Lab 4
Physical Properties of Stars, Photometric Color Systems

Boxes contain questions that you are expected to answer (in the box). You will also be asked to put computer output into special directories, so it can be graded. If you need more room for an answer, put it on the back of a page, with a pointer to it.

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Please read (again!) the pages from Nick Strobel's course notes about parallax and distances, about magnitudes, and about temperatures, colors, and luminosities. These three pages are linked from the Phys134 course web page.

Many factors influence the appearance of a star's emitted energy spectrum, but the most important is the stellar temperature. Here we will investigate how the visible-light spectrum changes with temperature, and how these changes are manifested in the stars' photometric colors.

In your home directory you will find a new data directory named **lab4**. That directory contains (among other things) three spectra of stars with differerent temperatures:
F05.dat = 9550K
F10.dat = 6110K
F15.dat = 4350K

These files contain the **energy flux** F_λ (proportional to erg/cm^2-s-AA), where wavelength λ is measured in Angstroms (AA), and 1 Angstrom = 0.1 nm. The flux values have been arbitrarily normalized so they have a value of 100 units near 5500 AA.

Make a plot showing all 3 spectra overplotted, in 3 different colors. As usual, label the axes, put your name in the title, and save the plot in the directory **/backup/results_YourName**. This first plot of Lab4 stored in your directory should be named **Lab4plot01.ps**.

2 Using a CCD detector with colored filters, we measure the energy flux from stars integrated over the bandpass of each filter. Thus, with a filter having transmission $T(\lambda)$, we measure an average flux $<F_\lambda>$ that is weighted by the filter transmission taken over the bandpass:

$$<F_\lambda> = \Sigma \; (F_\lambda \; T(\lambda) \, \Delta\lambda) / \Sigma \; (T(\lambda) \, \Delta\lambda)$$

Filter fluxes are however conventionally computed using sums over frequency ν , not wavelength. Since $d\nu = c \, d\lambda / \lambda^2$, we get

$$\cancel{<F_\nu> = \Sigma \; (F_\lambda \; T(\lambda) \, \Delta\lambda) / \Sigma \; (T(\lambda) \, \Delta\lambda / \lambda^2)}, \text{ so}$$

$$\cancel{<F_\nu> = <F_\lambda> \Sigma \; (T(\lambda) \, \Delta\lambda) / \Sigma \; (T(\lambda) \, \Delta\lambda / \lambda^2)}$$

All wrong. See notes.

Stellar magnitudes measured with these filters are then given by

$$M_\nu = -2.5 \log <F_\nu> - Z_\nu$$

For simplicity, let us take $T(\lambda)$ to be equal to unity within the nominal bandpass of the filter, and zero outside it. This makes the sums in the last equation into easy integrals over wavelength. We will concern ourselves with 2 standard astronomical filters, designated **B** and **V**. (B was named for "blue", and V for "visual", which is to say, yellow-green). Filter bandpasses for these filters (in AA), and their zero points (which depend on instrumental sensitivity, and hence are to a degree arbitrary) are:

B = {3910, 4890} $\quad$ **Z_B = -32.109**
V = {5050, 5950} $\quad$ **Z_V = -32.000**

These zero points give silly results when this calculation is done correctly, but let's use them anyway, for consistency.

Compute **B** and **V** magnitudes for each of the stars f05, f10, f15 that you just plotted. Do the sums of star fluxes over wavelength by using the xmgrace Data/Transformations/Integration tool, and evaluating the sum across the wavelength limits given above. Also compute the "color index" **(B-V)**. This is the difference of two stellar magnitudes, so it depends on the log of the ratio of the two energy fluxes. Show the key points of your calculation, and write your results on the next page.

$$\langle F_\nu \rangle = \sum F(\lambda) \cdot \Delta\lambda \cdot \left[c \frac{1}{\lambda}\Big|_{\lambda_1}^{\lambda_0} \right]^{-1} = \sum F(\lambda) \cdot \left\{ 10^{-18} \cdot \left[\frac{1}{\lambda}\Big|_{\lambda_1}^{\lambda_0} \right]^{-1} \right\} \equiv F(\lambda) / \alpha(\text{filter})$$

There are 3 cases:

