1$ ?*
    - $t_0 = 9.32$  Gyr [R 5.4.2].
- The 4.6 Gyr age of the solar system clearly rules out matter + curvature models with  $\Omega_m \gg 1$ .



## Elbbuh Niwde's Discovery (HW #4, [R] 5.7)

- You are asked to consider observations in a matter + curvature universe that is contracting,  $\Omega_0 > 1$ .
- *Why does Dr. Niwde observe that all the galaxies are moving towards her?*



Given measurements of  $H_0$  and  $\Omega_0$ , how much time remains before the Big Crunch? i.e., What is  $t_{\text{crunch}} - t_0$ ?

# Matter + Dark Energy + Flat Models. I.

- $\Omega_m + \Omega_\Lambda = 1$ ,  
*Why is this a statement that  $\kappa = 0$ ?*
  - Definition  $\Omega(t) = \varepsilon(t) / \varepsilon_{\text{crit}}(t)$
- Still a small subset of all models.
- We can get an age  
 $t_0 = 0.964H_0^{-1} = 13.5 \text{ Gyr}$   
using the measured values  
 $\Omega_{0,m}=0.3$  and  $\Omega_\Lambda=0.7$ .

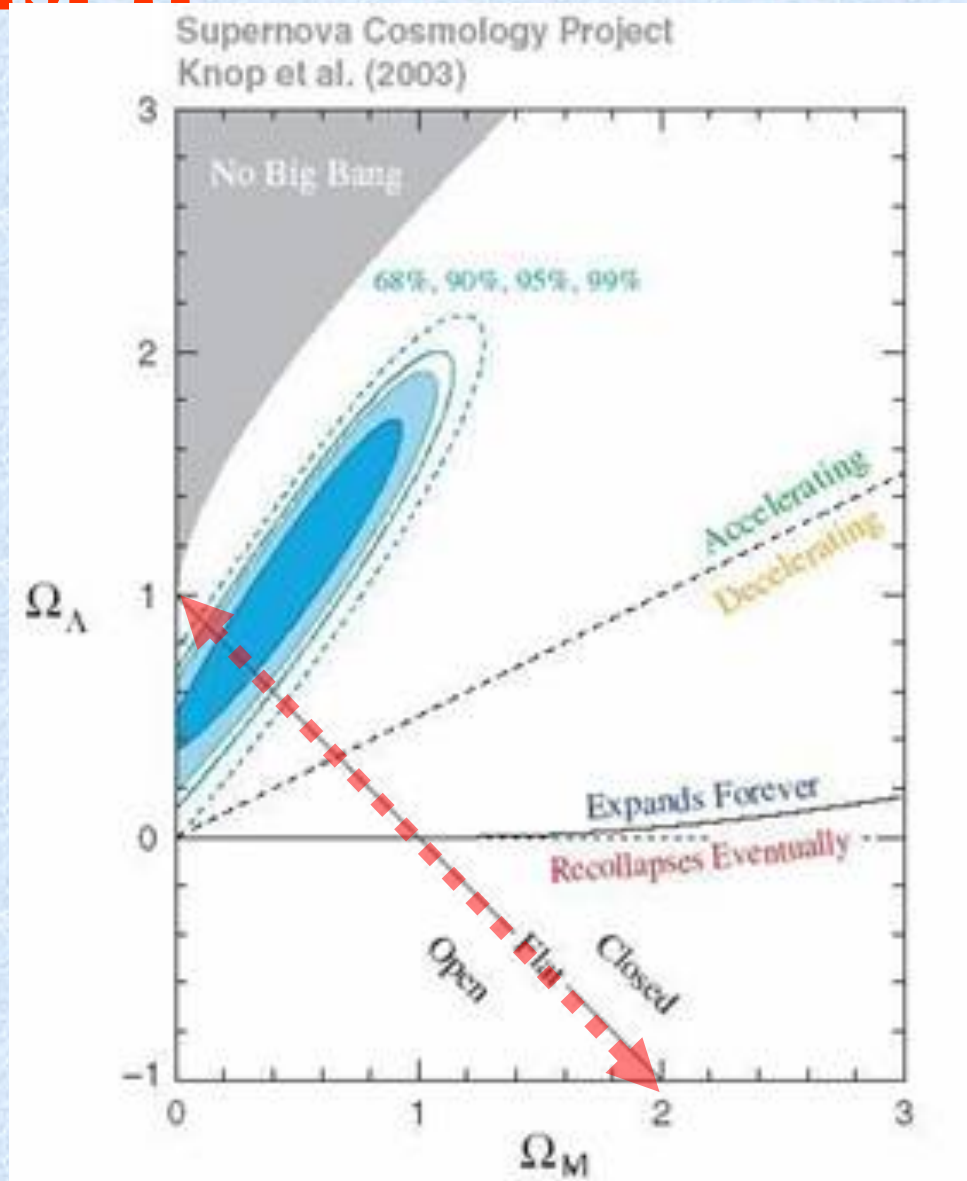
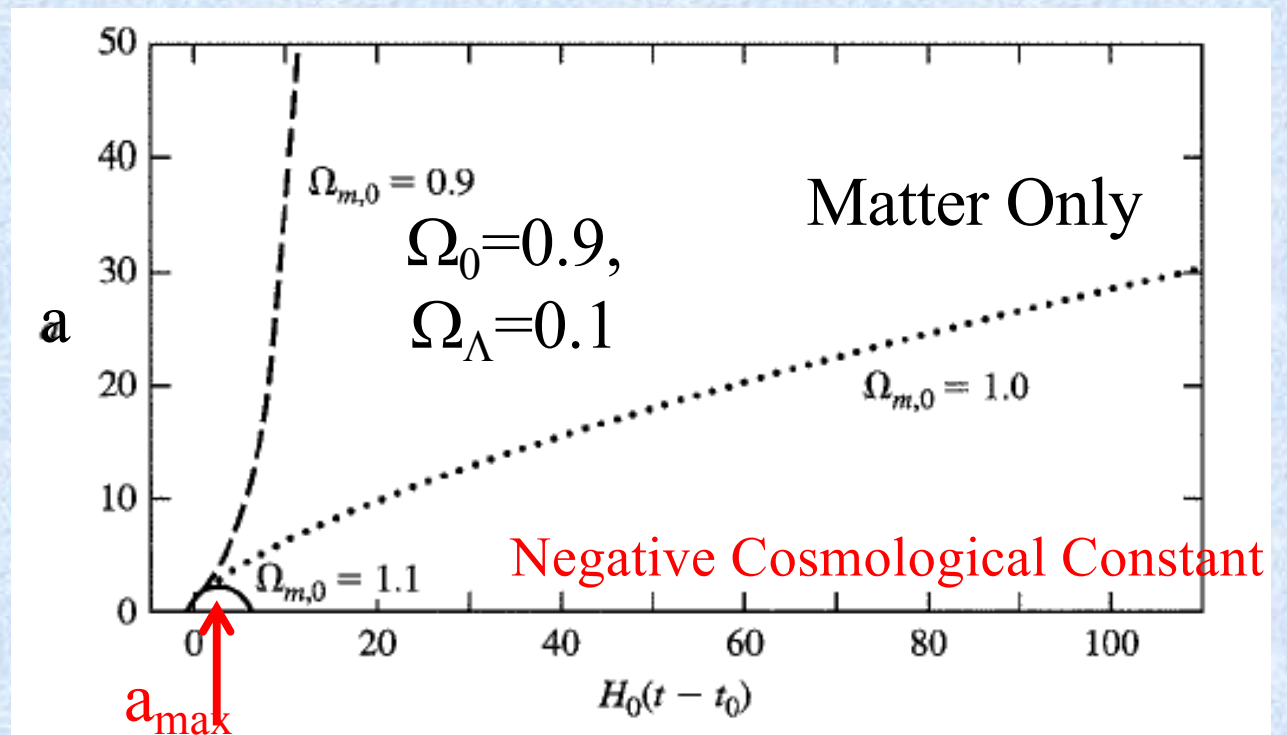


