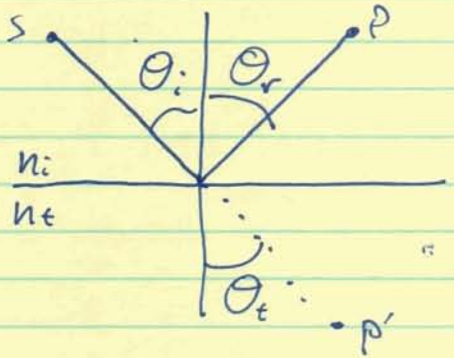
1) from treatment on p 107 in Hecht

$$\frac{\sin\theta_i}{v_i} = \frac{\sin\theta_t}{v_t}$$

but in our case $v_i = v_t$ so

$\theta_i = \theta_t$ or $\theta_i = \theta_r$

or take the same analogy with transmission case

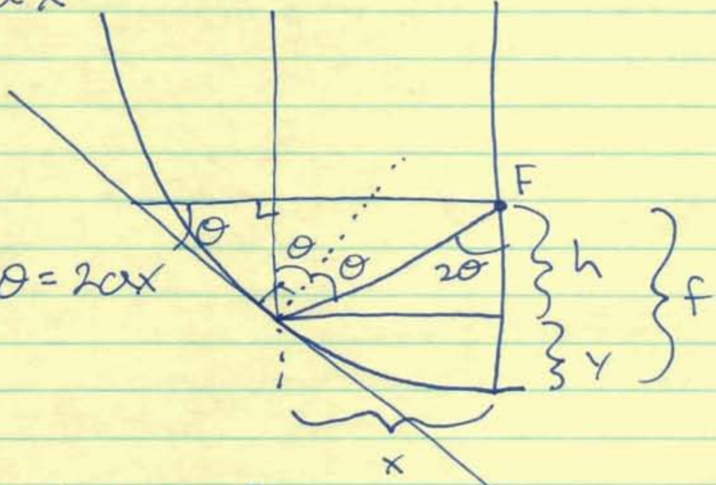


a straight line has $\theta_i = \theta_t$ since $n_i = n_t$

or $\theta_i = \theta_r$

2)

$y = ax^2$



$\tan\theta = 2ax$

$$\tan 2\theta = \frac{x}{h}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

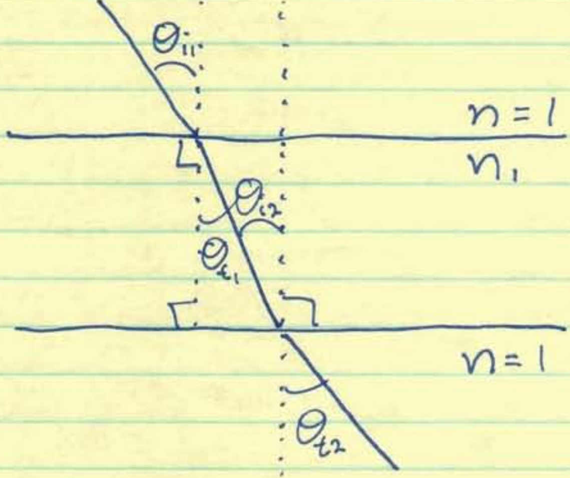
$$f = h + y = \frac{x}{\tan 2\theta} + ax^2$$

$$= \frac{\cancel{x}(1-4a^2x^2)}{4a\cancel{x}} + ax^2$$

$$= \frac{1}{4a} - \cancel{ax^2} + \cancel{ax^2} \quad \checkmark$$

The hight f
at which all rays (vertical)
cross the vertizal axis
is independent of x.

3)



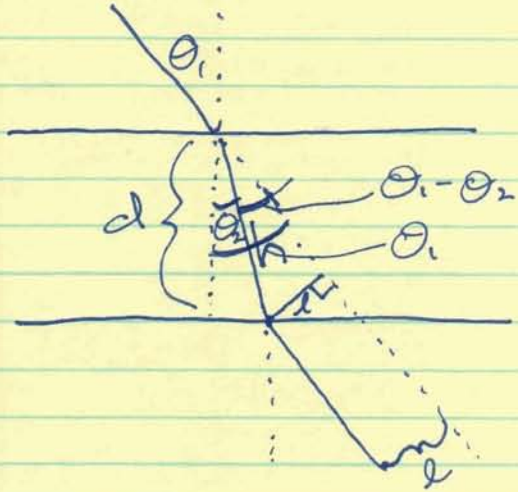
at the top of the plate

$$\sin\theta_{i1} = n_1 \sin\theta_{t1}$$

at the bottom of the plate

$$n_1 \sin\theta_{i2} = \sin\theta_{t2}$$

but $\theta_{t1} = \theta_{i2}$ so $\sin\theta_{i1} = \sin\theta_{t2}$ or $\theta_{i1} = \theta_{t2}$



$$\sin\theta_1 = n_1 \sin\theta_2$$

$$l = \sin(\theta_1-\theta_2)\,h$$

$$h = \frac{d}{\cos\theta_2}$$

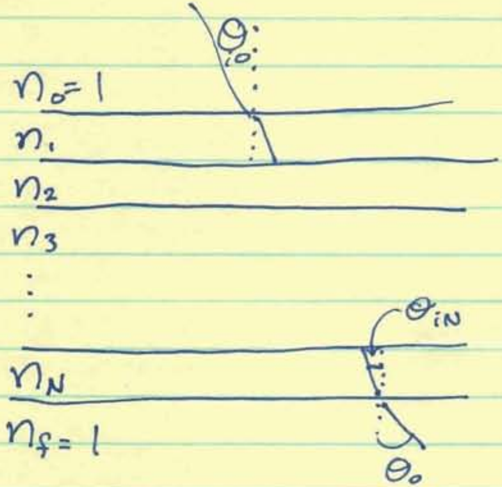
$$l = \frac{\sin(\theta_1-\theta_2)\,d}{\cos\theta_2}$$

$$l = \frac{(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)\,d}{\cos\theta_2}$$

$$l = \left[\sin\theta_1 - \frac{\cos\theta_1 \sin\theta_1/n_1}{(1-\sin^2\theta_1/n_1^2)^{1/2}}\right] d$$

$$= d\sin\theta_1\left[1 - \left(\frac{1-\sin^2\theta_1}{n_1^2-\sin^2\theta_1}\right)^{1/2}\right]$$

3.b)



$$\sin\theta_{i0} = n_1 \sin\theta_{t0} = n_1 \sin\theta_{i1} = n_2 \sin\theta_{t1} = \ldots$$

$$\ldots = n_m \sin\theta_{t\,m-1} = n_m \sin\theta_{im} = \ldots$$

$$\ldots = n_N \sin\theta_{t\,N-1} = n_N \sin\theta_{iN} = n_f \sin\theta_{tN}$$

$$\Rightarrow \quad \sin\theta_{i0} = \sin\theta_{tN} \quad , \quad \theta_{tN} = \theta_0$$

also $\quad n_m \sin\theta_{im} = \sin\theta_{i0}$

4) $$\theta_c = \sin^{-1}\frac{1}{2.4} = 0.43 \text{ (rad)}$$

solid angle transmitted

$$\Omega_T = \int_0^{2\pi}\int_0^{0.43} \sin\theta\, d\theta\, d\phi = 2\pi(1-\cos(0.43))$$
$$= 2\pi(1-0.909)$$

$$\frac{\Omega_T}{\Omega_E} = \frac{2\pi(0.091)}{4\pi} = \frac{0.091}{2} = 0.0455$$

so ~ 4.6 % of the light leaves the diamond