$\alpha(B) = 5.125 \times 10^{13}$

$\alpha(V) = 2.995 \times 10^{13}$

$\alpha(B \text{ shifted}) = 4.898 \times 10^{13}$

$\sum F(\lambda)$:	f05	f10	f15
	166342	106998	59922
	87825	90132	85349
	164037	110624	67233

	Magnitudes f05	Magnitudes f10	Magnitudes f15	Magnitudes (shifted) f05	Magnitudes (shifted) f10	Magnitudes (shifted) f15
B:	53.33	53.81	54.44	53.30	53.72	54.27
V:	53.33	53.30	53.36	53.33	53.30	53.36
B-V:	0.00	0.51	1.08	-0.03	0.42	0.91

Clearly, the magnitudes you compute depend on the wavelength bandpasses used. To illustrate this, shift the **B** filter bandpass by 100 AA to the red, ie, take the **B** filter bandpass to be:

B: {4010, 4990}

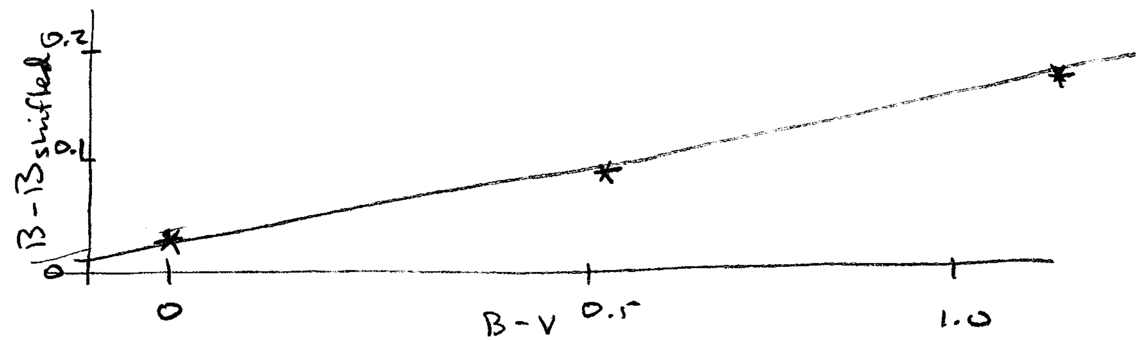
and recompute the **B** magnitudes and the **B-V** colors. Put these numbers into the "Magnitudes (shifted)" slots above. Explain the differences you notice:

The shifted B magnitudes are smaller (brighter) than the unshifted ones, and more so for cooler, redder stars. This is because, in this blue wavelength and temperature range, $F(\lambda)$ increases with increasing λ.

What is the qualitative relation between temperature and **(B-V)** color?

Higher temperature goes with smaller (bluer) B-V color.

Sketch a plot showing **B – B$_{\text{(shifted)}}$** as a function of **(B-V)**:



So colors provide a way to estimate stellar temperatures. Suppose we know the temperature -- then what else do we know? Stellar evolution theory provides relations between the characteristics of stars at the time they are born (mass and composition), and their properties (including temperature and luminosity) at any time in their later lives. For a quick introduction to (or refresher on) many of the basic notions, see http://www.tim-thompson.com/hr.html

For a more quantitative picture of stellar evolution, consider the files **yy08.cat, yy10.cat, yy13.cat, yy18.cat, yy25.cat**. These contain evolutionary tracks for stars with compositions similar to the Sun's and with masses of 0.8, 1.0, 1.3, 1.8, and 2.5 times the mass of the Sun. The columns of the tables contain log(Temperature (K)), log(Luminosity (solar units)), and Age (GY). Notice (from a vi inspection) that the range of ages in the various tables is fairly wide -- massive stars live for much shorter times than low-mass ones.

Make a plot that shows the evolutionary tracks for all 5 stars overplotted on one graph. Use different colors to distinguish the different masses, and (for once) connect the points with straight lines, using no symbols. Make the x-axis log(Temperature), but make Temperature increase from right to left. (Because that's the way astronomers do it. Don't ask.) Make the y-axis log(Luminosity), increasing from bottom to top. On the track for the 1-solar-mass star, mark the following points with text integers 1-5 the following locations: (1) Arrival on the main sequence. (2) Turnoff from the main sequence. (3) Beginning of the subgiant branch. (4) Base of the giant branch. (5) Tip of the giant branch. Do all the usual housekeeping (plot labels and such). Save your plot (plot02).