Fig. 2

# Matter + Dark Energy + Flat Models. II.

- A positive cosmological constant introduces new phenomena.
  - Most models expand forever, a.k.a. Big Chill
  - Many (but not all) models accelerate



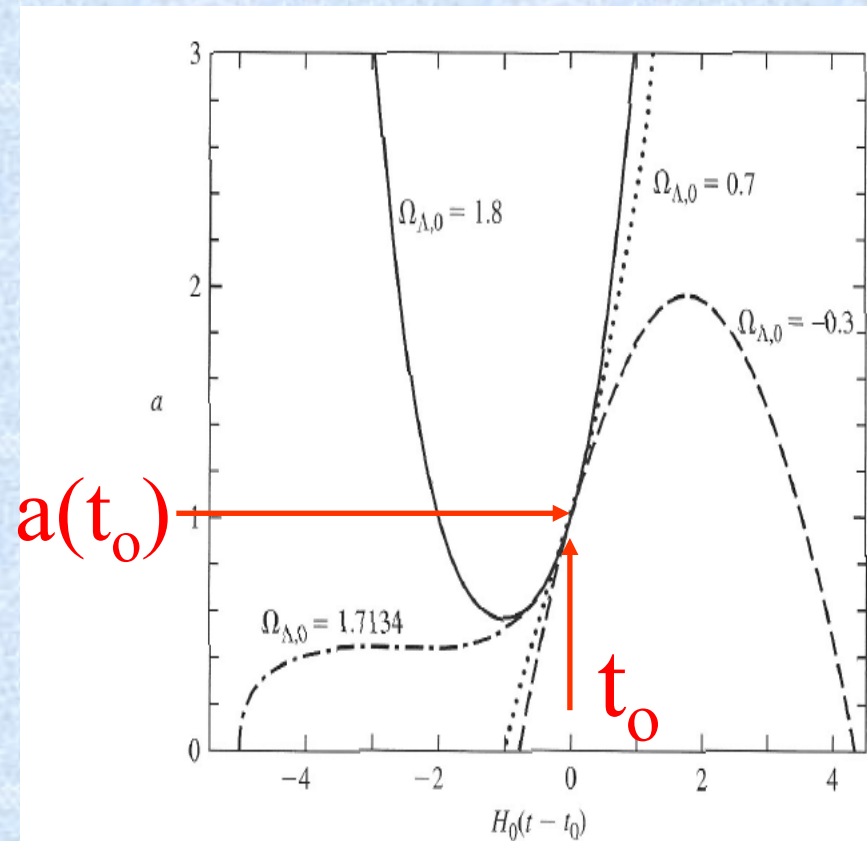


## Dark Energy + Curvature (HW #4, Prob. 3)

- Consider an expanding positively curved universe with  $\Omega_\Lambda > 1$ .
- This universe had **no Big Bang**. You are asked to show that this universe underwent a **Big Bounce** at a scale factor

$$a_{\text{bounce}} = [(\Omega_0 - 1) / \Omega_0]^{1/2}$$

- What observations could be made to determine whether we live in such a universe?
  - Measure matter content,  $\Omega_0$
  - Measure maximum redshift for galaxies (i.e., minimum  $a(t_e)$ ).

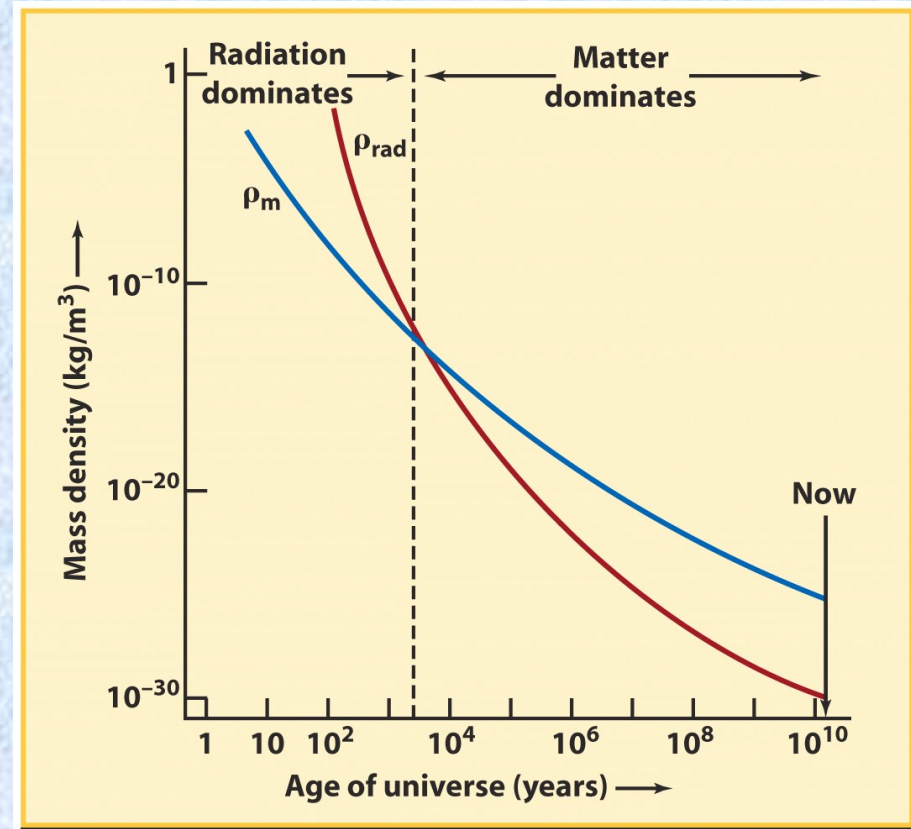


**FIGURE 6.4** The scale factor  $a$  as a function of  $t$  in four different universes, each with  $\Omega_{m,0} = 0.3$ . The dashed line shows a “Big Crunch” universe ( $\Omega_{\Lambda,0} = -0.3, \kappa = -1$ ). The dotted line shows a “Big Chill” universe ( $\Omega_{\Lambda,0} = 0.7, \kappa = 0$ ). The dot-dash line shows a loitering universe ( $\Omega_{\Lambda,0} = 1.7134, \kappa = +1$ ). The solid line shows a “Big Bounce” universe ( $\Omega_{\Lambda,0} = 1.8, \kappa = +1$ ).

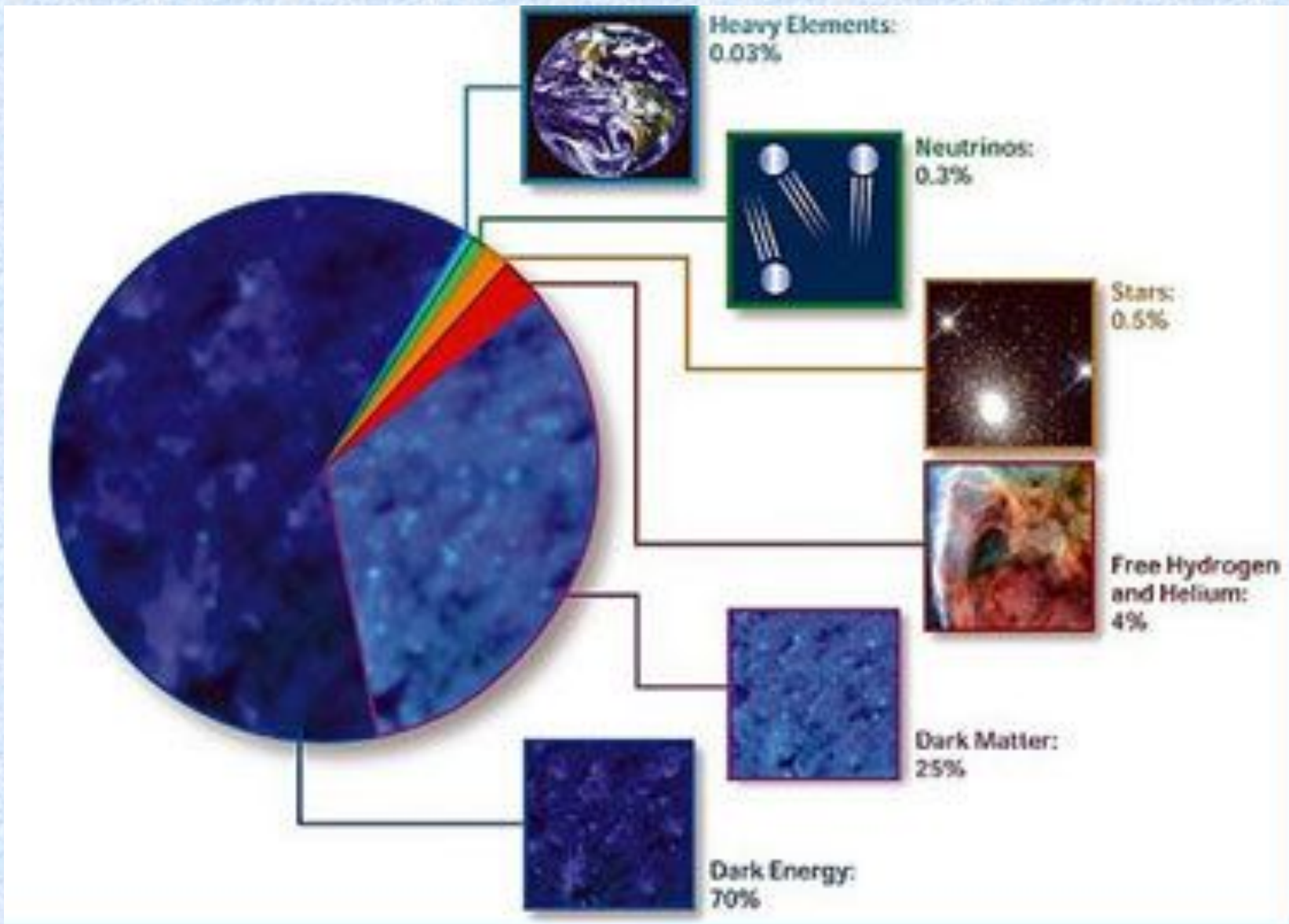
Note: Curve shown for  $\Omega_0 = 0.3$ .

# Radiation + Matter

- Energy density in radiation falls as  $\varepsilon(t)=\varepsilon_0 a(t)^{-4}$ .
- The early universe is effectively radiation dominated  $z > 3600$ .
  - What does  $a(t)$  look like?
  - Solution to the Friedmann Eqn.  $a(t)=(t/t_0)^{1/2}$  at  $z \gg 3600$ .
  - Solution transitions to  $a(t)=(t/t_0)^{2/3}$  at  $z < 3600$ .
- How do we calculate  $t_{\text{rm}}$ ?
  - $t_{\text{rm}} = 3.47e-6 H_0^{-1}$  [R 5.114]
  - Or 49,000 yr for  $H_0 = 70$  km/s/Mpc



# What Kind of Universe Do We Live In?



# Planck Collaboration 2015 Results

## Planck Collaboration: Cosmological parameters

**Table 3.** Parameters of the base  $\Lambda$ CDM cosmology computed from the 2015 baseline *Planck* likelihoods illustrating the consistency of parameters determined from the temperature and polarization spectra at high multipoles. Column [1] uses the *TT* spectra at low and high multipoles and is the same as column [6] of Table 1. Columns [2] and [3] use only the *TE* and *EE* spectra at high multipoles, and only polarization at low multipoles. Column [4] uses the full likelihood. The last column lists the deviations of the cosmological parameters determined from the *TT*+lowP and *TT,TE,EE*+lowP likelihoods.

Parameter	[1] <i>Planck</i> TT+lowP	[2] <i>Planck</i> TE+lowP	[3] <i>Planck</i> EE+lowP	[4] <i>Planck</i> TT,TE,EE+lowP	([1] – [4])/ $\sigma_{[1]}$
$\Omega_b h^2$ . . . . .	$0.02222 \pm 0.00023$	$0.02228 \pm 0.00025$	$0.0240 \pm 0.0013$	$0.02225 \pm 0.00016$	–0.1
$\Omega_c h^2$ . . . . .	$0.1197 \pm 0.0022$	$0.1187 \pm 0.0021$	$0.1150^{+0.0048}_{-0.0055}$	$0.1198 \pm 0.0015$	0.0
$100\theta_{MC}$ . . . . .	$1.04085 \pm 0.00047$	$1.04094 \pm 0.00051$	$1.03988 \pm 0.00094$	$1.04077 \pm 0.00032$	0.2
$\tau$ . . . . .	$0.078 \pm 0.019$	$0.053 \pm 0.019$	$0.059^{+0.022}_{-0.019}$	$0.079 \pm 0.017$	–0.1
$\ln(10^{10} A_s)$ . . . . .	$3.089 \pm 0.036$	$3.031 \pm 0.041$	$3.066^{+0.046}_{-0.041}$	$3.094 \pm 0.034$	–0.1
$n_s$ . . . . .	$0.9655 \pm 0.0062$	$0.965 \pm 0.012$	$0.973 \pm 0.016$	$0.9645 \pm 0.0049$	0.2
$H_0$ . . . . .	$67.31 \pm 0.96$	$67.73 \pm 0.92$	$70.2 \pm 3.0$	$67.27 \pm 0.66$	0.0
$\Omega_m$ . . . . .	$0.315 \pm 0.013$	$0.300 \pm 0.012$	$0.286^{+0.027}_{-0.038}$	$0.3156 \pm 0.0091$	0.0
$\sigma_8$ . . . . .	$0.829 \pm 0.014$	$0.802 \pm 0.018$	$0.796 \pm 0.024$	$0.831 \pm 0.013$	0.0
$10^9 A_s e^{-2\tau}$ . . . . .	$1.880 \pm 0.014$	$1.865 \pm 0.019$	$1.907 \pm 0.027$	$1.882 \pm 0.012$	–0.1