Now make the same plot again, only make all of the curves black, and do not provide any annotating integers. In each of the 5 tables, identify the point at which the age of the star is 10 million years (0.01 GY). In the text editor, create a table that you can read into xmgrace, giving log(Temperature) and log(Luminosity) for each of the stars at this age. Overplot this (jagged, because there are not very many points on it) curve, using a heavy green line. This is the 10-million-year *isochrone* (line of constant age). If you start with a group of stars (the members of a star cluster, say) that have the same initial composition and are formed at nearly the same time, then 10 million years later, you will see the stars of different masses strung out along this isochrone.

In the same way, plot isochrones for ages of 0.1 GY, 0.6 GY, and 5.0 GY. Do the usual labeling and save your plot (plot03).

What does it mean if the oldest age in a table is smaller than one of these isochrone ages?

A star of the given mass evolves so quickly that, by the isochrone age, it has evolved past the red giant stage to become a white dwarf.

Do a similar plot, but this time use only the data for the 1-solar-mass star (yy10.cat). Use a text editor and create a new text file that contains only the lines that lie as close as you can come to ages that are integral multiples of 1 GY, up to 12 GY. Overplot this table onto the original curve, using suitable symbols, but no lines connecting the points. Label, title, and save your plot (plot04).
About what fraction of this star's life is spent on the main sequence? What fraction on the giant branch?

The star spends about 10 of 12.2 GY on the main sequence, or ~ 80%

It spends less than 0.7 GY on the giant branch, or ~ 6%

Now we will use Sedgwick observations to measure colors of stars in the cluster M67. The filters on the Sedgwick telescope are not standard ones. Instead of the standard **B, V** filters that were discussed above, the Sedgwick CCD camera has filters called **Blue, Green**. The purpose of this part of the lab is to establish a relationship between magnitudes measured in Blue and Green filters and those measured in **B** and **V**. As we have seen above, we expect this relationship to depend upon the temperature and hence the color of the star observed.

First we must establish the standard **B,V** magnitudes of a large sample of stars in the cluster. This is information that can be gotten from the web. The details are messy, however, so I have done this part of it, producing a catalog **p134_montgomery.cat**, which contains, for about 400 stars near the center of M67, the (x,y) coordinates of each star, its V magnitude, and its B-V color.

We also have calibrated images of M67 in Sedgwick Blue, Green, and Red filters. These too are found in the lab4 directory, and are called **0427_M67_033B.fits, 0427_M67_033G.fits, 0427_M67_033R.fits**. For now we will ignore the Red image.

As images, the Sedgwick data are no good to us -- we need catalogs. To get them, run the SExtractor program. Do this by typing

sex 0427_M67_033B.fits –c default.sex –PARAMETERS_NAME sedgwick.a.param –CATALOG_NAME 0427_M67_033B.cat

and do something analagous for 0427_M67_033G.fit.

Next, match the "standard" p134_montgomery.cat catalog against the Sedgwick catalogs. Use **match_stars** for this:

match_stars p134_montgomery.cat 0427_M67_B.cat m67matchB.cat
match_stars p134_montgomery.cat 0427_M67_G.cat m67matchG.cat

Open the m67matchG.cat file with a text editor and note which columns contain which information. Those with suffix '0' come from the 'standard' catalog, while those with suffix '1' are from the Sedgwick data.

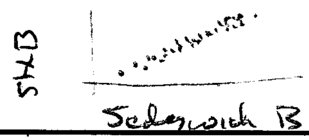
Check to be sure that the matching was successful. In xmgrace, import from m67matchG.cat the columns MAG_V0 (which is the V magnitude from the standard catalog) and MAG_ISOCOR1 (which is the Green magnitude from Sedgwick). Plot one against the other, label your plot in a sensible way, and save it. (plot05) If all is well, the two magnitudes should plot onto a straight line with almost unit slope, and little scatter. If they do not, recheck your previous steps until they do.

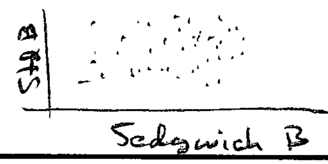
Do the same thing for M67matchB.cat, except for this, use xmgrace to create the standard B magnitudes from

B = (B-V) + V
= MAG_B-V0 + MAG_V0

Make a plot similar to plot05, except showing the comparison between the standard **B** and the Sedgwick **Blue** magnitudes. Label it as usual and save it. (plot06)

What would you expect to see in plot05 and plot06 if match_stars failed to work properly, and paired results from random stars together? The stars would not fall on a straight line, rather they would form a random scatter plot filling the whole range of magnitudes covered by each data source.