# Concordance cosmology (a few years earlier)

**table 28-2** | **Some Key Properties of the Universe**

Quantity	Significance	Value*
Hubble constant, $H_0$	Present-day expansion rate of the universe	$71^{+4}_{-3}$ km/s/Mpc
Density parameter, $\Omega_0$	Combined mass density of all forms of matter <i>and</i> energy in the universe, divided by the critical density	$1.02 \pm 0.02$
Matter density parameter, $\Omega_m$	Combined mass density of all forms of matter in the universe, divided by the critical density	$0.27 \pm 0.04$
Density parameter for ordinary matter, $\Omega_b$	Mass density of ordinary atomic matter in the universe, divided by the critical density	$0.044 \pm 0.004$
Dark energy density parameter, $\Omega_\Lambda$	Mass density of dark energy in the universe, divided by the critical density	$0.73 \pm 0.04$
Age of the universe, $T_0$	Elapsed time from the Big Bang to the present day	$(1.37 \pm 0.02) \times 10^{10}$ years
Age of the universe at the time of recombination	Elapsed time from the Big Bang to when the universe became transparent, releasing the cosmic background radiation	$(3.79^{+0.08}_{-0.07}) \times 10^5$ years
Redshift $z$ at the time of recombination	Since the cosmic background radiation was released, the universe has expanded by a factor $1 + z$	$1089 \pm 1$

\*Values are from the first year of WMAP data. (NASA/WMAP Science Team)

**Year 3 WMAP is very similar we will see in more detail later on...**

# Summary

- Multi-component universes (analytic models)
  - Illustrate the wide variety of cosmological models allowed by the Friedmann equation.
  - Predict some phenomena that observations firmly rule out.
  - Provide an approximation to the Concordance Model
- Review Concordance Model
  - The universe that we live in!

# Physics 133: Extragalactic Astronomy and Cosmology



Week 4

**Reminder:**

**Midterm exam (April 28): Ryden chapters 1 – 6.3**

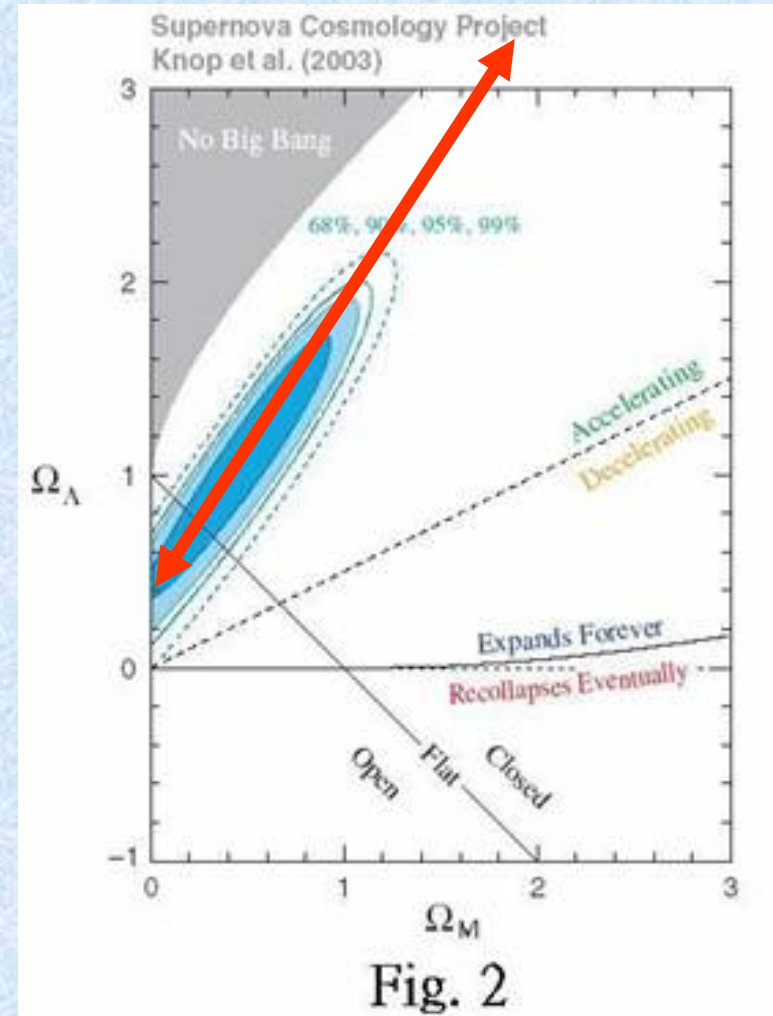
Chat Box: *What is a standard candle?*

# Outline: Measuring Cosmological Parameters

- Measuring kinematics of the universe determines its contents. We can use observations to describe  $a(t)$ .
- Proper distance is not directly measured. As observers we need distances that we can measure.
- Measuring cosmological parameters with standard candles, standard rods, and (new) standard sirens.

# Measuring Kinematics of the Universe Determines Its Content. 1. Acceleration.

- Measuring the acceleration of the universe determines the quantity  $q_0 = 0.5\Omega_{m,0} - \Omega_{\Lambda,0}$ ,
- Which is a line  
$$\Omega_{\Lambda} = 0.5\Omega_{m,0} + (-q_0)$$
in the density parameter plot.
- Let's see how this works.



# The Contents of the Universe Determine Its Acceleration. I.

- Measure  $H_0$  and the ***deceleration parameter***  $q_0$  (independent of any model)
- Use the Friedmann and acceleration equations to determine what  $\varepsilon(t)$  would give these parameters
- So the deceleration parameter is related to the difference between the density parameters for matter and lambda.

$$a(t) = a(t_0) + \left(\frac{da}{dt}\right)_{t=t_0} (t-t_0) + \frac{1}{2} \left(\frac{d^2a}{dt^2}\right)_{t=t_0} (t-t_0)^2 + O(t^3)$$

$$a(t) \sim 1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2$$

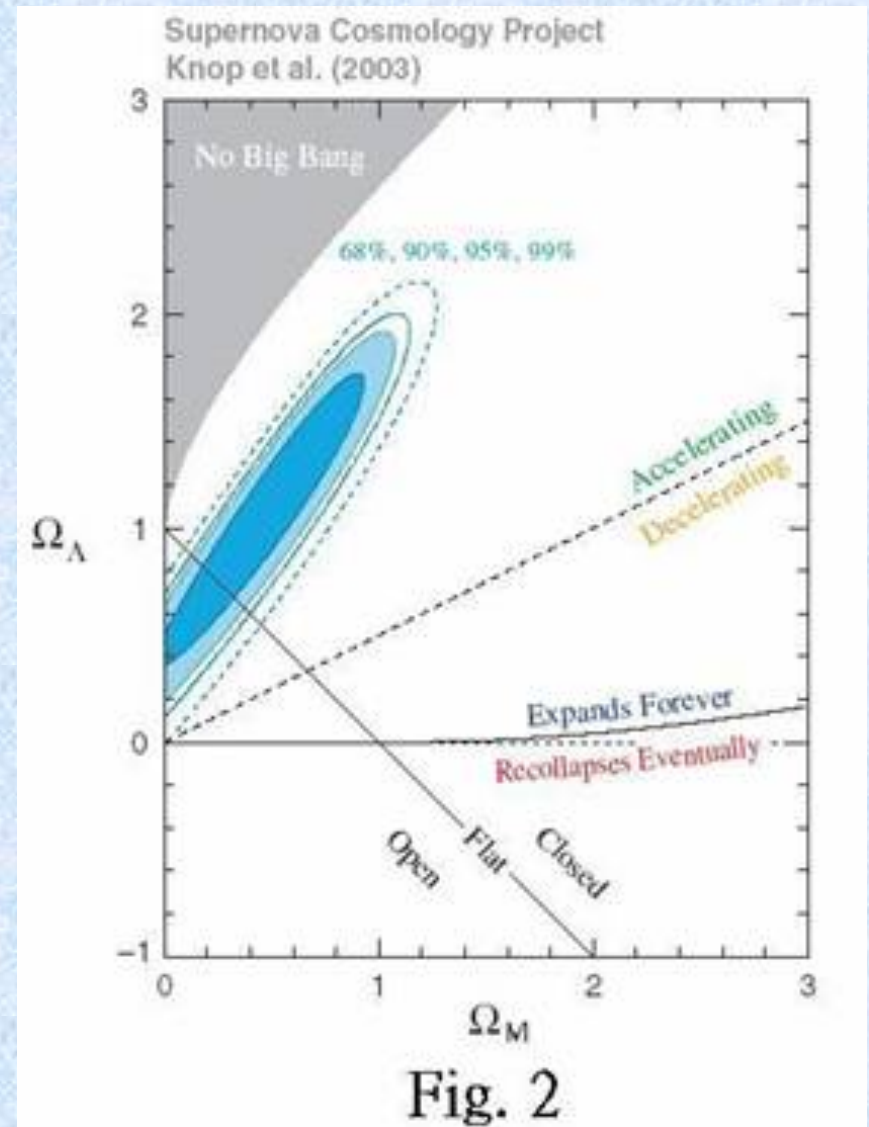
$$q_0 \equiv -\left(\frac{\ddot{a}}{\dot{a}^2}\right)_{t=t_0}$$

$$q_0 = \frac{1}{2} \sum_w \Omega_{w,0}(1+3w)$$

$$q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}$$

# Quiz #8: The Contents of the Universe Determine Its Acceleration

- What is the value of the deceleration parameter for the Benchmark model?
- Is universe the universe that we live in decelerating?



# How do we measure the kinematics of the Universe?

- Recall Hubble's law; redshift increases linearly with proper distance.
- This allows us to measure one parameter, maybe two.

[blackboard]

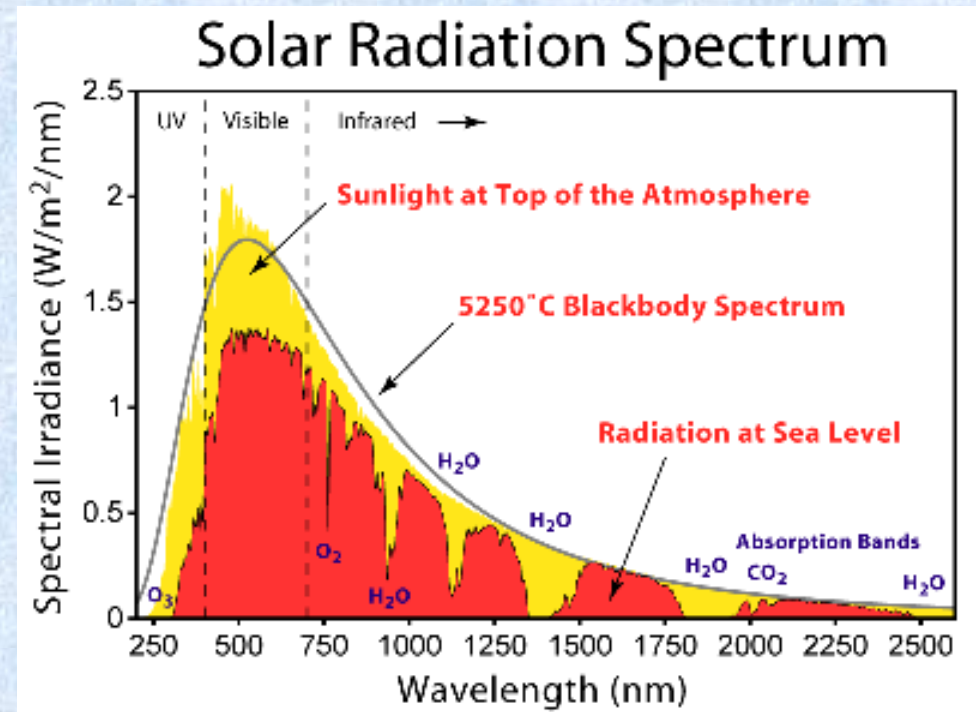
- But proper distance is hard to measure.
- Proper distance is hard to even define when we have incomplete knowledge of  $a(t)$ . Recall the definition.



*Let's define two distances that astronomers  
can actually measure.*

# Flux and Flux “Density”

- Astronomers talk about flux density because their detectors usually are not sensitive to photons of all energies.
- A spectrum shows the amount of flux per unit wavelength (or per unit frequency)
- $m - m_0 = -2.5 \log(F_\lambda / F_{\lambda,0})$
- $V = -2.5 \log(F_\lambda / 3.64 \times 10^{-9} \text{ erg/cm}^2/\text{s}/\text{\AA})$



# Convenient distances.

## Luminosity distance. I.

- We can measure Flux
- If we know the luminosity, we have a distance!
  - $F=L/4\pi (d_1)^2$
- **But what distance is this?**
  - **Depends on curvature**
  - **Depends on expansion**

[Blackboard]