Not


But


Now that we are satisfied that the standard and SExtracted magnitudes make sense, we wish to estimate relationships between the standard and Sedgwick colors that express (Blue, Green) magnitudes as linear functions of **(B,V)** magnitudes.

9 5 4

In xmgrace, make the difference **(Blue – B) = MAG_ISOCOR1 – (MAG_B-V0 + MAG_V0)** from m67matchB.cat. Plot this against **(B-V)**

Use xmgrace Data/Transformations/Regression to make a linear fit to these data. This will attempt to fit the data to a model **(Blue – B) = A1 + A2*(B-V).**
If there are a few highly discrepant points, use Edit/Regions to create a region of interest that selects only those points that allow a linear fit that is pleasing to the eye (but that does justice to the actual scatter in the data). Return the "fitted function" so you can see how well the fit succeeded. When you have a result that satisfies you, label and save your plot (plot07). The regression routine displays the fit coefficients A1 and A2. Write them here.

A1 = -25.121 A2 = -0.317

Do the same thing with **(Green – V)**, again plotted against **(B-V)**, using input data from m67matchG.cat. Label and save your plot (plot08), and write down the fit coefficients here.

$B1 = -25.003 \qquad B2 = +0.055$

To distinguish these values from the previous ones, let's now call them B1 and B2. We thus have 2 equations:

Blue – B = A1 + A2*(B-V)
Green - V = B1 + B2*(B-V)

Rewrite as

$(Blue - A1) = (1 + A2)\cdot B - A2 \cdot V$

$(Green - B1) = B2 \cdot B + (1 - B2)\cdot V$

2 linear eqns for B, V in terms of Blue, Green, A1, A2, B1, B2.

Solve these equations to give two expressions that give **(B,V)** in terms of **(Blue, Green)** and the constants **A1, A2, B1, B2**.

For 2 linear eqns, we have (solving by determinants)

$D \equiv (1 + A2)(1 - B2) + A2\, B2$

$(Blue - A1) = (1+A2)B - A2\, V$

$(Green - B1) = B2 \cdot B + (1-B2)\cdot V$

Then

$B = [(Blue - A1)(1 - B2) + (Green - B1)\, A2] / D$

$V = [(Green - B1)(1 + A2) - (Blue - A1)\, B2] / D$

$D = 0.683 \cdot .945 + .317 \cdot .055$

Put in the numbers **A1, A2, B1, B2** and write down the resulting expressions

$D = 0.6454 - .0174 = 0.628$

$B = [(Blue + 25.121)\cdot 0.945 + (Green + 25.003)(-)\, 0.317] / 0.628$
$= Blue \cdot 1.505 - Green \cdot 0.505 + 25.18$

$V = [(Green + 25.003)\cdot 0.683 - (Blue + 25.121)\, 0.055] / 0.628$
$= Green \cdot 1.087 - Blue \cdot 0.087 + 24.99$

9 Having derived a means of putting Sedgwick **(Blue, Green)** photometry on a standard scale, we would of course like to know how much we can trust our answer. The xmgrace linear regression routine is using essentially the same procedures as described in Taylor's Chapter 8, sections 8.1-8.4. Please re-read these sections to be sure you are familiar with the material.

Estimate the standard deviation of the **(Blue – B)** measurements from the scatter in the residuals about the linear fit. (This, of course, corresponds to Taylor's σ_y.) Do the same for the **(Green – V)** measurements. Write down these estimates.