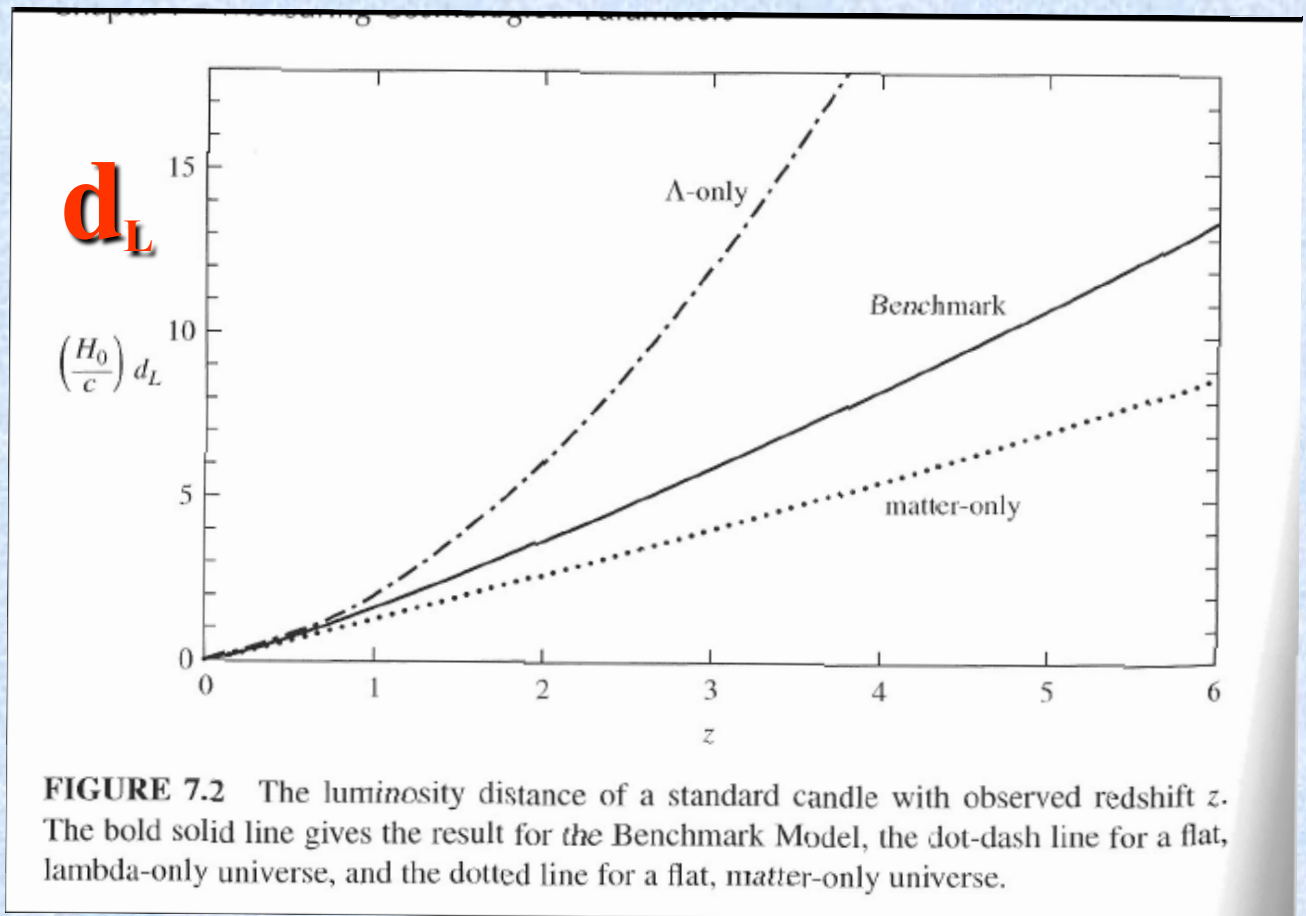


Standard candle

# Convenient distances.

## Luminosity distance. II

- At  $z \ll 1$ , luminosity distance is a good approximation to proper distance.