Blue – B is about 0.32 peak-peak, or (1/4 of this) 0.08 RMS

Green - V " " 0.22 0.055 RMS

Use these numbers and Taylor's Equations (8.16), (8.17) to estimate the errors in **A1, A2, B1, B2.** Doing this will require a good deal of clumsy messing around with creating new data sets (by formula) in xmgrace, especially to compute things like **Σ x²** and Taylor's denominator **Δ**. If you are fluent with some spreadsheet or scripting language (excel, perl, python) please feel free to use them. In any case, write the estimated errors in the fitted coefficients here. We need, for the Blue and Green data sets, N, $\Sigma(B-V)^2$, $\Sigma(B-V)$ and $\Delta = N\,\Sigma(B-V)^2 - [\Sigma(B-V)]^2$. For my selected points, these are:

	N	$\Sigma(B-V)^2$	$\Sigma(B-V)$	Δ		σ_1	σ_2	
Blue	166	76.12	108.	972.	$\Rightarrow$	0.022	0.033	(A1, A2)
Green	195	95.51	131.5	1332.		0.015	0.021	(B1, B2)

Propagate these errors through your general equations (on the previous page) to give the errors in **(B,V)** in terms of **(Blue, Green).** Evaluate these for the numerical values of **A1, A2, B1, B2** and write the result here.

It turns out that the errors in A1, A2, B1, B2 are large but strongly correlated with each other. Thus, as posed, this question does not make much sense, and it therefore becomes another FREEBIE.

These are equations describing the **color transformation** between our instrumental (Sedgwick) photometry system and the standard **(B,V)** photometric system. A lot of astronomical journal papers have been written describing just such color transformations.

Final Result (useful in the Project):

$$B = 1.505 \cdot \text{Blue} - 0.505 \cdot \text{Green} + 25.18$$

$$V = 1.087 \cdot \text{Green} - 0.087 \cdot \text{Blue} + 24.99$$

Errors in coefficients (except zero points) are about ± 0.02

P2, Lab4.

Ignore $T(\lambda)$ from the beginning, since we are only going to set it to unity.

We desire $\langle F_\nu \rangle = \int_{\nu_0}^{\nu_1} F(\nu)\, d\nu \Big/ \int_{\nu_0}^{\nu_1} d\nu$

Set $\lambda = \frac{c}{\nu}$, and note that with this definition,

$$\int_{\nu_0}^{\nu_1} F(\nu)\, d\nu = \int_{\lambda_0}^{\lambda_1} F(\lambda)\, d\lambda$$

So $\langle F_\nu \rangle = \int_{\lambda_0}^{\lambda_1} F(\lambda)\, d\lambda \Big/ \int_{\lambda_0}^{\lambda_1} -c \frac{d\lambda}{\lambda^2}$

$$\langle F_\nu \rangle = \int_{\lambda_0}^{\lambda_1} F(\lambda)\, d\lambda \cdot \left[-c \cdot \frac{1}{\lambda} \Big|_{\lambda_0}^{\lambda_1} \right]^{-1}$$

Written (approximately) as a sum, this is

$$\langle F_\nu \rangle = \sum_{\lambda_0}^{\lambda_1} F(\lambda)\Delta\lambda \cdot \left[c \frac{1}{\lambda} \Big|_{\lambda_1}^{\lambda_0} \right]^{-1}$$

xmgrace delivers $\sum_{\lambda_0}^{\lambda_1} F(\lambda)$, and by inspection of the

table of $F(\lambda)$ vs λ, we see that $\Delta\lambda = 3$ Å

For the B bandpass [3910, 4890] and star f05,

$$\langle F_\nu \rangle = (193679 - 27337) \cdot 3 \cdot \left[3\times10^{18} \cdot 5.126\times10^{-5} \right]^{-1}$$

$$\mathrm{erg}/(\mathrm{cm}^2\text{-Å-s}) \cdot \text{Å} \cdot \left[\text{Å}/\mathrm{s} \cdot \text{Å}^{-1} \right]^{-1}$$

$$= 3.245 \times 10^{-9}$$

$$\mathrm{erg}/(\mathrm{cm}^2\text{-s-Hz})$$

$-2.5 \log \langle F_\nu \rangle = 21.22$

Error Estimates for A1, A2, B1, B2

Blue: $\sigma_{A1} = 0.08\sqrt{\frac{\Sigma x^2}{\Delta}} = .08\sqrt{\frac{76.2}{972}} = 0.022$

$\sigma_{A2} = 0.08\sqrt{\frac{N}{\Delta}} = 0.08\sqrt{\frac{166}{972}} = 0.033$

Green σ_{B1} $0.055\sqrt{\frac{\Sigma x^3}{\Delta}} = .055\sqrt{\frac{95.51}{1332}} = 0.015$

σ_{B2} $0.055\sqrt{\frac{N}{\Delta}} = .055\sqrt{\frac{195}{1332}} = 0.021$