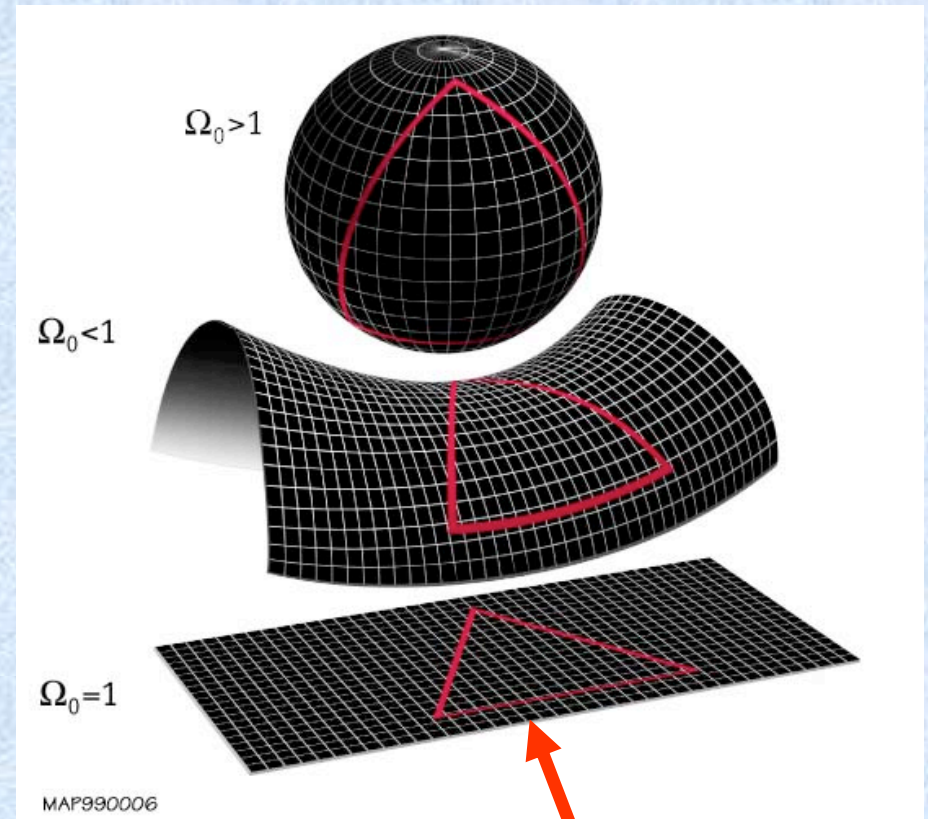


As  $z \Rightarrow 0$ , we have  $d_L \sim d_p(t_0) \sim c H_0^{-1} z$

# Convenient distances.

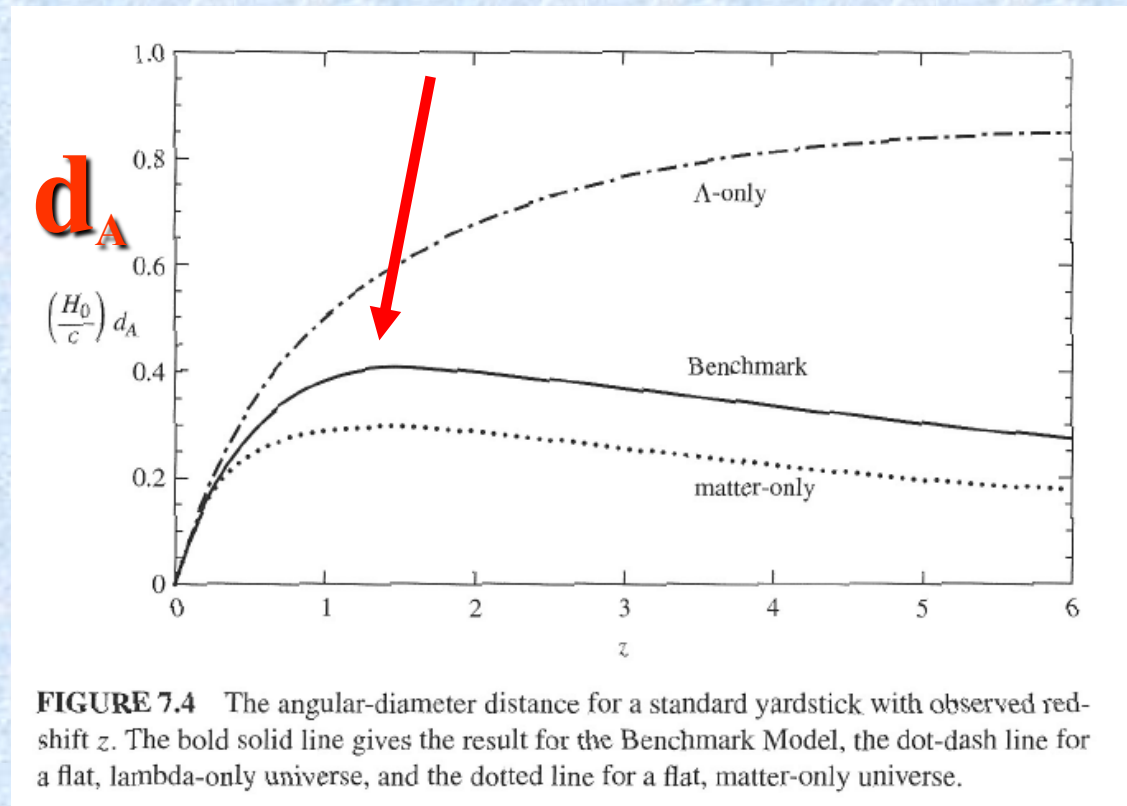
## Angular diameter distance. I

- We can measure angles
- If we know the size of an object, we have a distance!
  - $\theta=L/d_A$
- But what distance is this?
- [Blackboard]
- Distant objects appear larger than they really are because photons were emitted at a time when the object was closer.
- At low redshift,  $d_A \sim d_p(t_e)$



## Angular Diameter Distance. II.

HW #4 [R] 6.3 (flat, any  $w$ , find max  $d_A$ )



- As  $z \implies 0$ , we have  $d_A \sim d_p(t_0) \sim c H_0^{-1} z$
- As  $z \implies \text{infinity}$ , we have  $d_A$  goes to  $d_{\text{hor}}(t_0) / z$
- Benchmark model has **maximum  $d_A = 1800$  Mpc at  $z=1.6$ .**

# Summary of Distances

1. Co-moving Distance
2. Proper Distance
3. Luminosity Distance
4. Angular-Diameter Distance

- **As  $z \implies 0$ , we have**

$$d_A \sim d_L \sim d_p(t_0) \sim c H_0^{-1} z$$

- **As  $z \implies \text{infinity}$ , we have**

$$d_p(t_0) \text{ goes to } d_{\text{hor}}(t_0)$$

$$d_L \text{ goes to } z d_{\text{hor}}(t_0)$$

$$d_A \text{ goes to } d_{\text{hor}}(t_0) / z$$

- **Benchmark model has maximum  $d_A = 1800 \text{ Mpc}$  at  $z=1.6$ ; and object subtends the smallest angle**

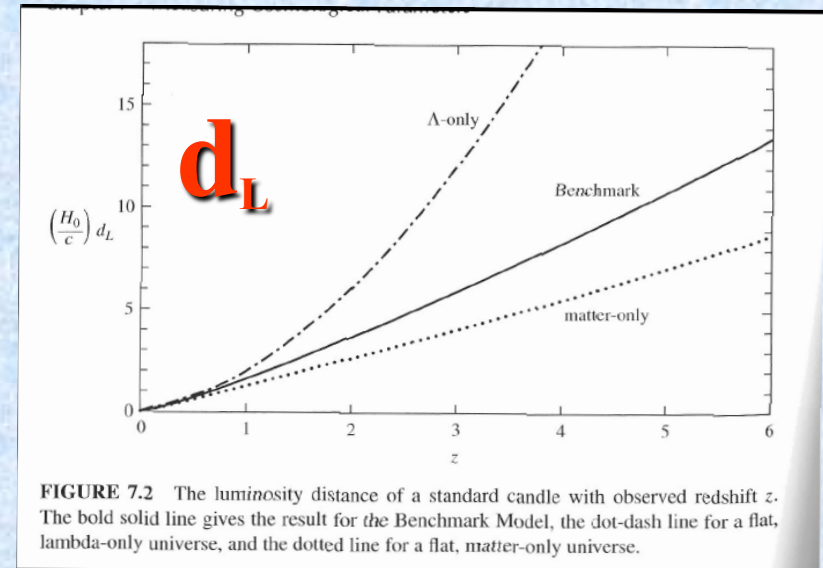


FIGURE 7.2 The luminosity distance of a standard candle with observed redshift  $z$ . The bold solid line gives the result for the Benchmark Model, the dot-dash line for a flat, lambda-only universe, and the dotted line for a flat, matter-only universe.

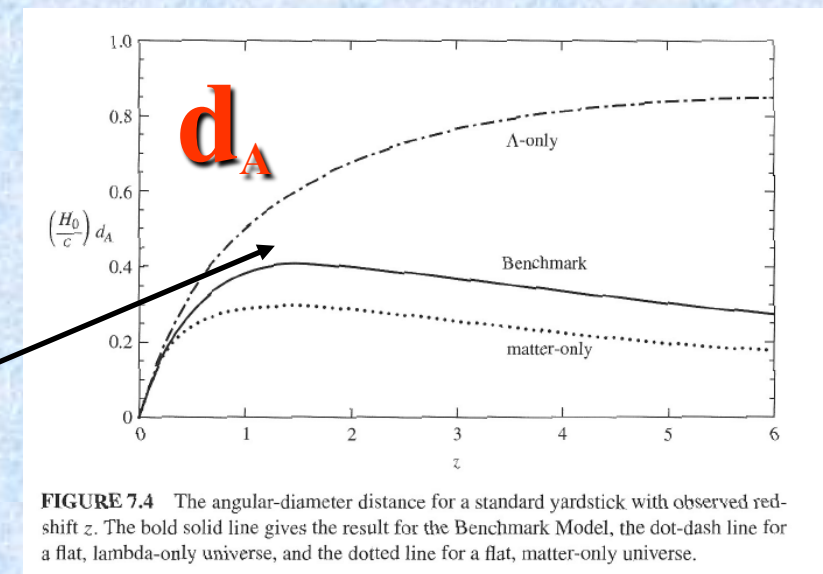
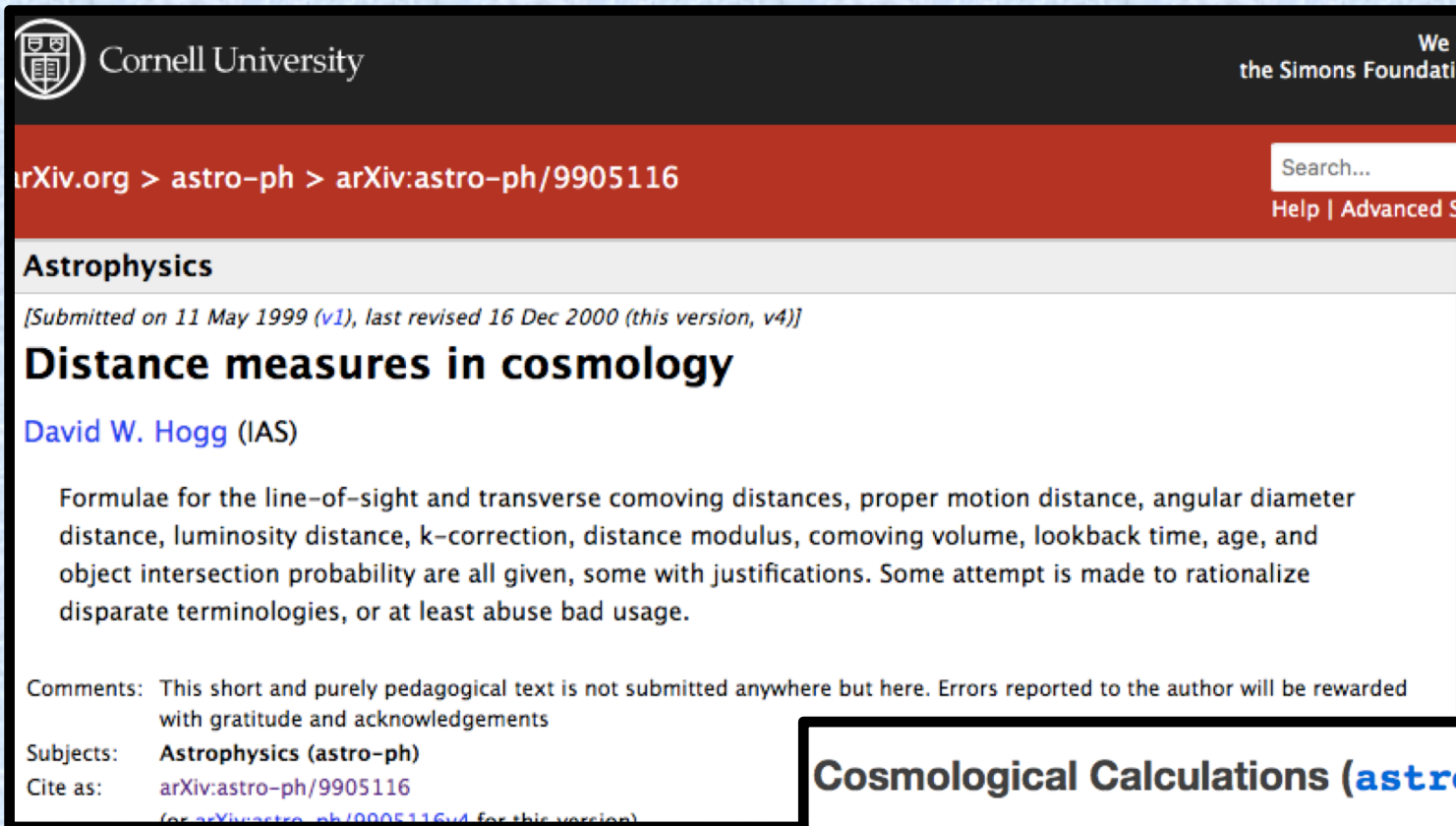


FIGURE 7.4 The angular-diameter distance for a standard yardstick with observed redshift  $z$ . The bold solid line gives the result for the Benchmark Model, the dot-dash line for a flat, lambda-only universe, and the dotted line for a flat, matter-only universe.

# Extra: For the Budding Astrophysicist

*Links available at the bottom of the course web page.*



The screenshot shows the top portion of an arXiv page. At the top left is the Cornell University logo and name. At the top right is the text 'We are grateful to the Simons Foundation'. Below this is a red navigation bar with the text 'arXiv.org > astro-ph > arXiv:astro-ph/9905116' on the left and a search box with 'Search...' and 'Help | Advanced Search' on the right. The main content area has a grey header 'Astrophysics' and a sub-header '[Submitted on 11 May 1999 (v1), last revised 16 Dec 2000 (this version, v4)]'. The title 'Distance measures in cosmology' is in large bold black font. Below the title is the author 'David W. Hogg (IAS)' in blue. The abstract text follows: 'Formulae for the line-of-sight and transverse comoving distances, proper motion distance, angular diameter distance, luminosity distance, k-correction, distance modulus, comoving volume, lookback time, age, and object intersection probability are all given, some with justifications. Some attempt is made to rationalize disparate terminologies, or at least abuse bad usage.' Below the abstract are 'Comments', 'Subjects: Astrophysics (astro-ph)', and 'Cite as: arXiv:astro-ph/9905116 (or arXiv:astro-ph/9905116v4 for this version)'.

Cornell University

We are grateful to the Simons Foundation

arXiv.org > astro-ph > arXiv:astro-ph/9905116

Search...  
Help | Advanced Search

**Astrophysics**

[Submitted on 11 May 1999 (v1), last revised 16 Dec 2000 (this version, v4)]

## Distance measures in cosmology

David W. Hogg (IAS)

Formulae for the line-of-sight and transverse comoving distances, proper motion distance, angular diameter distance, luminosity distance, k-correction, distance modulus, comoving volume, lookback time, age, and object intersection probability are all given, some with justifications. Some attempt is made to rationalize disparate terminologies, or at least abuse bad usage.

Comments: This short and purely pedagogical text is not submitted anywhere but here. Errors reported to the author will be rewarded with gratitude and acknowledgements

Subjects: **Astrophysics (astro-ph)**

Cite as: **arXiv:astro-ph/9905116**  
(or **arXiv:astro-ph/9905116v4** for this version)

## Cosmological Calculations ([astropy.cosmology](https://github.com/dwheeler/astropy.cosmology))

### Introduction

The [astropy.cosmology](https://github.com/dwheeler/astropy.cosmology) sub-package contains classes for representing cosmologies and utility functions for calculating commonly used quantities that depend on a cosmological model. This includes distances, ages, and lookback times corresponding to a measured redshift or the transverse separation corresponding to a measured angular separation.

### Getting Started

# Summary: Measuring Cosmological Parameters

- Measuring kinematics of the universe determines its contents. We can use observations to describe  $a(t)$ .
  - Acceleration of the universe is related to the density parameter.
  - Proper distance depends on redshift via the Hubble constant, to first order
- Proper distance is not directly measured. As observers we need distances that we can measure.
  - Luminosity Distance
  - Angular Diameter Distance
- *Measuring cosmological parameters with standard candles and standard rods (rulers)*